International Journal of Engineering & Technology, 7 (4.10) (2018) 306-309

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

MHD Peristaltic Transport of a Dusty Couple Stress Fluid Through a Symmetric Porous Channel

E. P. Siva[1]*, J. Prakash[2], A.Govindarajan[3] and M.Vidhya[4]

[1,3]Department of Mathematics, SRM Institute of Science and Technology, Kattankulathur -603 203, Tamilnadu, India
[2]Department of Mathematics, Srinivasa Ramanujan Center, SASTRA Deemed University, Kumbakonam – 612001, Tamilnadu, India
[4]Department of Mathematics, Sathiyabama Institute of Science and Technology, Sholinganallur - 600 119, Tamilnadu, India
**Corresponding author E-mail: siva.e@ktr.srmuniv.ac.in*

Abstract

This paper agreement with the MHD peristalsis flow of a dusty couple stress liquid in a 2-D channel. Analytical solutions are got for pressure gradient and axial velocity in the fluid using perturbation technique. The results obtained for velocity profile, pressure gradient and skin friction are graphically represented. It is noted the velocity profiles increases with the increase in time averaged flow rate (Q). The velocity profiles reduce with the raise in couple stress parameter (S) and Permeability parameter (k).

Keywords: *Peristaltic flow, Couple stress fluid, Long Wavelength approximation, Perturbation*

1. Introduction

The problem on peristalsis of two phase system with the effect of magnetic field through a porous medium attracted several scientists due to its play an important role in bio, aero and industrial engineering. Dissolved micro-molecules of filament deferral in paper creation, flow of blood through arteries, propulsion and combustion in rockets, corrosion of objects due to nonstop impingement of suspended particles in the atmosphere are few real life examples of such flow. A various systems in the body like circulatory, nervous and urinary system are stimulated by Magnetotherapy. Chaturani [1] developed the Poiseulle flow (non Newtonian) mathematical model. An extended work has been studied by Chaturani and Rathod [2]. Dulal et al. [3] studied blood circulation of a couple stress model. Two phase fluid system is the suitable model to define blood is a binary system. Brown and Hung [4] discussed the two phase fluid system on peristaltic motion. Subba Reddy et al. [5] investigate the non uniform dusty plasma in the rotational waves. Few researchers (Ravi kumar et al. [6], Raghunath Rao and Prasada Rao [7], Mekheimer et al. [8], Sobh [9]) have been analyzed the peristalsis of couple stresses under different physical conditions. Valanis and Sun [10] also discussed a micro circulation of a blood flow. The major purpose of this chapter is to study the peristalsis of a dusty non Newtonian (couple stress model) fluid in a porous channel under lubrication approach. An approximate solution is obtained for flow variables and the influence of pertinent parameters is discussed numerically and graphically.

2. Formulation of the Problem

We consider the two dimensional porous channel with speed of the wave is c, channel wavelength l and the channel amplitude a(as shown in Figure 1).
The wall equation is:

$$\bar{\eta}(x,t') = \pm\left[d + a\sin\left(\frac{2\pi}{\lambda}(X - ct')\right)\right]$$

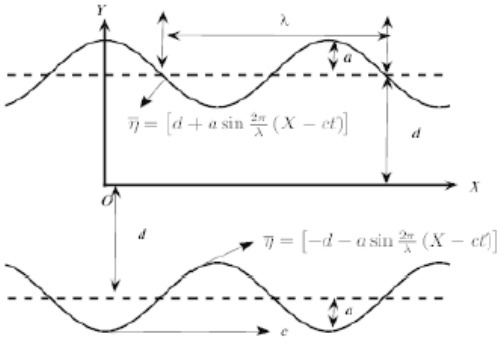


Figure 1:

3. Governing Equations

All paragraphs must be justified alignment. With justified alignment, both sides of the paragraph are straight.

