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Research paper

MHD Peristaltic Transport of a Dusty Couple Stress Fluid Through a Symmetric Porous Channel

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Abstract

This paper agreement with the MHD peristalsis flow of a dusty couple stress liquid in a 2-D channel. Analytical solutions are got for pressure gradient and axial velocity in the fluid using perturbation technique. The results obtained for velocity profile, pressure gradient and skin friction are graphically represented. It is noted the velocity profiles increases with the increase in time averaged flow rate (Q). The velocity profiles reduce with the raise in couple stress parameter (S) and Permeability parameter (k).

Keywords: *Peristaltic flow, Couple stress fluid, Long Wavelength approximation, Perturbation*

1. Introduction

The problem on peristalsis of two phase system with the effect of magnetic field through a porous medium attracted several scientists due to its play an important role in bio, aero and industrial engineering. Dissolved micro-molecules of filament deferral in paper creation, flow of blood through arteries, propulsion and combustion in rockets, corrosion of objects due to nonstop impingement of suspended particles in the atmosphere are few real life examples of such flow. A various systems in the body like circulatory, nervous and urinary system are stimulated by Magnetotherapy. Chaturani [1] developed the Poiseulle flow (non Newtonian) mathematical model. An extended work has been studied by Chaturani and Rathod [2]. Dulal et al. [3] studied blood circulation of a couple stress model. Two phase fluid system is the suitable model to define blood is a binary system. Brown and Hung [4] discussed the two phase fluid system on peristaltic motion. Subba Reddy et al. [5] investigate the non uniform dusty plasma in the rotational waves. Few researchers (Ravi kumar et al. [6], Raghunath Rao and Prasada Rao [7], Mekheimer et al. [8], Sobh [9]) have been analyzed the peristalsis of couple stresses under different physical conditions. Valanis and Sun [10] also discussed a micro circulation of a blood flow. The major purpose of this chapter is to study the peristalsis of a dusty non Newtonian (couple stress model) fluid in a porous channel under lubrication approach. An approximate solution is obtained for flow variables and the influence of pertinent parameters is discussed numerically and graphically.

2. Formulation of the Problem

We consider the two dimensional porous channel with speed of the wave is c, channel wavelength l and the channel amplitude a(as shown in Figure 1).
The wall equation is:

$$\bar{\eta}(x,t') = \pm\left[d + a\sin\left(\frac{2\pi}{\lambda}\left(X - ct'\right)\right)\right]$$

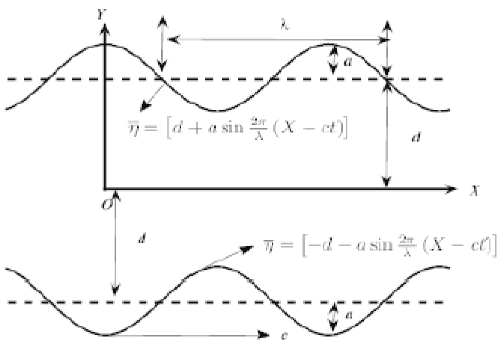

Figure 1:

3. Governing Equations

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3.1. Governing Equations of Fluid Phase

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{2}$$

$$\rho\left(\frac{\partial U}{\partial t'} + \bar{u}\frac{\partial U}{\partial X} + \bar{v}\frac{\partial U}{\partial Y}\right) = -\frac{\partial P}{\partial X} + \mu\nabla^2 U \tag{3}$$
$$+\eta^*\nabla^4 U + KN(U_p - U) - \sigma B_0^2 U - \frac{\upsilon}{k_0}U$$

$$\rho\left(\frac{\partial V}{\partial t'}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}\right)=-\frac{\partial P}{\partial Y}+\mu\nabla^2V+\eta^*\nabla^4V \tag{4}$$
$$+KN(V_p-V)-\frac{\upsilon}{k_0}$$

