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Research paper

K-Graceful, Odd-Even Graceful, Heronian Mean and Analytic Mean Labeling for the Extended Duplicate Graph of Kite Graph

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Abstract

In this paper, we investigate the extended duplicate graph of kite graph $KI_{3,m}$ admits K-graceful, Odd-Even graceful, Heronian mean and Analytic mean labeling.

Keywords: *Graceful; Kite graph; Duplicate graph; Heronian; Analytic.*

1. Introduction

Rosa has introduced the graph labeling schemes found their origin with graceful labeling [5]. R.B. Gnanajothi was introduced the idea of odd graceful labeling of some graph [2]. The concept of Odd-Even was introduced by Sridevi, Naveethakrishnan, K.Nagarajan and A.Nagarajan in 2012 [4]. For more related results on duplicate graphs we refer some articles [6,7]. The concept of heronian mean and analytic mean labeling was studied and refer [8,9]. The present work is aimed to prove K-graceful ,Odd-Even graceful, Heronian mean, Analytic mean labeling for the extended duplicate graph of kite graph.

2. Preliminaries

In this section, we provide some basic definitions relevant to this paper.

Definition 2.1 Kite Graph

The kite graph is obtained by attaching a path of length 'm' with cycle of length 'n' and it is denoted as $KI_{n,m.}$ Kite graphs is also known as the Dragon Graphs or Canoe Paddle Graphs.

Kite Graph

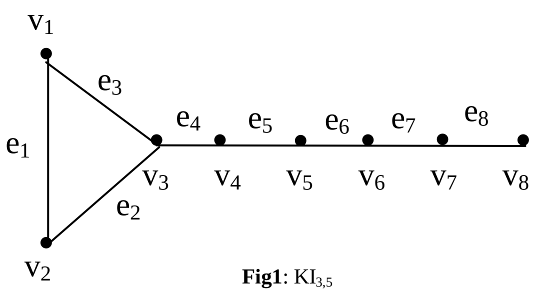


Fig1: $KI_{3,5}$

Definition 2.2 Duplicate Graph

If G(V,E) is a simple graph, then the duplicate graph of G is DG = (V_1, E_1) ,where the vertex set V_1 is the union of V and V′ and the intersection of V and V′ is ϕ and *C* is bijective (for $v \in V$, we write $f(v) = v'$) and the set E_1 called the edge set of DG is defined as the edge *ab* is in E if and only if both *ab′* and *a′b* are edges in E_1.

Duplicate graph of kite graph

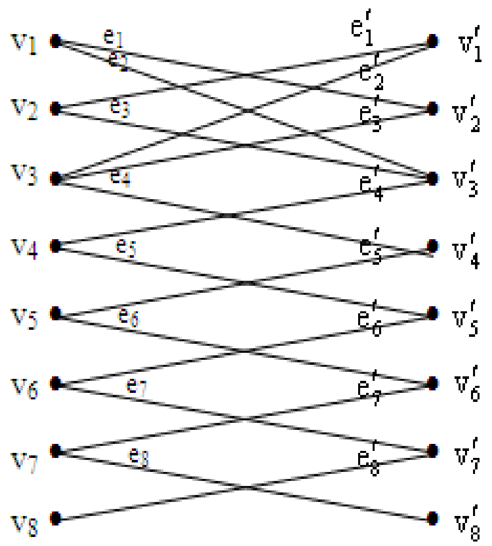


Fig 2: *DG ($KI_{3,5}$)*

Definition 2.3 Extended duplicate graph of Kite graph

Let DG = (V_1,E_1) be a duplicate graph of the kite graph G(V,E). Extended duplicate graph of kite graph is obtained by adding the edge $v_2 v_2'$ to the duplicate graph. It is denoted by EDG($KI_{3,m}$), m ≥ 1. The vertex set and edge set of EDG($KI_{3,m}$) given as follows: V={v_1, v_2,.., v_{m+3}, v'_1, v'_2,.., v'_{m+3}}, E={$e_1=v_1v'_2$, $e_2=v_1v'_3$, $e_{m+4}=v_2v'_2$, $e'_1=v'_1v_2$, $e'_2=v'_1v_3$}U {$e_{i+2}=v_{i+1}v'_{i+2}$, $e'_{i+2}=v'_{i+1}v_{i+2}$/1≤ i ≤ m+1}. Clearly it has 2m+6 vertices and 2m+7 edges.