3.1. Governing Equations of Fluid Phase

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{2}$$

$$\rho\left(\frac{\partial U}{\partial t'} + \bar{u}\frac{\partial U}{\partial X} + \bar{v}\frac{\partial U}{\partial Y}\right) = -\frac{\partial P}{\partial X} + \mu\nabla^2 U \tag{3}$$
$$+\eta^*\nabla^4 U + KN(U_p - U) - \sigma B_0^2 U - \frac{\upsilon}{k_0}U$$

Copyright © 2018 Authors. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

$$\rho\left(\frac{\partial V}{\partial t'}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}\right)=-\frac{\partial P}{\partial Y}+\mu\nabla^2 V+\eta^*\nabla^4 V \quad (4)$$
$$+KN(V_p-V)-\frac{\upsilon}{k_0}$$

3.2. Governing Equations of Dust Phase

$$\frac{\partial N}{\partial X}+N\left(\frac{\partial U_p}{\partial X}+\frac{\partial V_p}{\partial Y}\right)=0 \quad (5)$$

$$\left(\frac{\partial V_p}{\partial t'}+U_p\frac{\partial V_p}{\partial X}+V_p\frac{\partial V_p}{\partial Y}\right)=\frac{k}{m}(V-V_p) \quad (6)$$

$$\left(\frac{\partial V_p}{\partial t'}+U_p\frac{\partial V_p}{\partial X}+V_p\frac{\partial V_p}{\partial Y}\right)=\frac{k}{m}(V-V_p) \quad (7)$$

$$\nabla^2=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right),\nabla^4=\nabla^2\nabla^2 \quad (8)$$

where $\bar{u},\bar{v}$ is the velocity of the fluid particles, $\bar{u}_p,\bar{v}_p$ is the velocity of the dust particles, $\bar{p}$ is the fluid pressure, σ is the electrical conductivity of the fluid, B_0 is the applied magnetic field, ρ is the density of the fluid, υ is the kinematic coefficient of the viscosity of fluid, and $K=6\pi\mu r$, r being the particle radius, is the Stoke's drag coefficient for the dust particles(a constant), m is the mass of the solid particles, η is the coefficient of couple stress, N is the number density of the particle, μ is the coefficient of viscosity, k is the stokes resistance coefficient. The corresponding boundary conditions are

$$U=0 \quad at \quad Y=\pm\eta \;,\; \frac{\partial^2 U}{\partial Y^2}=0 \quad at \quad Y=\pm\eta$$

$$V=0 \quad at \quad Y=0$$

Introducing the following non dimensional quantities

$$x=\frac{X}{\lambda},\ y=\frac{Y}{d},\ \bar{\eta}=\frac{\eta}{d},\ v=\frac{V}{c\delta},\delta=\frac{d}{\lambda},u=\frac{U}{c},$$
$$u_p=\frac{U_p}{c},v_p=\frac{V_p}{c\delta},\ t=\frac{c\bar{t}}{\lambda},\ \varepsilon=\frac{a}{d},\mathrm{Re}=\frac{\rho_f cd}{\mu},$$
$$p=\frac{d^2P}{\mu c\lambda},k=\frac{k_0}{d^2},M^2=\frac{\sigma^* B_0^2 d^2}{\mu},S^2=\frac{\mu d^2}{\eta^*},$$
$$\tau^*=\frac{cm}{kd},t=\frac{ct'}{\lambda}.$$

Applying all the non dimensional quantities in the governing equations (Equations (2) - (7)) for the fluid and particle phase, and introducing the stream function ψ, ϕ

$$u=\frac{\partial\psi}{\partial y},v=\frac{\partial\psi}{\partial x},u_s=\frac{\partial\phi}{\partial y},v_s=\frac{\partial\phi}{\partial x}$$

