



Soret and Dufor Effects on a Semi Infinite Inclined Plate

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Abstract

A numerical technique is used to study a thermo-diffusion and diffusion-thermo effects on a two dimensional, laminar, unsteady convective flow past an inclined plate. The basic equations of the flow problem are non-linear, coupled and integro partial differential equations, these equations are transformed into a dimensionless form and solve it numerically using well known Crank-Nicolson implicit finite difference method. Thermo-diffusion and diffusion-thermo effects on velocity, temperature, concentration, shearing stress, rate of heat and mass transfer are studied in detail.

Keywords: Concentation, Non-linear, finite difference, Thermo-diffusion, velocity.

1. Introduction

Heat transfer due to convection is one of the major kind of heat and mass transfer in fluids. Combined heat and mass transfer process in natural convection have drawn the attention of numerous researchers due to their applications in many divisions of science and engineering. The event of free convection happens in the fluid when temperature varies and which causes density difference leading to buoyancy force acting on the elements of the fluid. It can be observed in our day to day life in the atmospheric flow which is obsessed by difference in temperature. There are many transport developments happening in nature due to differences in temperature and species concentration.

Gebhart and Pera [1] analysed the problem of natural convection on a vertical plate with non-uniform surface temperature and mass diffusion in steady state. The governing equations were solved by using similarity variables. Callahan and Marner [2] solved numerically by using explicit finite difference method the problem of simultaneous heat and mass transfer effects on a vertical plate. Implicit finite difference analysis of free convective flow past a semi-infinite vertical plate with heat and mass transfer was analyzed by Soundalgekar and Ganesan [3]. Ekambavannan and Ganesan [4] considered the free convection flows over an inclined plate with variable surface temperature mass diffusion. Ganesan and Palani [5] numerically studied the unsteady natural convection flow past an inclined plate with variable surface heat and mass flux.

Fourier's law, for instance, portrays the relation between temperature gradient and energy flux. In other features, Fick's law was established by the correlation of concentration gradient and mass flux. Moreover, it was observed that the energy flux can also be generated by composition gradients, pressure gradients, or body forces. The energy flux rooted by a composition gradient was discovered in 1873 by Dufour and was referred as the Dufour effect. It was also known as diffusion-thermo effect. On the other

hand, a mass flux can also be developed by a temperature gradient, and it was ascertained by Soret. This is known as the thermal-diffusion effect. In general, the soret and Dufour effects were of smaller order of magnitude than the effects described by Fourier's or Fick's law and were frequently neglected in heat and mass transfer processes. The thermal-diffusion effect has been used for isotope separation and in mixtures among gases with very light molecular weight (H₂, He) and medium molecular weight (N₂, air), the Dufour effect was known to be of such a magnitude that it cannot be omitted in certain conditions. In their book, Eckert and Drake [6] have ascertained several cases where the Dufour effect cannot be omitted.

Baron [7] studied soret effects in mass transfer. Sparrow *et al.* [8] considered Dufour effects in a stagnation-point flow of air with injection of gases of various molecular weights into the boundary layer. Sparrow *et al.* [9] analyzed transpiration induced buoyancy and thermal diffusion-diffusion thermo in a helium-air free convection boundary layer. Jha and Singh [10] examined the problem of free convection heat and mass transfer flow over an infinite vertical plate moving impulsively in its own place by considering the effects of Dufour and Soret. Transient free convection heat and mass transfer flow of an electrically conducting, viscous incompressible fluid past an infinite vertical porous plate with transverse magnetic field was studied by Shariful Alam *et al.* [11]. Steady, laminar, heat and mass transfer with MHD mixed convection from a semi-infinite, isothermal, vertical and permeable surface immersed in a uniform porous medium in the presence of thermal radiation and Dufour and Soret effects was studied by Chamkha and Abdullatif Ben-Nakhi [12]. The impact of suction/injection on thermophoretic particle deposition in free convection in a vertical plate embedded in a fluid saturated non-Darcy porous medium using similarity variables was examined by Partha.

Now, we intend to study the problem of natural convection effects on a semi-infinite inclined plate in the presence of Soret and Dufour effects. The flow equations are transformed into a dimensionless form and their solutions are obtained by an implicit finite difference method.