Extended duplicate graph of kite graph

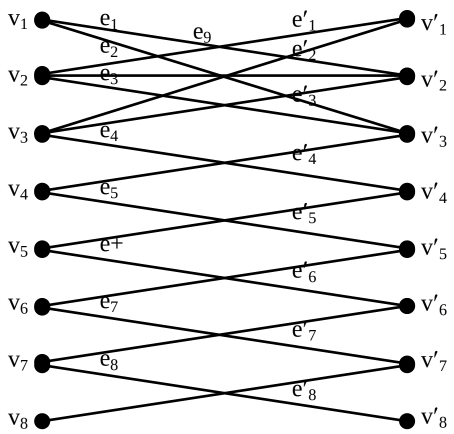


Fig3: *EDG ($KI_{3,5}$)*

Definition 2.4 Graceful labeling

We call a function f is graceful of a graph G, if f : V → {0,1,2,3,...,q} in such a way that every edge v_iv_j is assigned the label $|f(v_i)-f(v_j)|$ where $v_i, v_j \in$ V and the edge labels are distinct numbers {1,2,3,4,......q}. A graph with a graceful labeling is called Graceful graph.

Definition 2.5 K-Graceful labeling

We call a function f is graceful of a graph G, if f : V → {0,1,2,3,...,k+q-1} in such a way that every edge v_iv_j is assigned the label $|f(v_i)-f(v_j)|$ where $v_i, v_j \in$ V and the edge labels are distinct numbers {k, k+1, k+2,.....k+q-1}. A graph with a graceful labeling is called graceful graph.

Definition 2.6 Odd-Even Graceful labeling

The odd-even graceful labeling of a graph G with q edges means that there is an injection f: V(G) to { 1, 3, 5,…, 2q+1} such that when each edge uv is assigned the label $|f(u)-f(v)|$, the resulting edge labels are {2,4,6,…,2q}. A graph which admits an odd-even graceful is called odd-even graceful graph.

Definition 2.6 Heronian Mean labeling

A function f is called a Heronian mean labeling of a graph G=(V,E) with p vertices and q edges if it is possible to label the vertices x ∈v with distinct labels f(x) from 1,2,…,q+1 in such a way that when each edge e=uv is labeled with f(e=uv) $=\left\lceil\frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3}\right\rceil$ or $\left\lfloor\frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3}\right\rfloor$, then the edge labels are distinct.

Definition 2.6 Analytic Mean labeling

A (p,q) graph G=(V,E) is said to be an analytic mean graph if it is possible to label the vertices v in V with distinct from 0,1,2…,p-1 in such a way that when each edge e=uv is labeled with f(e=uv) = $\frac{|(f(u))^2-(f(v))^2|}{2}$ $if\ |(f(u))^2-(f(v))^2|$ is even and $\frac{|(f(u))^2-(f(v))^2|+1}{2}$ $if\ |(f(u))^2-(f(v))^2|$ is odd and the edge labels are distinct.

3. Main Results

In this section, we provide algorithms to prove the existence of K-graceful, odd-even graceful, heronian mean and Analytic mean labeling for the extended duplicate graph of kite graph.

Algorithm: 3.1

Procedure (K-Graceful labeling for EDG ($KI_{3,m}$, m ≥ 1)

V ← {v_1, v_2, … ,v_{m+3}, $v'_1, v'_2,..., v'_{m+3}$ }

E ← {e_1, e_2,…,e_{m+3}, e_{m+4}, $e'_1, e'_2,..., e'_{m+3}$ }

if $m \geq 1$

for i= 0 to $\left\lfloor\frac{m+2}{2}\right\rfloor$ **do**

$v_{2i+1} \leftarrow i$

$v'_{2i+1} \leftarrow m+k+3+i$

end for

for i= 0 to $\left\lfloor\frac{m+1}{2}\right\rfloor$ **do**

$v_{2i+2} \leftarrow m+3-i$

$v'_{2i+2} \leftarrow 2m+k+6-i$

end for

end if

end procedure

Theorem 3.1:

The extended duplicate graph of kite graph $KI_{3,m}$, m ≥ 1 is K-graceful .

Proof:

We know that, the extended duplicate graph of kite graph $KI_{3,m}$, m ≥ 1 has 2m+6 vertices and 2m+7 edges. The vertices are labeled by defining a function f : V → {0,1,2,3,...,k+(2m+7)-1} as given algorithm 3.1.

$f(v_{2i+1}) = i$, $f(v'_{2i+1}) = m+k+3+i$ for $0 \leq i \leq \left\lfloor\frac{m+2}{2}\right\rfloor$

and $f(v_{2i+2}) = m+3-i$, $f(v'_{2i+2}) = 2m+k+6-i$ for $0 \leq i \leq \left\lfloor\frac{m+1}{2}\right\rfloor$.