4. Solution of the Problem

Zeroth and first order in δ can be expressed as follows

4.1 Zeroth Order in δ

$$0=-\frac{\partial p}{\partial x}+\frac{\partial^3\psi_0}{\partial y^3}-\frac{1}{S^2}\left(\frac{\partial^5\psi_0}{\partial y^5}\right) \quad (2)$$
$$-\left(\frac{1}{k}+M^2\right)\frac{\partial\psi_0}{\partial y}+\frac{\mathrm{Re}\,\alpha^*}{\tau^*}\left(\frac{\partial\phi_0}{\partial y}-\frac{\partial\psi_0}{\partial y}\right)$$

$$0=-\frac{\partial p}{\partial y}$$
$$0=\frac{1}{\tau^*}\left(\frac{\partial\psi_0}{\partial y}-\frac{\partial\phi_0}{\partial y}\right)$$

The respective zeroth order boundary conditions are

$$\psi_0=\pm\frac{F}{2},\frac{\partial\psi_0}{\partial y}=0,\frac{\partial^3\psi_0}{\partial y^3}=0 \quad at \quad y=\pm\eta$$
$$\frac{\partial\psi_0}{\partial x}=0 \quad at \quad y=0$$

4.2. First Order in δ

$$\mathrm{Re}\left(\frac{\partial}{\partial t}\frac{\partial\psi_0}{\partial y}+\frac{\partial\psi_0}{\partial y}\frac{\partial^2\psi_0}{\partial x\partial y}-\frac{\partial\psi_0}{\partial x}\frac{\partial^2\psi_0}{\partial y^2}\right)=$$
$$-\frac{\partial p}{\partial x}+\frac{\partial^3\psi_1}{\partial y^3}-\frac{1}{S^2}\left(\frac{\partial^5\psi_1}{\partial y^5}\right)$$
$$-\left(\frac{1}{k}+M^2\right)\frac{\partial\psi_1}{\partial y}+\frac{\mathrm{Re}\,\alpha^*}{\tau^*}\left(\frac{\partial\phi_1}{\partial y}-\frac{\partial\psi_1}{\partial y}\right)$$

$$0=-\frac{\partial p}{\partial y}$$
$$\delta\left(\frac{\partial}{\partial t}\frac{\partial\phi_0}{\partial y}+\frac{\partial\phi_0}{\partial y}\frac{\partial}{\partial x}\left(\frac{\partial\phi_0}{\partial y}\right)-\frac{\partial\phi_0}{\partial x}\frac{\partial}{\partial y}\left(\frac{\partial\phi_0}{\partial y}\right)\right)=$$
$$\frac{1}{\tau^*}\left(\frac{\partial\psi_1}{\partial y}-\frac{\partial\phi_1}{\partial y}\right)$$
$$\delta\left(-\frac{\partial}{\partial t}\frac{\partial\phi_0}{\partial x}-\frac{\partial\phi_0}{\partial y}\frac{\partial}{\partial x}\left(\frac{\partial\phi_0}{\partial x}\right)+\frac{\partial\phi_0}{\partial x}\frac{\partial}{\partial y}\left(\frac{\partial\phi_0}{\partial y}\right)\right)$$
$$=\frac{1}{\tau^*}\left(\frac{\partial\psi_1}{\partial x}-\frac{\partial\phi_1}{\partial x}\right)$$

The respective zeroth order boundary conditions are

$$\psi_1=0,\frac{\partial\psi_1}{\partial y}=0,\frac{\partial^3\psi_1}{\partial y^3}=0 \quad at \quad y=\pm\eta$$
$$\frac{\partial\psi_1}{\partial x}=0 \quad at \quad y=0$$

4.3 Zeroth Order Solutions

$$\psi_0=C_1+C_2e^{k_1y}+C_3e^{-k_1y}+C_4e^{k_2y}+C_5e^{-k_2y}+C_6k_3y$$
$$k_1^2=\left(\frac{S\sqrt{S^2-4H^2}}{2}+\frac{S^2}{2}\right),k_2^2=\left(-\frac{S\sqrt{S^2-4H^2}}{2}+\frac{S^2}{2}\right),$$
$$k_3=\frac{1}{H^2S^2}$$