3.2. Governing Equations of Dust Phase

$$\frac{\partial N}{\partial X}+N\left(\frac{\partial U_p}{\partial X}+\frac{\partial V_p}{\partial Y}\right)=0 \tag{5}$$

$$\left(\frac{\partial V_p}{\partial t'}+U_p\frac{\partial V_p}{\partial X}+V_p\frac{\partial V_p}{\partial Y}\right)=\frac{k}{m}(V-V_p) \tag{6}$$

$$\left(\frac{\partial V_p}{\partial t'}+U_p\frac{\partial V_p}{\partial X}+V_p\frac{\partial V_p}{\partial Y}\right)=\frac{k}{m}(V-V_p) \tag{7}$$

$$\nabla^2=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right),\nabla^4=\nabla^2\nabla^2 \tag{8}$$

where $\overline{u},\overline{v}$ is the velocity of the fluid particles, $\overline{u}_p,\overline{v}_p$ is the velocity of the dust particles, $\overline{p}$ is the fluid pressure, σ is the electrical conductivity of the fluid, B_0 is the applied magnetic field, ρ is the density of the fluid, υ is the kinematic coefficient of the viscosity of fluid, and $K=6\pi\mu r$, r being the particle radius, is the Stoke's drag coefficient for the dust particles(a constant), m is the mass of the solid particles, η is the coefficient of couple stress, N is the number density of the particle, μ is the coefficient of viscosity, k is the stokes resistance coefficient. The corresponding boundary conditions are

$$U=0 \quad at \quad Y=\pm\eta, \quad \frac{\partial^2U}{\partial Y^2}=0 \quad at \quad Y=\pm\eta$$

$$V=0 \quad at \quad Y=0$$

Introducing the following non dimensional quantities

$$x=\frac{X}{\lambda},\ y=\frac{Y}{d},\ \overline{\eta}=\frac{\eta}{d},\ v=\frac{V}{c\delta},\delta=\frac{d}{\lambda},u=\frac{U}{c},$$
$$u_p=\frac{U_p}{c},v_p=\frac{V_p}{c\delta},\ t=\frac{c\overline{t}}{\lambda},\ \varepsilon=\frac{a}{d},\mathrm{Re}=\frac{\rho_f cd}{\mu},$$
$$p=\frac{d^2P}{\mu c\lambda},k=\frac{k_0}{d^2},M^2=\frac{\sigma^*B_0^2d^2}{\mu},S^2=\frac{\mu d^2}{\eta^*},$$
$$\tau^*=\frac{cm}{kd},t=\frac{ct'}{\lambda}.$$

Applying all the non dimensional quantities in the governing equations (Equations (2) - (7)) for the fluid and particle phase, and introducing the stream function ψ, ϕ

$$u=\frac{\partial\psi}{\partial y},v=\frac{\partial\psi}{\partial x},u_s=\frac{\partial\phi}{\partial y},v_s=\frac{\partial\phi}{\partial x}$$

4. Solution of the Problem

Zeroth and first order in δ can be expressed as follows

4.1 Zeroth Order in δ

$$0=-\frac{\partial p}{\partial x}+\frac{\partial^3\psi_0}{\partial y^3}-\frac{1}{S^2}\left(\frac{\partial^5\psi_0}{\partial y^5}\right) \tag{2}$$
$$-\left(\frac{1}{k}+M^2\right)\frac{\partial\psi_0}{\partial y}+\frac{\mathrm{Re}\,\alpha^*}{\tau^*}\left(\frac{\partial\phi_0}{\partial y}-\frac{\partial\psi_0}{\partial y}\right)$$

$$0=-\frac{\partial p}{\partial y}$$
$$0=\frac{1}{\tau^*}\left(\frac{\partial\psi_0}{\partial y}-\frac{\partial\phi_0}{\partial y}\right)$$

The respective zeroth order boundary conditions are

$$\psi_0=\pm\frac{F}{2},\frac{\partial\psi_0}{\partial y}=0,\frac{\partial^3\psi_0}{\partial y^3}=0 \quad at \quad y=\pm\eta$$
$$\frac{\partial\psi_0}{\partial x}=0 \quad at \quad y=0$$

