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Research paper



A New Approach to Find an Optimal Solution of a Fuzzy Linear Programming Problem by Fuzzy Dynamic Programming

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Abstract

This paper mainly focuses on a new approach to find an optimal solution of a fuzzy linear programming problem with the help of Fuzzy Dynamic Programming. Linear programming deals with the optimization of a function of variables called an objective function, subject to a set of linear inequalities called constraints. The objective function may be maximizing the profit or minimizing the cost or any other measure of effectiveness subject to constraints imposed by supply, demand, storage capacity, etc., Moreover, it is known that fuzziness prevails in all fields. Hence, a general linear programming problem with fuzzy parameters is considered where the variables are taken as Triangular Fuzzy Numbers. The solution is obtained by the method of FDP by framing fuzzy forward and fuzzy backward recursive equations. It is observed that the solutions obtained by both the equations are the same. This approach is illustrated with a numerical example. This feature of the proposed approach eliminates the imprecision and fuzziness in LPP models. The application of Fuzzy set theory in the field of dynamic Programming is called Fuzzy Dynamic Programming.

Keywords: *Fuzzy dynamic Programming; Triangular Fuzzy numbers; Fuzzy Linear Programming Problem; Fuzzy Recursive Equations; Fuzzy Optimal Solution.*

1. Introduction

Dynamic Programming is a mathematical technique or tool which is used for making a sequence of interrelated decisions. It provides a systematic procedure to determine the combination of decisions which maximizes the overall effectiveness. Dynamic Programming does not have a standard mathematical formulation like Linear Programming, but it is a general type of approach to solve a problem by framing equations to fit each individual situation. The solution of each stage is a decision and it is sequenced by all the stages as a decision policy. Each decision policy is combined together with some return in the form of costs or benefits. The objective in DP is to select a decision policy which will maximize the returns.

In all our human activities, we come across multi stage decision making problems. FDP is an eminent tool to handle such type of uncertainty and fuzziness persisting in the analysis of MCDM problems. The application of fuzzy set theory in decision making process by Bellman[3] and Zadeh[10] attracted many researchers to contribute in this field. They started to apply fuzzy concepts in all the fields. Many researchers like Kacprzyk[6], Esogbue[5] contributed more to the literature. Kacprzyk[6] detailed the development in the theory and applications of FDP. Esogbue[5] provided a brief review of basic problem classes and developments of FDP. Schweickardt and Miranda[9] presented a FDP model in the evaluation of expansion distribution cost in fuzzy environments. Later on, Alkan[2] developed a FDP technique with fuzzy state variables and decision variables. He applied it to solve a ballistic missile tracking problem with new fuzzy optimal control technique. Chung-Ching Su[4] developed a new approach using FDP for the unit commitment of a Power System. Abo-Sinna[1] discussed about his survey and some applications of multiple objective FDP problems. Other excellent applications of FDP also appear in the literature[7, 8].

The purpose of this paper is to find an optimal solution of a fuzzy linear programming problem by FDP. Here the variables are taken as Triangular fuzzy numbers. Fuzzy forward and fuzzy backward recursive equations are framed and fuzzy optimal solution is also obtained. It is observed that optimal solution obtained by both the equations are the same. The crisp value is also obtained by the Ranking method.

2. Definition and Preliminaries

2.1. Fuzzy Set

Mathematically, a fuzzy set can be defined by assigning to each possible individual in the universe of discourse, a value representing its grade of membership function in the fuzzy set. The membership function of a fuzzy set $\tilde{A}$ is denoted by $\mu_{\tilde{A}}$. (i.e) $\mu_{\tilde{A}}$: X $\rightarrow$ [0, 1].

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2.2 Triangular Fuzzy Number:

A Triangular fuzzy number (TFN) denoted by $\tilde{A}$ is defined as (a_1, a_2, a_3) where the membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2}, & a_2 \le x \le a_3 \\ 0, & otherwise. \end{cases}$$

2.3 Properties of Triangular Fuzzy Numbers (TFN)

Let $\widetilde{A}$ = (a, b, c) and $\widetilde{B}$ = (p, q, r) be two positive triangular fuzzy numbers with the membership functions defined above.