$$\psi_1 = C_7 + (C_8 + L_{13})e^{k_1 y} + (C_9 + L_{14})e^{-k_1 y}$$
$$+(C_{10} + L_{15})e^{k_2 y} + (C_{11} + L_{16})e^{-k_2 y} + C_{12}k_3 y$$
$$+m_2(L_{17}e^{(k_1+k_2)y} + L_{18}e^{-(k_1+k_2)y})$$
$$+m_2(L_{19}e^{(k_1-k_2)y} + L_{20}e^{-(k_1-k_2)y} + L_{21}ye^{k_1 y}$$
$$+L_{22}ye^{-k_1 y} + L_{23}ye^{k_2 y} + L_{24}ye^{-k_2 y})$$

The velocity of fluid phase is given by

$$u = u_0 + \delta u_1 + ...$$

In which

$$u_0 = C_2k_1e^{k_1 y} - C_3k_2e^{-k_1 y} + C_4k_2e^{k_2 y} - C_5k_2e^{-k_2 y} + C_6k_3$$
$$u_1 = (C_8 + L_{13})k_1e^{k_1 y} - (C_9 + L_{14})k_1e^{-k_1 y}$$
$$+(C_{10} + L_{15})k_2e^{k_2 y} - (C_{11} + L_{16})e^{-k_2 y} + C_{12}k_3$$
$$+m_2(L_{17}(k_1 + k_2)e^{(k_1+k_2)y} - L_{18}(k_1 + k_2)e^{-(k_1+k_2)y})$$
$$+m_2(L_{19}(k_1 - k_2)e^{(k_1-k_2)y} - L_{20}(k_1 - k_2)e^{-(k_1-k_2)y}$$
$$+L_{21}(k_1ye^{k_1 y} + e^{k_1 y})) - m_2(L_{22}(yk_1e^{-k_1 y} + e^{-k_1 y})$$
$$+L_{23}(yk_2e^{k_2 y} + e^{k_2 y}) - L_{24}(yk_2e^{-k_2 y} + e^{-k_2 y})$$

where C_1 - C_{12} , L_1 - L_{24} are known constants. The values of these constants are not given due to the sake of brevity but, these values are taken into account while drawing the profiles.

5. Results and Discussion

This part provides the manners of a variety of parameters concerned in the expressions of u, dp/dx and τ. In particular, the variations of M, k, Θ and S were tested.

5.1. Velocity Profile

The behavior of parameters like S, k, M and flow rate involved in velocity profile (u) is shown in Figures 2 - 5. It is examined that the velocity profiles are parabolic in nature. The influence of M on u is plotted in Figure 2. Velocity (u) diminishes with the boost in M. It is clear through Figure 3 that u raises as S decreases. Figure 4 illustrates the effect of porous parameter (k) on u. It is clear from Figure 4 that the velocity raises as k increases. Figure 5 shows the influence of flow rate on velocity. It is observed that the increasing the flow rate leads to increase in the fluid velocity.

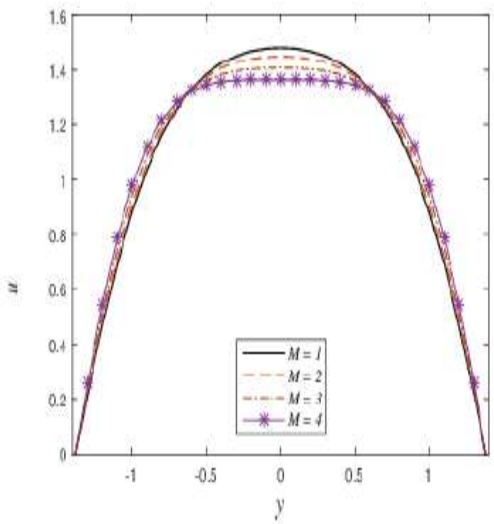


Figure 2: Hartmann number effect on u for k = 0.1, S = 2, Re = 1, τ^* = 0.3, Θ = 2, ε = 0.4, δ = 0.1.