Hence all 2m+6 vertices labeled as {0,1,2,3,...,k+2m+6} which are all distinct.

The resulting edge labels are given as follows:

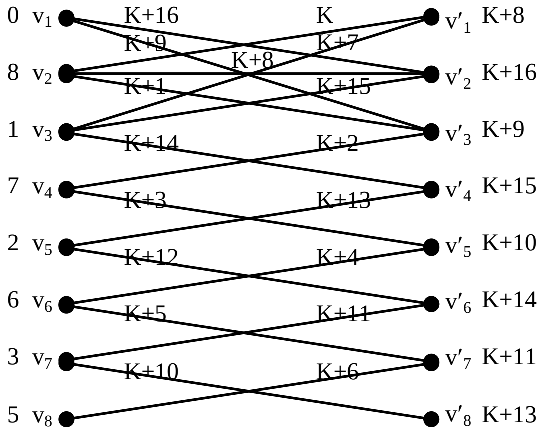
$f(e_{m+4}) = 2m+7$, $f(e_1) = 2m+k+6$, $f(e_2) = m+k+4$, $f(e'_1) = k$, $f(e'_2) = k+3$; for $1 \leq i \leq \left\lfloor\frac{m+2}{2}\right\rfloor$, $f(e_{2i+1}) = k+2i-1$ and $f(e'_{i+1}) = 2m+k+7-2i$ for $1 \leq i \leq \left\lfloor\frac{m+1}{2}\right\rfloor$,

$f(e_{2i+2}) = 2m+k+6-2i$, $f(e'_{i+2}) = 2i+k$

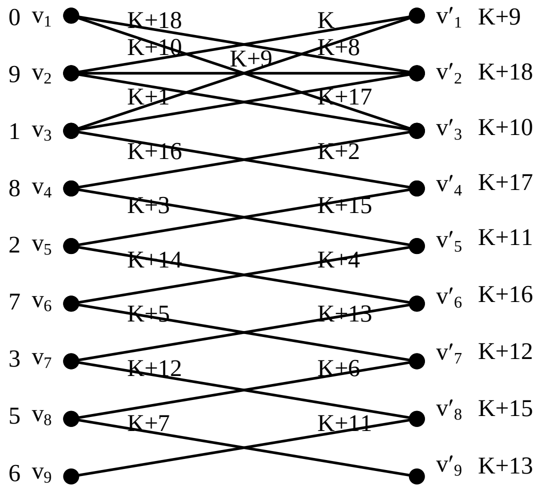
Hence all 2m+7 edges are labeled as {k, k+1, k+2,.....k+q-1} which are all distinct.

Thus the extended duplicate graph of kite graph $KI_{3,m}$, m ≥ 1 is K-graceful .

K-graceful labeling in extended duplicate graph for $KI_{3,5}$



K-graceful labeling in extended duplicate Graph for $KI_{3,6}$



Algorithm: 3.2

Procedure (Odd-Even Graceful labeling for EDG ($KI_{3,m}$, $m \geq 1$)

$V \leftarrow \{v_1, v_2, \ldots, v_{m+3}, v'_1, v'_2, \ldots, v'_{m+3}\}$

$E \leftarrow \{e_1, e_2, \ldots, e_{m+3}, e_{m+4}, e'_1, e'_2, \ldots, e'_{m+3}\}$

if $m \geq 1$

for i= 0 to $\left\lceil \frac{m+3}{2} \right\rceil$ do

$v_{2i-1} \leftarrow 2i-1$

$v'_{2i-1} \leftarrow 2m+7+2i$

end for

for i= 0 to $\left\lfloor \frac{m+3}{2} \right\rfloor$ do

$v_{2i} \leftarrow 2m+9-2i$

$v'_{2i} \leftarrow 4m+17-2i$

end for

end if

end procedure

Theorem 3.2

The extended duplicate graph of kite graph $KI_{3,m}$, $m \geq 1$ is odd-even graceful .

Proof:

We know that, the extended duplicate graph of kite graph $KI_{3,m}$, $m \geq 1$ has 2m+6 vertices and 2m+7 edges. The vertices are labeled by defining a function $f : V \rightarrow \{1,3,5,...,2q+1\}$ as given algorithm 3.2

$f(v_{2i-1}) = 2i-1$, $f(v'_{2i-1}) = 2m+7+2i$ for $0 \leq i \leq \left\lceil \frac{m+3}{2} \right\rceil$ and $f(v_{2i}) = 2m+9-2i$, $f(v'_{2i}) = 4m+17-2i$ for $0 \leq i \leq \left\lfloor \frac{m+3}{2} \right\rfloor$.