1. Addition of two TFNs
   $\widetilde{A}$ + $\widetilde{B}$ = (a + p, b + q, c + r)
2. Subtraction of two TFNs
   $\widetilde{A}$ - $\widetilde{B}$ = (a − r, b − q, c − p)
3. Multiplication of two TFNs
   $\widetilde{A} * \widetilde{B}$ = (minT, bq, maxT) where T = {ap, ar, cp, cr}
4. Scalar Multiplication of TFN

k$\widetilde{A}$ = (ka, kb, kc) if k∈R and k ≥ 0
k$\widetilde{A}$ = (kc, kb, ka) if k∈R and k < 0

2.4 Ranking Methodology of Triangular Fuzzy Numbers (TFN)

Several approaches for the ranking of fuzzy numbers have been proposed in the literature. An efficient approach for comparing the fuzzy number is by the use of a ranking function based on their graded means. The ranking Methodology of Triangular Fuzzy Numbers is used to defuzzify the Triangular Fuzzy Numbers to find the crisp solutions.

That is for every $\tilde{A}$ = (a, b, c) ∈ F(R) the ranking function $R(\tilde{A})$ = F(R)→R by graded means is defined by the following function.

$$R(\tilde{A}) = \left(\frac{a_1 + 4a_2 + a_3}{6}\right)$$

3. Fuzzy Dynamic Programming

3.1. Fuzzy Linear Programming Problem

The general form of Fuzzy Linear Programming Problem is given by

Maximize $\tilde{Z} = c_1\tilde{x}_1 + c_2\tilde{x}_2 + c_3\tilde{x}_3 + \cdots + c_n\tilde{x}_n$

Subject to the constraints,

$a_{11}\tilde{x}_1 + a_{12}\tilde{x}_2 + \cdots + a_{1n}\tilde{x}_n \le \tilde{b}_1$
$a_{21}\tilde{x}_1 + a_{22}\tilde{x}_2 + \cdots + a_{2n}\tilde{x}_n \le \tilde{b}_2$
$a_{31}\tilde{x}_1 + a_{32}\tilde{x}_2 + \cdots + a_{3n}\tilde{x}_n \le \tilde{b}_3$
$a_{m1}\tilde{x}_1 + a_{m2}\tilde{x}_2 + \cdots + a_{mn}\tilde{x}_n \le \tilde{b}_m$
$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \ldots \tilde{x}_n \ge 0$.

This problem consists of m resources and n decision variables that can be formulated as a Fuzzy dynamic programming problem as follows:

Each activity j (j = 1, 2,… n) is considered as stage. Therefore, the problem is regarded as n-stage problem and decision variables i.e. alternatives are the levels of activities $\tilde{x}_j \ge 0$ at the stage j. Since $\tilde{x}_j$ is continuous, there exists infinite number of alternatives (values of $\tilde{x}_j$) within the feasible space.

The problems of allocation are denoted as a particular type of Linear Programming problems that require allocation of available resources to the activities. The constants $\tilde{b}_1, \tilde{b}_2, \tilde{b}_3, \ldots \tilde{b}_m$ which are taken as Triangular fuzzy numbers are the amounts of available resources. The state of the system at any stage is nothing but the amount allocated at that stage and left for the remaining stages. The m-dimensional vector $(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3, \ldots \tilde{b}_m)$ would be considered as the state.

The optimal fuzzy value of the objective function defined above for the stages $\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \ldots \tilde{x}_n$ and for the states $(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3, \ldots \tilde{b}_m)$ is given by $\tilde{f}_n(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3, \ldots \tilde{b}_m)$.

3.2. Fuzzy Forward Recursive Equations

The fuzzy forward recursive equation is thus given as

$$\tilde{f}_1(\tilde{b}_1, \tilde{b}_2, \ldots \tilde{b}_m) = \max_{\tilde{0} \le \tilde{x}_1 \le \tilde{b}} \{c_1\tilde{x}_1\}$$

$$\tilde{f}_j(\tilde{b}_1, \tilde{b}_2, \ldots \tilde{b}_m) = \max_{\tilde{0} \le \tilde{x}_j \le \tilde{b}} \{c_j\tilde{x}_j + \tilde{f}_{j-1}(\tilde{b}_1 - a_{1j}\tilde{x}_j, \tilde{b}_2 - a_{2j}\tilde{x}_j, \ldots \tilde{b}_m - a_{mj}\tilde{x}_j)\}$$

where the maximum value of $\tilde{b}$ that $\tilde{x}_j$ can assume is given by

$$\tilde{b} = Min\left[\frac{\tilde{b}_1}{a_{1j}}, \frac{\tilde{b}_2}{a_{2j}}, \ldots \frac{\tilde{b}_m}{a_{mj}}\right] \text{ and } j = 2,3, \ldots n.$$