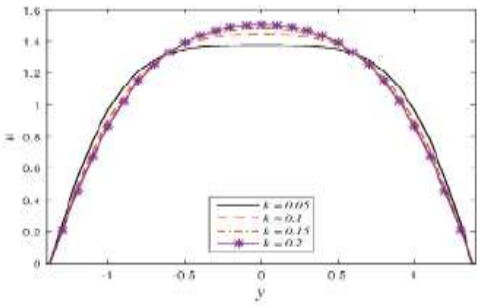


Figure 3: Couple stress parameter effect on u for k = 0.1, M = 2, Re = 1, τ^* = 0.3, Θ = 2, ε = 0.4, δ = 0.1.

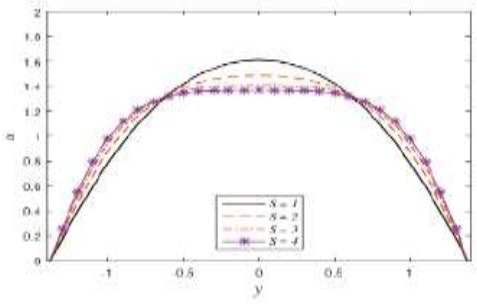


Figure 4: Porous parameter effect on u for M = 2, Re = 1, τ^* = 0.3, Θ = 2, ε = 0.4, δ = 0.1.

5.2. Pressure Gradient

Figures 6 to 8 illustrate the variation of M, k and S on dp/dx. dp/dx is plotted over one wave length. The effect of M on dp/dx is discussed in Figure 6. From this figure we get the information that in the wide part of the channel, the movement of the fluid is very easily through without any difficulty

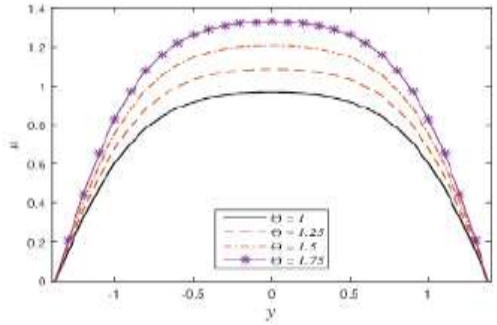


Figure 5: Mean flow rate effect on u for M = 2, Re = 1, τ^* = 0.3, ε = 0.4, δ = 0.1, k = 0.1.

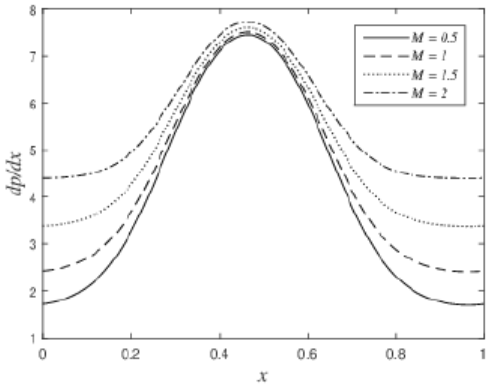


Figure 6: Effect of Hartmann number (M) on pressure gradient for ε = 0.4,Re = 1, t = 0.2, k = 0.1, S = 2, α = 0.4, Re = 1, τ^* = 0.3, Θ = 2, δ = 0.1.

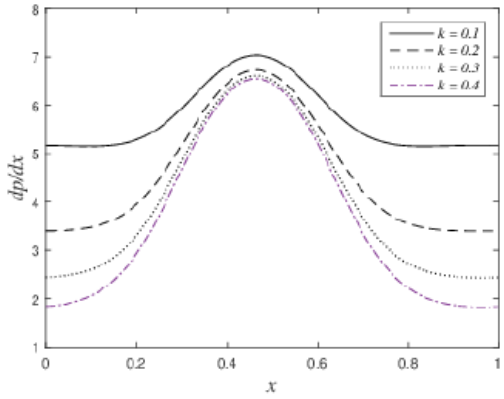


Figure 7: Effect of k on pressure gradient for ε = 0.4,Re = 1, t = 0.2, M = 2, S = 2, α = 0.4, Re = 1, τ^* = 0.3, Θ = 2, δ = 0.1.