2. Mathematical Analysis

Soret and Dufour effects on unsteady natural convection flow of a viscous incompressible fluid past a semi-infinite inclined plate is considered here-with. The x -axis is considered along the plate and y -axis is taken upward normal to the plate. t' is the time, ϕ is the angle inclination of the plate with the horizontal axis. The effect of viscous dissipation is neglected in the energy equation. The acceleration due to gravity g acts vertically downward. Initially, it is considered that the plate and the fluid are at the same ambient temperature T'_∞ and the species concentration C'_∞ . As time advances the temperature and concentration of the plate are T'_w and C'_w respectively. The concentration of the diffusing species is taken to be very low in the binary mixture and hence there is no chemical reaction between the diffusing species and the fluid.

The governing boundary layer equation of mass, momentum, energy and species concentration under usual Boussinesq's approximation are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t'} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = g\beta \cos \phi \frac{\partial}{\partial x} \int_y^\infty (T' - T'_\infty) dy$$

$$+ g\beta \sin \phi (T' - T'_\infty) + g\beta^* \cos \phi \frac{\partial}{\partial x} \int_y^\infty (C' - C'_\infty) dy$$

$$+ g\beta^* \sin \phi (C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} \quad (2)$$

$$\frac{\partial T'}{\partial t'} + u \frac{\partial T'}{\partial x} + v \frac{\partial T'}{\partial y} = \alpha \frac{\partial^2 T'}{\partial y^2} + \frac{D_m K_T}{c_s c_p} \frac{\partial^2 C'}{\partial y^2} \quad (3)$$

$$\frac{\partial C'}{\partial t'} + u \frac{\partial C'}{\partial x} + v \frac{\partial C'}{\partial y} = D \frac{\partial^2 C'}{\partial y^2} + \frac{D_m K_T}{T'_m} \frac{\partial^2 T'}{\partial y^2} \quad (4)$$

$$t' \leq 0: \quad u=0, \quad v=0, \quad T'=T'_\infty, \quad C'=C'_\infty \quad \text{for all } x \text{ and } y$$

$$t' > 0: \quad u=0, \quad v=0, \quad T'=T'_w, \quad C'=C'_w \quad \text{at } y=0$$

$$u=0, \quad T'=T'_\infty, \quad C'=C'_\infty \quad \text{at } x=0 \quad (5)$$

$$u \rightarrow 0, \quad T' \rightarrow T'_\infty, \quad C' \rightarrow C'_\infty \quad \text{as } y \rightarrow \infty$$

where u and v are the components of velocity along x and y directions respectively, β is the volumetric coefficient of thermal expansion, β^* is coefficient of expansion with concentration, ν is the kinematic viscosity, α is the thermal diffusivity, D_m is the coefficient of mass diffusivity, T'_m is the mean field temperature, K_T is the thermal diffusion ratio, c_p is the specific heat at constant pressure, c_s is the concentration susceptibility.

The last term on the right-hand side of the thermal equation (3) and the species equation (4) signifies the Dufour effect and the Soret or thermal-diffusion effect respectively.

On introducing the following non-dimensional quantities:

$$X = \frac{x}{L}, \quad Y = \frac{y}{L} Gr_L^{1/4}, \quad U = \frac{uL}{\nu} Gr_L^{-1/2}, \quad V = \frac{vL}{\nu} Gr_L^{-1/4},$$

$$t = \frac{vt'}{L^2} Gr_L^{1/2}, \quad T = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty},$$

$$Gr_L = \frac{g\beta L^3 (T'_w - T'_\infty)}{\nu^2}, \quad Gr_L^* = \frac{g\beta L^3 (C'_w - C'_\infty)}{\nu^2}, \quad (6)$$

$$Pr = \frac{\nu}{\alpha}, \quad S = \frac{\nu}{D}, \quad N = \frac{Gr^*}{Gr}, \quad K = \frac{K'L^2}{\nu} Gr_L^{-1/2}$$

$$D_f = \frac{D_m K_T (C'_w - C'_\infty)}{\nu c_s c_p (T'_w - T'_\infty)}, \quad S_r = \frac{D_m K_T (T'_w - T'_\infty)}{\nu T'_m (C'_w - C'_\infty)}$$

Governing equations reduces to the following form:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (7)$$

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = Gr_L^{-1/4} \cos \phi \frac{\partial}{\partial X} \int_Y^\infty T dY$$

$$+ N Gr_L^{-1/4} \cos \phi \frac{\partial}{\partial X} \int_Y^\infty C dY + T \sin \phi + N C \sin \phi + \frac{\partial^2 U}{\partial Y^2} \quad (8)$$

$$\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{1}{Pr} \frac{\partial^2 T}{\partial Y^2} + D_f \frac{\partial^2 C}{\partial Y^2} \quad (9)$$

$$\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} + S_r \frac{\partial^2 T}{\partial Y^2} \quad (10)$$