3.3. Fuzzy Backward Recursive Equations

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$$\tilde{f}_j(\tilde{b}_1, \tilde{b}_2, \ldots \tilde{b}_m) = \max_{\tilde{0} \le \tilde{x}_j \le \tilde{b}} \{c_j\tilde{x}_j + \tilde{f}_{j+1}(\tilde{b}_1 - a_{1j}\tilde{x}_j, \tilde{b}_2 - a_{2j}\tilde{x}_j, \ldots \tilde{b}_m - a_{mj}\tilde{x}_j)\}$$

where the maximum value of $\tilde{b}$ that $\tilde{x}_j$ can assume is given by

$$\tilde{b} = Min\left[\frac{\tilde{b}_1}{a_{1j}}, \frac{\tilde{b}_2}{a_{2j}}, \ldots \frac{\tilde{b}_m}{a_{mj}}\right] \text{ and } j = 1,2, \ldots n-1.$$

4. Illustrative Example

The following Fuzzy Linear Programming Problem is considered where the unknown variables are taken as Triangular Fuzzy Numbers. The concept of Fuzzy Dynamic Programming is applied to solve the following FLPP.

Maximize $\tilde{Z} = 3\tilde{x}_1 + 5\tilde{x}_2$
Subject to $\tilde{x}_1 \le (3,4,5)$,
$\tilde{x}_2 \le (5,6,7)$,
$3\tilde{x}_1 + 2\tilde{x}_2 \le (15,18,21)$,
$\tilde{x}_1, \tilde{x}_2 \ge 0$ where $\tilde{0} = (0,0,0)$.

As the FLPP contains two variables, it is treated as a two stage FDP problem. Both the variables $\tilde{x}_1$ and $\tilde{x}_2$ represent the infinite number of alternatives within the feasible space. The three constraints are considered as three resources which are $\tilde{b}_1$, $\tilde{b}_2$ and $\tilde{b}_3$. Therefore these three resources will be allocated to the variables $\tilde{x}_1$ and $\tilde{x}_2$ at various stages. The state of the system at a stage will be given by the amount of resources allocated at that stage and left for the remaining stages. The states of the Fuzzy Dynamic Programming Problem are $\tilde{b}_1$ = (3,4,5), $\tilde{b}_2$ = (5,6,7) and $\tilde{b}_3$ = (15,18,21). The FLPP is solved by the method of Fuzzy Forward Recursive equations method.

5. Solution by Fuzzy Forward Recursive Equations

Stage 1

The optimal value of $\tilde{f}_1(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3)$ at stage 1 is given by $\tilde{f}_1(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) = \max_{\tilde{0} \le \tilde{x}_1 \le \tilde{b}}\{3\tilde{x}_1\}$, where $\tilde{b}_1$ = (3,4,5), $\tilde{b}_2$ = (5,6,7) and $\tilde{b}_3$ = (15,18,21). The feasible value of $\tilde{x}_1$ is non-negative and satisfies all the three constraints.

But the maximum value of $\tilde{b}$ that $\tilde{x}_1$ can assume is given by

$\text{Min}\left[\left(\frac{3}{1},\frac{4}{1},\frac{5}{1}\right),\left(\frac{5}{0},\frac{6}{0},\frac{7}{0}\right),\left(\frac{15}{3},\frac{18}{3},\frac{21}{3}\right)\right]$ which is (3, 4, 5).

Therefore,

$\tilde{f}_1(\tilde{b}_1,\tilde{b}_2,\tilde{b}_3) = \max_{\tilde{0}\le\tilde{x}_1\le(3,4,5)}\{3\tilde{x}_1\}$

$\Rightarrow \tilde{f}_1(\tilde{b}_1,\tilde{b}_2,\tilde{b}_3) = 3\ Min\left[(3,4,5),\frac{(15,18,21)-2\tilde{x}_2}{3}\right]$

Where $\tilde{x}_1{}^* = Min\left[(3,4,5),\frac{(15,18,21)-2\tilde{x}_2}{3}\right]$

Stage 2

The fuzzy recursive equation for optimization of this problem of two-stage is given by

$\tilde{f}_2(\tilde{b}_1,\tilde{b}_2,\tilde{b}_3) = \max_{\tilde{0}\le\tilde{x}_2\le\tilde{b}}\{5\tilde{x}_2+3\tilde{x}_1\}$

$\Rightarrow \tilde{f}_2(\tilde{b}_1,\tilde{b}_2,\tilde{b}_3) = \max_{\tilde{0}\le\tilde{x}_2\le\tilde{b}}\left[5\tilde{x}_2+3\ Min\left\{(3,4,5),\frac{(15,18,21)-2\tilde{x}_2}{3}\right\}\right]$

But the maximum value of $\tilde{b}$ that $\tilde{\boldsymbol{x}}_2$ can assume is given by

$\text{Min}\left[\left(\frac{3}{0},\frac{4}{0},\frac{5}{0}\right),\left(\frac{5}{1},\frac{6}{1},\frac{7}{1}\right),\left(\frac{15}{2},\frac{18}{2},\frac{21}{2}\right)\right]$ which is (5, 6, 7).