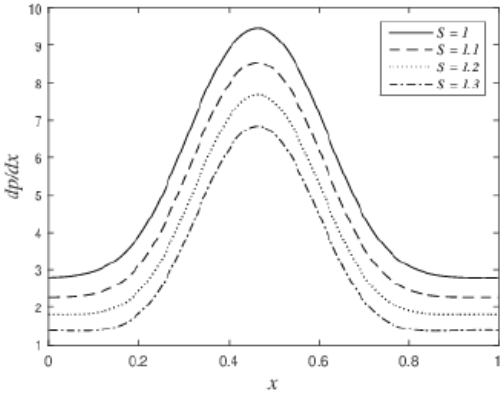


Figure 8: Effect of S on pressure gradient for ε = 0.4,Re = 1, t = 0.2, k = 0.1, α = 0.4, Re = 1, τ^* = 0.3, Θ = 2, δ = 0.1, M = 2.

6. Conclusion

The prime ambition of this section is to explore the influence of MHD on the two-dimensional peristalsis of a couple stress incompressible viscous fluid with solid particles. Here with list out all the obtained results of physical quantities on u, dp/dx and skin friction t .

1. Lorentz force defines that magnetic reduce the velocity by its force and velocity drop out with raise up M.
2. The profiles of velocity boost in time averaged flow rate (Q). The velocity profiles reduces with raise in the value of S and k.
3. In particular, we observed that by raising the value of M, which raise the value of dp/dx) more over the reverse trend observed for different values of k.
4. Decrease the value of skin friction with an enhancement of S and it increases while increasing M.

References

[1] Chaturani, P., (1978), "Viscosity of Poiseulle flow of fluid with couplestress with applications to blood flow," Biorheology, 15, pp. 119-128.

[2] Chaturani, P. and Rathod, V. P., (1981), "A circular steady of Poiseulle flow of Couple stress with applications to blood flow," Biorheology, 18, pp. 235-244.

[3] Dulal Pal, Rudraiah, N., and Devanathan, R., (1988), "A couple stress model of blood flow in the microcirculation," Bulletin of Mathematical Biology, 50, pp. 329-344.

[4] Brown, T.D., and Hung, T. K., (1977), "Computational and experimental investigations of two-dimensional nonlinear peristaltic flows," Journal of Fluid Mechanics, 83, pp. 249-272.

[5] Subba Reddy, M. H., Jayarami Reddy, B., Nagendra. N and Swaroopa, B., (2012), " Slip Effects on the peristaltic motion of a Jeffrey fluid through a porous medium in an asymmetric channel under the effect magnetic field," Journal of Applied Mathematics and Fluid Mechanics, 4, pp. 59-72.

[6] Ravikumar, S., and Siva Prasad, R., (2010), " Interaction of pulsatile flow on the peristaltic motion of couple stress fluid through porous medium in a flexible channel," European Journal of Pure and Applied Mathematics, 3, pp. 213-226.

[7] Raghunath Rao, T., and Prasada Rao, D. R. V., (2007), " Peristaltic transport of a couple stress fluid permeated with suspended particles," International Journal of Advances in Applied Mathematics and Mechanics, 1, pp. 86-102.

[8] Mekheimer, Kh .S., and Abd elmaboud, Y., (2008), " The influence of heat transfer and magnetic field on peristaltic transport of a Newtonian fluid in a vertical annulus: application of an endoscope," Physics Letters A, 372 , pp. 1657-1665.

[9] Sobh, A. M., (2008), " Interaction of couple stresses and slip flow on peristaltic transport in uniform and non-uniform channels," Turkish Journal Engineering and Environmental Science, 32, pp. 117-123.

[10] Valanis, J., and Sun, C. T., (1969), " Poiseulle flow of fluid with couple stress with applications to blood flow," Biorheology, 6, pp. 85-97.