4.2. First Order in δ

$$\mathrm{Re}\left(\frac{\partial}{\partial t}\frac{\partial\psi_0}{\partial y}+\frac{\partial\psi_0}{\partial y}\frac{\partial^2\psi_0}{\partial x\partial y}-\frac{\partial\psi_0}{\partial x}\frac{\partial^2\psi_0}{\partial y^2}\right)=$$
$$-\frac{\partial p}{\partial x}+\frac{\partial^3\psi_1}{\partial y^3}-\frac{1}{S^2}\left(\frac{\partial^5\psi_1}{\partial y^5}\right)$$
$$-\left(\frac{1}{k}+M^2\right)\frac{\partial\psi_1}{\partial y}+\frac{\mathrm{Re}\,\alpha^*}{\tau^*}\left(\frac{\partial\phi_1}{\partial y}-\frac{\partial\psi_1}{\partial y}\right)$$

$$0=-\frac{\partial p}{\partial y}$$
$$\delta\left(\frac{\partial}{\partial t}\frac{\partial\phi_0}{\partial y}+\frac{\partial\phi_0}{\partial y}\frac{\partial}{\partial x}\left(\frac{\partial\phi_0}{\partial y}\right)-\frac{\partial\phi_0}{\partial x}\frac{\partial}{\partial y}\left(\frac{\partial\phi_0}{\partial y}\right)\right)=$$
$$\frac{1}{\tau^*}\left(\frac{\partial\psi_1}{\partial y}-\frac{\partial\phi_1}{\partial y}\right)$$
$$\delta\left(-\frac{\partial}{\partial t}\frac{\partial\phi_0}{\partial x}-\frac{\partial\phi_0}{\partial y}\frac{\partial}{\partial x}\left(\frac{\partial\phi_0}{\partial x}\right)+\frac{\partial\phi_0}{\partial x}\frac{\partial}{\partial y}\left(\frac{\partial\phi_0}{\partial y}\right)\right)$$
$$=\frac{1}{\tau^*}\left(\frac{\partial\psi_1}{\partial x}-\frac{\partial\phi_1}{\partial x}\right)$$

The respective zeroth order boundary conditions are

$$\psi_1=0,\frac{\partial\psi_1}{\partial y}=0,\frac{\partial^3\psi_1}{\partial y^3}=0 \quad at \quad y=\pm\eta$$
$$\frac{\partial\psi_1}{\partial x}=0 \quad at \quad y=0$$

4.3 Zeroth Order Solutions

$$\psi_0=C_1+C_2e^{k_1y}+C_3e^{-k_1y}+C_4e^{k_2y}+C_5e^{-k_2y}+C_6k_3y$$
$$k_1^2=\left(\frac{S\sqrt{S^2-4H^2}}{2}+\frac{S^2}{2}\right),k_2^2=\left(-\frac{S\sqrt{S^2-4H^2}}{2}+\frac{S^2}{2}\right),$$
$$k_3=\frac{1}{H^2S^2}$$

$$\psi_1 = C_7 + \left(C_8 + L_{13}\right)e^{k_1 y} + \left(C_9 + L_{14}\right)e^{-k_1 y}$$
$$+\left(C_{10} + L_{15}\right)e^{k_2 y} + \left(C_{11} + L_{16}\right)e^{-k_2 y} + C_{12}k_3 y$$
$$+m_2(L_{17}e^{\left(k_1+k_2\right)y} + L_{18}e^{-\left(k_1+k_2\right)y})$$
$$+m_2(L_{19}e^{\left(k_1-k_2\right)y} + L_{20}e^{-\left(k_1-k_2\right)y} + L_{21}ye^{k_1 y}$$
$$+L_{22}ye^{-k_1 y} + L_{23}ye^{k_2 y} + L_{24}ye^{-k_2 y})$$

The velocity of fluid phase is given by

$$u = u_0 + \delta u_1 + ...$$

In which

$$u_0 = C_2 k_1 e^{k_1 y} - C_3 k_2 e^{-k_1 y} + C_4 k_2 e^{k_2 y} - C_5 k_2 e^{-k_2 y} + C_6 k_3$$
$$u_1 = \left(C_8 + L_{13}\right)k_1 e^{k_1 y} - \left(C_9 + L_{14}\right)k_1 e^{-k_1 y}$$
$$+\left(C_{10} + L_{15}\right)k_2 e^{k_2 y} - \left(C_{11} + L_{16}\right)e^{-k_2 y} + C_{12}k_3$$
$$+m_2(L_{17}\left(k_1 + k_2\right)e^{\left(k_1+k_2\right)y} - L_{18}\left(k_1 + k_2\right)e^{-\left(k_1+k_2\right)y})$$
$$+m_2(L_{19}\left(k_1 - k_2\right)e^{\left(k_1-k_2\right)y} - L_{20}\left(k_1 - k_2\right)e^{-\left(k_1-k_2\right)y}$$
$$+L_{21}(k_1 ye^{k_1 y} + e^{k_1 y})) - m_2(L_{22}(yk_1 e^{-k_1 y} + e^{-k_1 y})$$
$$+L_{23}(yk_2 e^{k_2 y} + e^{k_2 y}) - L_{24}(yk_2 e^{-k_2 y} + e^{-k_2 y})$$