$\tilde{f}_2(\tilde{b}_1,\tilde{b}_2,\tilde{b}_3) = \max_{\tilde{0}\le\tilde{x}_2\le(5,6,7)}\left[5\tilde{x}_2+3\ Min\left\{(3,4,5),\frac{(15,18,21)-2\tilde{x}_2}{3}\right\}\right]$

We have,

$Min\left\{(3,4,5),\frac{(15,18,21)-2\tilde{x}_2}{3}\right\}$

$= \begin{cases}(3,4,5), & if\ \tilde{0}\le\tilde{x}_2\le(2,3,4)\\ \frac{(15,18,21)-2\tilde{x}_2}{3}, & if\ (2,3,4)<\tilde{x}_2\le(5,6,7)\end{cases}$

$5\tilde{x}_2+3\ Min\left\{(3,4,5),\frac{(15,18,21)-2\tilde{x}_2}{3}\right\}$

$= \begin{cases}5\tilde{x}_2+(9,12,15), & if\ \tilde{0}\le\tilde{x}_2\le(2,3,4)\\ 3\tilde{x}_2+(15,18,21), & if\ (2,3,4)<\tilde{x}_2\le(5,6,7)\end{cases}$

$\Rightarrow \tilde{f}_2(\tilde{b}_1,\tilde{b}_2,\tilde{b}_3) = Max\begin{cases}(19,27,35), & when\ \tilde{x}_2=(2,3,4)\\ (30,36,42), & when\ \tilde{x}_2=(5,6,7)\end{cases}$

$\Rightarrow \tilde{f}_2(\tilde{b}_1,\tilde{b}_2,\tilde{b}_3) = (30, 36, 42).$

Therefore we have, $\tilde{x}_2{}^* = (5,6,7)$ and Max $\tilde{Z} = (30,36,42)$.

$\tilde{x}_1{}^* = Min\left[(3,4,5),\frac{(15,18,21)-2\tilde{x}_2}{3}\right]$

$\tilde{x}_1{}^* = Min\left[(3,4,5),\frac{(15,18,21)-2(5,6,7)}{3}\right]$

$\Rightarrow \tilde{x}_1{}^* = \left(\frac{5}{3},2,\frac{7}{3}\right).$

The above FLPP is solved by Fuzzy forward recursive equation. By the Ranking method of Triangular Fuzzy Numbers, we have the crisp solution of the above FLPP as $x_1{}^* = 2, x_2{}^* = 6, Max\ Z = 36$. Now let us solve the above FLPP by Fuzzy Backward Recursive Equations.

6. Solution by Fuzzy Backward Recursive Equations

Stage 2

The optimal value of $\tilde{f}_2(\tilde{b}_1,\tilde{b}_2,\tilde{b}_3)$ at stage 2 is given by $\tilde{f}_2(\tilde{b}_1,\tilde{b}_2,\tilde{b}_3) = \max_{\tilde{0}\le\tilde{x}_2\le\tilde{b}}\{5\tilde{\boldsymbol{x}}_2\}$, where $\tilde{b}_1$ = (3,4,5), $\tilde{b}_2$ = (5,6,7) and $\tilde{b}_3$ = (15,18,21). The feasible value of $\tilde{x}_2$ is non-negative and satisfies all the three constraints.

But the maximum value of $\tilde{b}$ that $\tilde{\boldsymbol{x}}_2$ can assume is given by

$\text{Min}\left[\left(\frac{3}{0},\frac{4}{0},\frac{5}{0}\right),\left(\frac{5}{1},\frac{6}{1},\frac{7}{1}\right),\left(\frac{15}{2},\frac{18}{2},\frac{21}{2}\right)\right]$ which is (5, 6, 7).