The corresponding initial and boundary conditions in a dimensionless form are given by

$$t \leq 0: \quad U=0, \quad V=0, \quad T=0, \quad C=0 \quad \text{for all } X \text{ and } Y$$

$$t > 0: \quad U=0, \quad V=0, \quad T=1, \quad C=1 \quad \text{at } Y=0$$

$$U=0, \quad T=0, \quad C=0 \quad \text{at } X=0 \quad (11)$$

$$U \rightarrow 0, \quad T \rightarrow 0, \quad C \rightarrow 0 \quad \text{as } Y \rightarrow \infty$$

Non-dimensional form of local as well as average skin friction, Nusselt number and Sherwood number is given by

$$\tau_x = Gr_L^{3/4} \left(\frac{\partial U}{\partial Y} \right)_{Y=0} \quad (12)$$

$$\bar{\tau} = Gr_L^{3/4} \int_0^1 \left(\frac{\partial U}{\partial Y} \right)_{Y=0} dX \quad (13)$$

$$Nu_x = \frac{X Gr_L^{1/4}}{T_{Y=0}} \left(- \frac{\partial T}{\partial Y} \right)_{Y=0} \quad (14)$$

$$\overline{Nu} = -Gr_L^{1/4} \int_0^1 \left(\frac{\partial T}{\partial Y} \right)_{Y=0} dX \quad (15)$$

$$Sh_x = \frac{X Gr_L^{1/4}}{C_{Y=0}} \left(- \frac{\partial C}{\partial Y} \right)_{Y=0} \quad (16)$$

$$\overline{Sh} = -Gr_L^{1/4} \int_0^1 \left(\frac{\partial C}{\partial Y} \right)_{Y=0} dX \quad (17)$$

3. Numerical Techniques

The dimensionless basic equations(7)-(10) represents non-linear, coupled and integro partial differential equations, which are solved numerically under the initial and boundary conditions(11) using an efficient implicit finite difference scheme of Crank-Nicolson method. For calculation, we consider the region of integration as a rectangles with sides $X_{\max} = 1.0$ and $Y_{\max} = 18.0$. The appropriate mesh sizes for calculation are taken as $\Delta X = 0.50$, $\Delta Y = 0.25$ and $\Delta t = 0.01$. The Crank-Nicolson implicit method is unconditionally stable, more accurate and fast convergent. The derivatives and integration involved in the equations (12)-(17) are evaluated respectively by five point approximation formula and Newton Cotes closed integration formulae.

4. Results and Discussion

The non-dimensional parameter Dufour D_f and Soret S_r which represent the diffusion-thermo and thermal-diffusion effect, can take arbitrary values, provided that the value of their product is kept to be constant (Kafoussis and Williams). Hence, in this problem we assumed that their product value is 0.06, provided that the mean temperature and the reference temperature are kept as a constant.

Transient period of velocity, temperature and concentration for various values of Soret and Dufour (their product is constant) are calculated numerically which are shown graphically in the figures 1-3. More time is required to reach the steady state solution for lower values of Dufour. From the figure, we see that the velocity and concentration profile increases, but the temperature profile decreases with increasing value of Soret number. This behaviour is a direct consequence of the Soret effect, which produces a mass flux from lower to higher solute concentration driven by the temperature gradient. Also, when Soret number is high enough, the thermal and the solutal buoyancy forces combine their actions to enhance the convection which leads to the increase in the velocity. Whereas the opposite trend is noticed for the increasing value of Dufour number.

Local as well as average values of skin friction, Nusselt number and Sherwood number are depicted graphically in the figures 4-9. From the figures, we see that local and average skin friction is found to increase either by increasing value of Soret number or decreasing value of Dufour number. The same trend is noticed for local and average Nusselt number. But the reverse trend is noticed for local and average Sherwood number.