where C_1 - C_{12} , L_1 - L_{24} are known constants. The values of these constants are not given due to the sake of brevity but, these values are taken into account while drawing the profiles.

5. Results and Discussion

This part provides the manners of a variety of parameters concerned in the expressions of u, dp/dx and τ. In particular, the variations of M, k, Θ and S were tested.

5.1. Velocity Profile

The behavior of parameters like S, k, M and flow rate involved in velocity profile (u) is shown in Figures 2 - 5. It is examined that the velocity profiles are parabolic in nature. The influence of M on u is plotted in Figure 2. Velocity (u) diminishes with the boost in M. It is clear through Figure 3 that u raises as S decreases. Figure 4 illustrates the effect of porous parameter (k) on u. It is clear from Figure 4 that the velocity raises as k increases. Figure 5 shows the influence of flow rate on velocity. It is observed that the increasing the flow rate leads to increase in the fluid velocity.

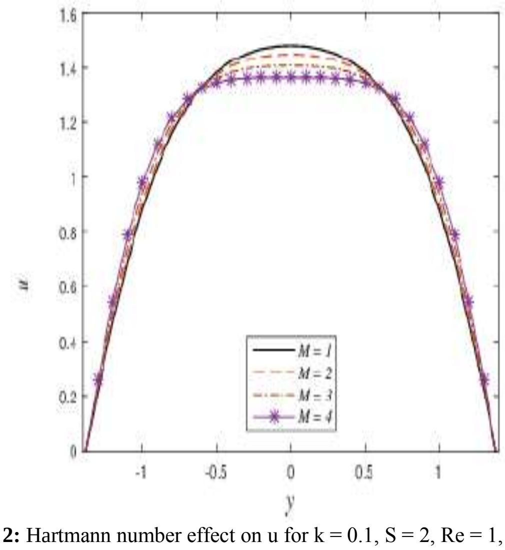

Figure 2: Hartmann number effect on u for k = 0.1, S = 2, Re = 1, τ^* = 0.3, Θ = 2, ε = 0.4, δ = 0.1.

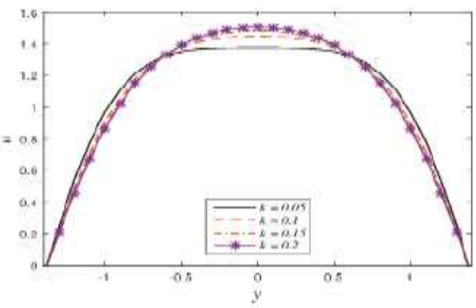

Figure 3: Couple stress parameter effect on u for k = 0.1, M = 2, Re = 1, τ^* = 0.3, Θ = 2, ε = 0.4, δ = 0.1.

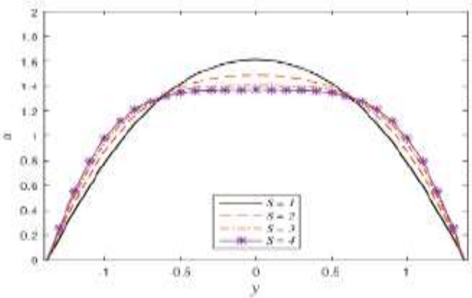

Figure 4: Porous parameter effect on u for M = 2, Re = 1, τ^* = 0.3, Θ = 2, ε = 0.4, δ = 0.1.