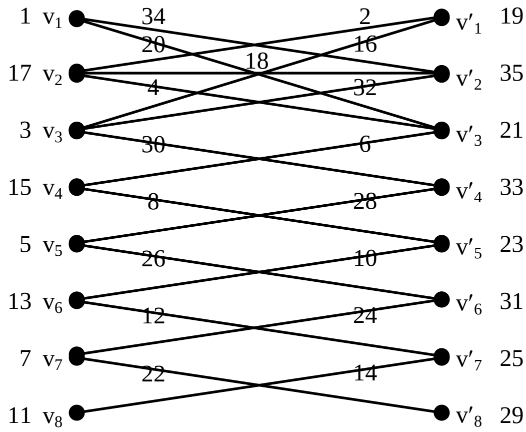
Hence all 2m+6 vertices labeled as {1,3,…,2q+1} which are all distinct.

The resulting edge labels are given as follows:

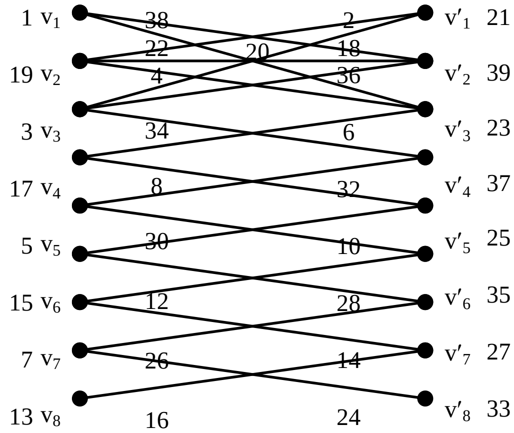
$f(e_{m+4}) = 2m+8$, $f(e_1) = 4m+14$, $f(e_2) = 2m+10$, $f(e'_1) = 2$, $f(e'_2) = 2m+6$; for $1 \leq i \leq \left\lfloor \frac{m+2}{2} \right\rfloor$, $f(e_{2i+1}) = 4i$, $f(e'_{2i+1}) = 4m+16-4i$ and for $1 \leq i \leq \left\lfloor \frac{m+1}{2} \right\rfloor$, $f(e_{2i+2}) = 4m+14-4i$, $f(e'_{2i+2}) = 4i+2$

Hence all 2m+7 edges are labeled as {2,4,…,2q} which are all distinct. Thus the extended duplicate graph of kite graph $KI_{3,m}$, $m \geq 1$ is odd-even graceful .

Odd-Even graceful labeling in extended duplicate graph of $KI_{3,5}$



Odd-Even graceful labeling in extended duplicate graph of $KI_{3,6}$



Algorithm: 3.3

Procedure (Heronian mean labeling for EDG ($KI_{3,m}$, m ≥ 1)

$V \leftarrow \{v_1, v_2, \dots, v_{m+3}, v'_1, v'_2, \dots, v'_{m+3}\}$

$E \leftarrow \{e_1, e_2, \dots, e_{m+3}, e_{m+4}, e'_1, e'_2, \dots, e'_{m+3}\}$

if $m \geq 1$

for i= 1 to $\left\lfloor \frac{m+4}{2} \right\rfloor$ do

$v_{2i-1} \leftarrow 2i - 1$

$v'_{2i-1} \leftarrow 2m + 1o - 2i$

end for

for i= 1 to $\left\lfloor \frac{m+3}{2} \right\rfloor$ do

$v_{2i} \leftarrow 2m + 9 - 2i$

$v'_{2i} \leftarrow 2i$

end for

end if

end procedure

Theorem 3.3

The extended duplicate graph of kite graph $KI_{3,m}$, m ≥ 1 admits heronian mean labeling.