References

- [1] B.Gebhart and L.Pera, "The nature of vertical natural convection flows resulting from the combined buoyancy effects of thermal and mass diffusion", *Int. J. Heat Mass Transfer*, 14(1971), 2025-2050.
- [2] G.D. Callahan and W.J. Marner, "Transient free convection with mass transfer on an isothermal vertical flat plate" *Int. J. Heat Mass Transfer*, 19(1976),165-174.
- [3] Soundalgekar V.M., and Ganesan P., Finite-Difference Analysis of transient free convection with mass transfer on an isothermal vertical flat plate, *Int.J.Engg.Sci.*, 19(1981), 757-770.
- [4] Ekambavanna K., and Ganesan P., Finite difference analysis of unsteady natural convection along an inclined plate with variable surface temperature and mass diffusion, *Warme Stoffurtrag*, 31(1995), 17-24.
- [5] Ganesan P., and Palani G., Convective flow over an inclined plate with variable heat and mass flux, in: 'Proceedings of the Fourth ISHMT/ASME Heat and Mass Transfer Conference, Pune, India, January 12-14, 2000, 323-329.
- [6] Eckert E.R.G., and Drake R.M., *Analysis of Heat and Mass Transfer*, Mc Graw Hill, New York, 1972.
- [7] Baron J.R., Thermal Diffusion Effects in Mass Transfer, *Int.J.Heat Mass Transfer*, 6(1963), 1025-1033.

- [8] Sparow E.M., Minkowycz W.J., and Eckert E.R., Diffusion-Thermo Effects in Stagnation-Point flow of air with injection of gases of various molecular weights into the boundary layer, *AIAA J.*, 2(1964), 652-659.
- [9] Sparow E.M., Minkowycz W.J., and Eckert E.R., Transpiration induced buoyancy and thermal diffusion-diffusion thermo in a Helium-Air free convection boundary layer, *J.Heat Mass Transfer*, 64(1964), 508-513.
- [10] Jha B.K., and Singh A.K., Soret effects on Free Convection and Mass Transfer flow in Stokes problem for an infinite vertical plate, *Astrophysics Space Sci.*, 173(1990), 251-255.
- [11] Alam M.S., Rahman M.m., and Samad M.A., Dufour and Soret Effects on Unsteady MHD free convection and mass transfer flow past a vertical porous plate in a porous medium. *Nonlin.Anal.Modell.Control*, 11(3), (2006), 217-226.
- [12] Chamkha A.J., and Ben-Nakhi A., MHD mixed convection-radiation interaction along a permeable surface immersed in a porous medium in the presence of Soret and Dufour's Effects, *Heat Mass Transfer*, 44(2008), 845-846.
- [13] Partha M.K., Suction/injection effects on thermophoresis particle deposition in a non-Darcy porous medium under the influence of Soret, Dufour effects, *Int.J.Heat and Mass Transfer*, 52(2009), 1971-1979.
- [14] Kafoussias N.G., and Williams, E.W., *Int.J.Engg.Sci.*, 33(1995), 1369.

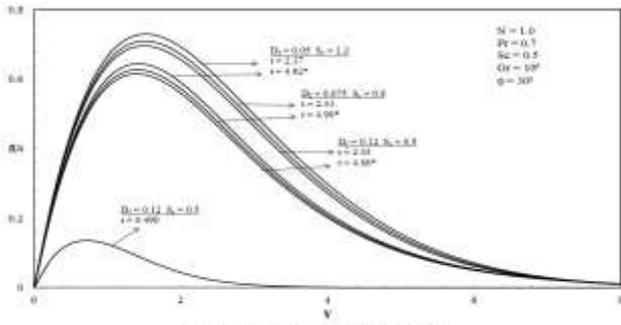


Fig.1 Transient velocity profiles at X=1.0 for different D_b and S_b (∞ -steady state)

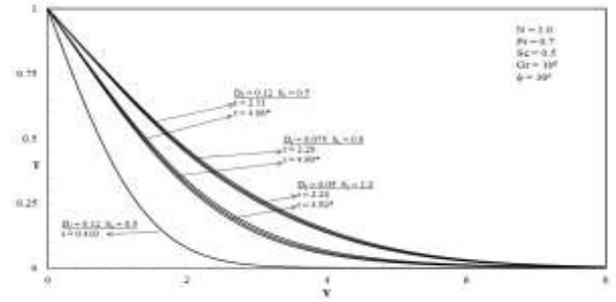


Fig.2 Transient temperature profiles at X=1.0 for different D_b and S_b (∞ -steady state)

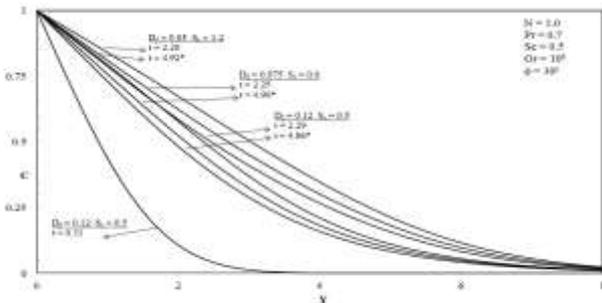


Fig.3 Transient concentration profiles at X=1.0 for different D_b and S_b (∞ -steady state)