5.2. Pressure Gradient

Figures 6 to 8 illustrate the variation of M, k and S on dp/dx. dp/dx is plotted over one wave length. The effect of M on dp/dx is discussed in Figure 6. From this figure we get the information that in the wide part of the channel, the movement of the fluid is very easily through without any difficulty

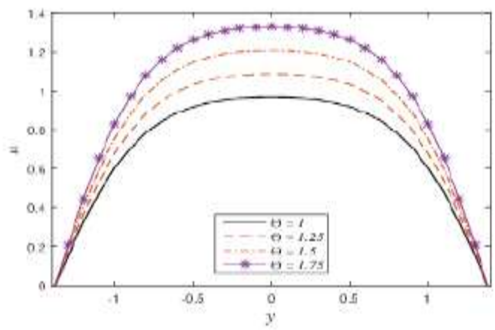

Figure 5: Mean flow rate effect on u for M = 2, Re = 1, τ^* = 0.3, ε = 0.4, δ = 0.1, k = 0.1.

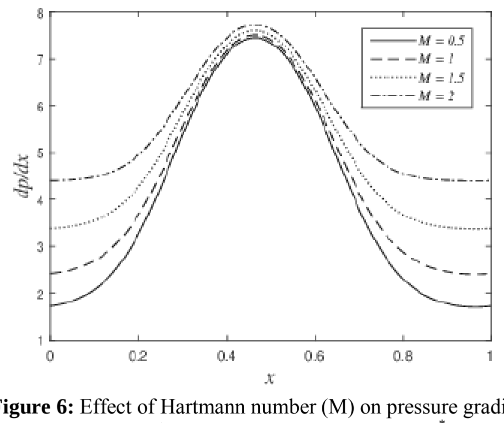

Figure 6: Effect of Hartmann number (M) on pressure gradient for ε = 0.4,Re = 1, t = 0.2, k = 0.1, S = 2, α = 0.4, Re = 1, τ^* = 0.3, Θ = 2, δ = 0.1.

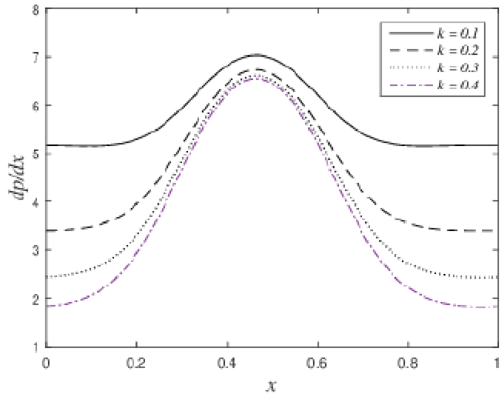

Figure 7: Effect of k on pressure gradient for ε = 0.4,Re = 1, t = 0.2, M = 2, S = 2, α = 0.4, Re = 1, τ^* = 0.3, Θ = 2, δ = 0.1.

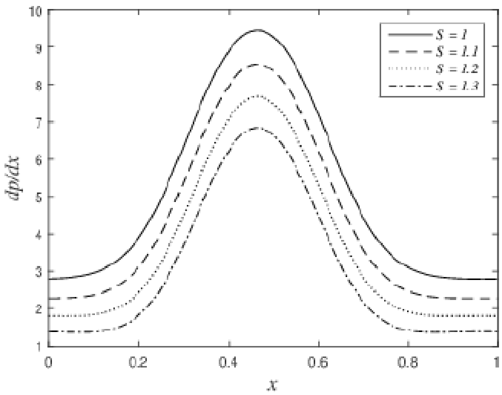

Figure 8: Effect of S on pressure gradient for ε = 0.4,Re = 1, t = 0.2, k = 0.1, α = 0.4, Re = 1, τ^* = 0.3, Θ = 2, δ = 0.1, M = 2.

6. Conclusion

The prime ambition of this section is to explore the influence of MHD on the two-dimensional peristalsis of a couple stress incompressible viscous fluid with solid particles. Here with list out all the obtained results of physical quantities on u, dp/dx and skin friction t .

1. Lorentz force defines that magnetic reduce the velocity by its force and velocity drop out with raise up M.
2. The profiles of velocity boost in time averaged flow rate (Q). The velocity profiles reduces with raise in the value of S and k.
3. In particular, we observed that by raising the value of M, which raise the value of dp/dx) more over the reverse trend observed for different values of k.
4. Decrease the value of skin friction with an enhancement of S and it increases while increasing M.

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