Therefore,

$\tilde{f}_2(\tilde{b}_1,\tilde{b}_2,\tilde{b}_3) = \max_{\tilde{0}\le\tilde{x}_2\le(5,6,7)}\{5\tilde{\boldsymbol{x}}_2\}$

$\Rightarrow \tilde{f}_2(\tilde{b}_1,\tilde{b}_2,\tilde{b}_3) = 5\ Min\left[(5,6,7),\frac{(15,18,21)-3\tilde{x}_1}{2}\right]$

Where $\tilde{x}_2{}^* = Min\left[(5,6,7),\frac{(15,18,21)-3\tilde{x}_1}{2}\right]$

Stage 1

The fuzzy recursive equation for optimization of this problem of two-stage is given by

$\tilde{f}_1(\tilde{b}_1,\tilde{b}_2,\tilde{b}_3) = \max_{\tilde{0}\le\tilde{x}_1\le\tilde{b}}\{3\tilde{\boldsymbol{x}}_1+5\tilde{x}_2\}$

$\Rightarrow \tilde{f}_1(\tilde{b}_1,\tilde{b}_2,\tilde{b}_3) = \max_{\tilde{0}\le\tilde{x}_1\le\tilde{b}}\left[3\tilde{x}_1+5\ Min\left\{(5,6,7),\frac{(15,18,21)-3\tilde{x}_1}{2}\right\}\right]$

But the maximum value of $\tilde{b}$ that $\tilde{\boldsymbol{x}}_1$ can assume is given by

$\text{Min}\left[\left(\frac{3}{1},\frac{4}{1},\frac{5}{1}\right),\left(\frac{5}{0},\frac{6}{0},\frac{7}{0}\right),\left(\frac{15}{3},\frac{18}{3},\frac{21}{3}\right)\right]$ which is (3, 4, 5).

$\tilde{f}_1(\tilde{b}_1,\tilde{b}_2,\tilde{b}_3) = \max_{\tilde{0}\le\tilde{x}_1\le(3,4,5)}\left[3\tilde{x}_1+5\ Min\left\{(5,6,7),\frac{(15,18,21)-3\tilde{x}_1}{2}\right\}\right]$

We have,

$Min\left\{(5,6,7),\frac{(15,18,21)-3\tilde{x}_1}{2}\right\}$

$= \begin{cases}(5,6,7), & if\ \tilde{0}\le\tilde{x}_1\le(1,2,3)\\ \frac{(15,18,21)-3\tilde{x}_1}{2}, & if\ (1,2,3)<\tilde{x}_1\le(3,4,5)\end{cases}$

$3\tilde{x}_1+5\ Min\left\{(5,6,7),\frac{(15,18,21)-3\tilde{x}_1}{2}\right\}$

$= \begin{cases}3\tilde{x}_1+(25,30,35), & if\ \tilde{0}\le\tilde{x}_1\le(1,2,3)\\ \frac{-9\tilde{x}_1}{2}+\left(\frac{75}{2},45,\frac{105}{2}\right), & if\ (1,2,3)<\tilde{x}_1\le(3,4,5)\end{cases}$

$\Rightarrow \tilde{f}_1(\tilde{b}_1,\tilde{b}_2,\tilde{b}_3) = Max\begin{cases}(28,36,44), & when\ \tilde{x}_1=(1,2,3)\\ (24,27,30), & when\ \tilde{x}_1=(3,4,5)\end{cases}$

$\Rightarrow \tilde{f}_1(\tilde{b}_1,\tilde{b}_2,\tilde{b}_3) = (28, 36, 44).$

Therefore we have, $\tilde{x}_1{}^* = (1,2,3)$ and Max $\tilde{Z} = (28,36,44)$.

$\tilde{x}_2{}^* = Min\left[(5,6,7),\frac{(15,18,21)-3\tilde{x}_1}{2}\right]$

$\tilde{x}_2{}^* = Min\left[(5,6,7),\frac{(15,18,21)-3(1,2,3)}{2}\right]$

$\Rightarrow \tilde{x}_2{}^* = (5,6,7).$

The above FLPP is also solved by Fuzzy backward recursive equation. By the Ranking method of Triangular Fuzzy Numbers, we have the crisp solution of the above FLPP as $x_1{}^* = 2, x_2{}^* = 6$, $Max\ Z = 36$. The crisp solution obtained by applying fuzzy backward recursive equation is same as the one obtained by fuzzy forward recursive equation.

7. Conclusion

In this paper, mathematical formulation of fuzzy Linear Programming problem and the procedure for finding optimal solution by fuzzy recursive equations are discussed with the numerical example. In particular, the values of variables are expressed as triangular fuzzy numbers. Fuzzy Dynamic Programming (FDP) is an eminent tool for dealing with a large volume of multi-stage decision-making problems in a fuzzy environment. Any problem in the optimization world such as Transportation Problem, Advertising Media problem, Production Planning problem, Network Problem etc., can be expressed as a Linear Programming Problem. The proposed approach will help the researchers to solve the LPPs involving multi-stage decision process in a fuzzy environment. The approach would also bring about many new interesting applications.

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