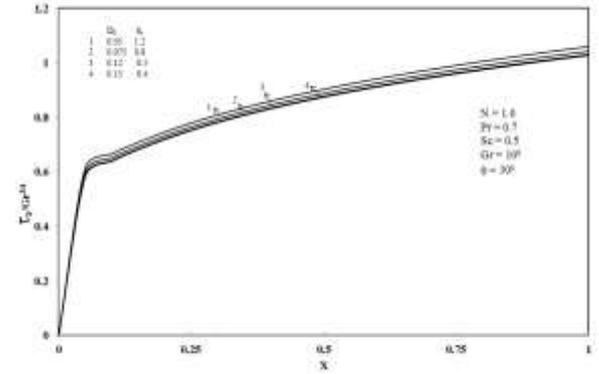


Fig.4 Local skin friction

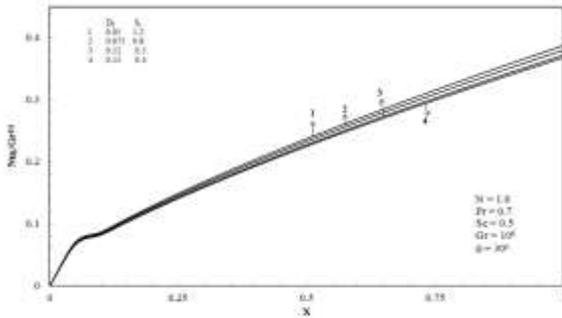


Fig.5 Local Nusselt number

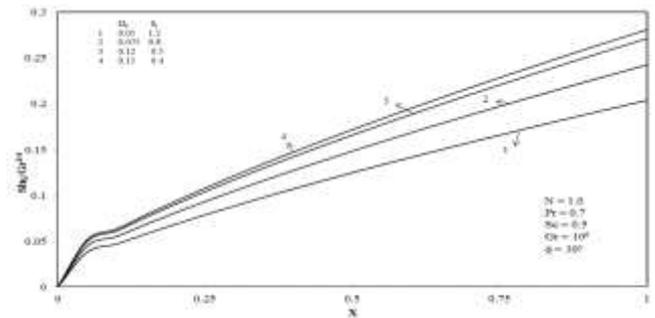


Fig.6 Local Sherwood number

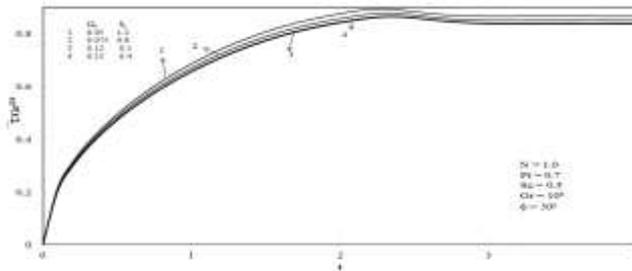


Fig.7 Average skin friction

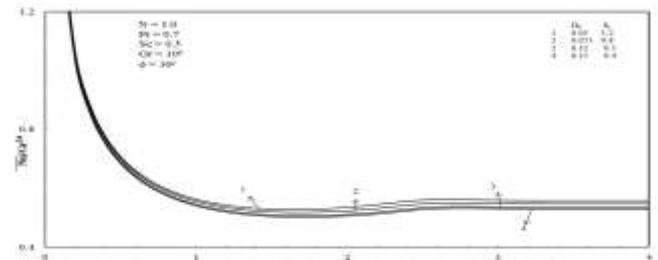


Fig.8 Average Nusselt number

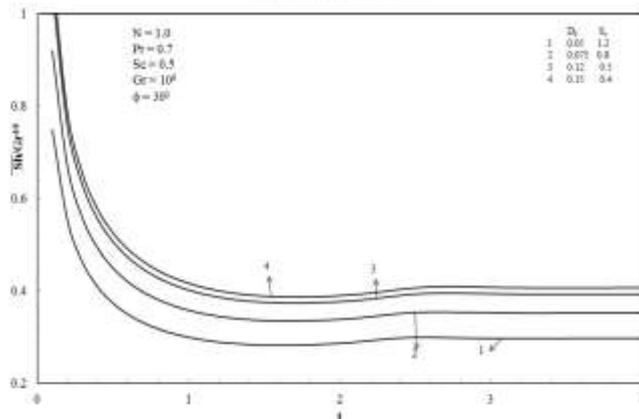


Fig.9 Average Sherwood number