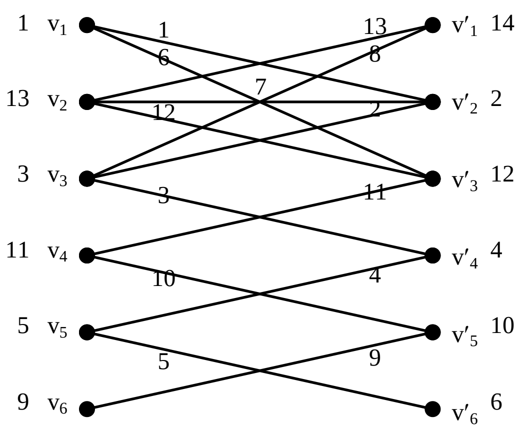
Proof:

We know that, the extended duplicate graph of kite graph $KI_{3,m}$, m ≥ 1 has 2m+6 vertices and 2m+7 edges. The vertices are labeled by defining a function $f : V \rightarrow \{1,2,3,...,q+1\}$ as given algorithm 3.3.The resulting edge labels are obtained as follows;

$f(e_1)=1$, $f(e_2)=m+3$, $f(e'_1)=2m+7$, $f(e'_2)=m+5$, $f(e_{m+4})=m+4$

for i= 1 to $\left\lfloor \frac{m+2}{2} \right\rfloor$, $f(e_{2i+1})=2m+8-2i$, $f(e'_{2i+1})=2i$; for i= 1 to $\left\lfloor \frac{m+1}{2} \right\rfloor$, $f(e_{2i+2})=2i+1$, $f(e'_{2i+2})=2m+7-2i$. Then the edge labels are distinct. Hence the extended duplicate graph of kite graph $KI_{3,m}$, m ≥ 1 admits heronian mean labeling.

Heronian mean labeling in extended duplicate graph of $KI_{3,3}$



Algorithm: 3.4

Procedure (Analytic mean labeling for EDG ($KI_{3,m}$, m ≥ 1)

$V \leftarrow \{v_1, v_2, \dots, v_{m+3}, v'_1, v'_2, \dots, v'_{m+3}\}$

$E \leftarrow \{e_1, e_2, \dots, e_{m+3}, e_{m+4}, e'_1, e'_2, \dots, e'_{m+3}\}$

if $m \geq 1$

for i= 1 to $\left\lfloor \frac{m+4}{2} \right\rfloor$ do

$v_{2i-1} \leftarrow 2i - 2$

$v'_{2i-1} \leftarrow 2m + 7 - 2i$

end for

for i= 1 to $\left\lfloor \frac{m+3}{2} \right\rfloor$ do

$v_{2i} \leftarrow 2m + 6 - 2i$

$v'_{2i} \leftarrow 2i - 1$

end for

end if

end procedure

Theorem 3.4

The extended duplicate graph of kite graph $KI_{3,m}$, m ≥ 1 admits analytic mean labeling

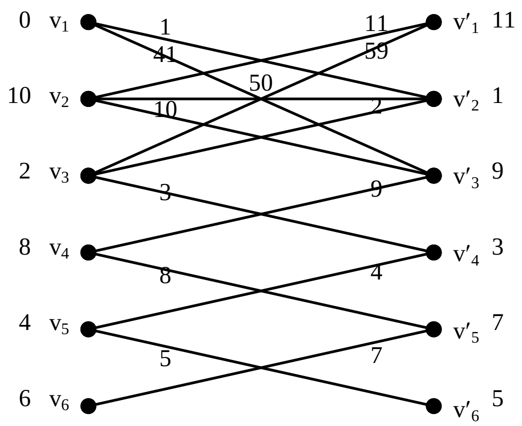
Proof:

We know that, the extended duplicate graph of kite graph $KI_{3,m}$, m ≥ 1 has 2m+6 vertices and 2m+7 edges. The vertices are labeled by defining a function $f : V \rightarrow \{0,1,2,...,p-1\}$ as given algorithm 3.4. The resulting edge labels are obtained as follows;

$f(e_1)=1$, $f(e_2)= \frac{(|v|-2)(|v|-4)}{2}+1$, $f(e'_1)=2m+5$, $f(e'_2)= \frac{(|v|)(|v|-2)}{2}-1$, $f(e_{m+4})= \frac{(|v|)(|v|-4)}{2}+2$

for i= 1 to $\left\lfloor \frac{m+2}{2} \right\rfloor$, $f(e_{2i+1})=2m+6-2i$, $f(e'_{2i+1})=2i$; for i= 1 to $\left\lfloor \frac{m+1}{2} \right\rfloor$, $f(e_{2i+2})=2i+1$, $f(e'_{2i+2})=2m+5-2i$. Then the edge labels are distinct. Hence the extended duplicate graph of kite graph $KI_{3,m}$, m ≥ 1 admits analytic mean labeling.

Analytic mean labeling in extended duplicate graph of $KI_{3,3}$



4. Conclusion

In this paper, we have constructed algorithms for labeling the vertices and edges and also proved the existence of K-graceful, odd-even graceful, heronian mean, analytic mean labeling for EDG($K_{3,m}$) , m ≥ 1

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