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Research paper



Batch Arrival Queueing Model with Unreliable Server

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Abstract

The unreliable server with provision of temporary server in the context of application has been investigated. A temporary server is installed when the primary server is over loaded i.e., a fixed queue length of K-policy customers including the customer with the primary server has been build up. The primary server may breakdown while rendering service to the customers; it is sent for the repair. This type of queuing system has been investigated using Matrix Geometric Method to obtain the probabilities of the system steady state.
AMS subject classification number— 60K25, 60K30 and 90B22.

Keywords: *Busy State ; Idle state ; Matrix Geometric Method ; Repair State ; Stationary distribution,; Server breakdown.*

1. Introduction

In our daily routine activities that the provision of temporary server in case when the server load increases, can work a significant role to reduce the system capacity. The provision of additional temporary server is commonly done to improve the work load on a single server; this may also be useful in improving the waiting time of the customers. In our real life congestion problems the concept of installing temporary server finds many application such as telecommunication systems, computer protocols, web servers, admission counters, message transmission, dispensaries and so many other type of situations.

From the real life situations, heavy traffic analysis has been a challenging topic of investigation to the queue theorists. Eager et al (1986) studied a comparison of receiver-initiated and sender-initiated adaptive load sharing system using queue theoretic approach. Kushner (2001) introduced the analysis of heavy traffic of controlled queuing and communication networks. Leite and Fragoso (2010) analyzed the number of customers of a parallel system of two queues, operating under heavy traffic by formulating reflected stochastic differential equation. Also the theoretical works have been appeared on the queues with more than one server with respect to heavy traffic. Normally the secondary server is installed with an aim to reduce the waiting time of the customers and to increase the efficiency of the system in terms of faster service rendered. Neuts and Lucantoni (1979) analysed a Markovian queue with N servers subject to breakdowns and repairs.

A two server Markovian queue was studied by Krishna Kumar and Madheswari (2005) by using matrix geometric method. A multi server queuing model with Markovian arrival was analyzed by Chakravarthy (2007) and Parthasarathy and Sudhesh(2008), and Gharbi and Ioualalen(2010). Recently, Ke et al (2013) analyzed a multi server queuing system with multiple groups of servers. Unreliable queueing with repeated orders was introduced by Aissani (1993). Vinod (1985) studied unreliable queueing system. The stationary analysis of M/M/4/4+1 queueing model was studied by Kalayanraman and Seenivasan(2010). Renisagaya Raj. and Chandrasekar (2016) analysed Matrix – Geometric Method for queueing model with state-Dependent arrival of an unreliable server and PH service. Haridass and Arumuganathan (2008) studied analysis of a bulk queue with unreliable server and single vacation. They computed various system performance measures using matrix analytical method and also developed stability condition.

Queues associated with reattempt are a common phenomenon of day-to-day congestion situations. These queues are associated with the fact that a customer when deprived of service, is forced to join the virtual queue of the customers called orbit from where he can try again and again to get served. The matrix geometric method and provided a number of solutions to a variety of queuing problems using matrix geometric approach. Several queue theorists developed repetitive matrix block structured model to obtain solutions of various queuing problems using matrix geometric approach was introduced by Neuts (1978, 1981). Jain and Jain (2010) analyzed working vacation queues using matrix geometric approach. Bhargava and Jain (2013) employed matrix geometric method to investigate multi server unreliable queuing system with modified vacation policy. Indra and vijay rajan (2017) proposed the queueing analysis of markovian queue having two heterogeneous servers with catastrophes using matrix Geometric technique. The stationary analysis of m/m/5/5+1 queueing model with linear retrial rate was analyzed by Kalyanaraman and Seenivasan (2014). Renisagaya Raj and Chandrasekar (2015) studied Matrix – Geometric Method for queueing model with subject to breakdown and N-Policy vacations.

The content of this paper is the provision of additional temporary server and a fixed queue length of K-policy customers including the customer with the primary server may breakdown while rendering service to the customers; it is sent for the repair. These types of the model have been investigated. For this queue the Markov Process $\{ X(t) = (S(t), C_1(t), C_2(t)); t \geq 0\}$ has been defined and the infinitesimal matrix Q has been obtained. Using the matrix equation $XQ = 0$ and $Xe = 1$, the solution vector X is steady state, obtained using Matrix Geometric Method. The rest of the paper is organized as follows: The model description and governing equations are presented in section 2. In section 3, contains the analysis of the queuing model. The model analyzed using the numerical examples to given particular values of parameters in section 4. The last section contains a brief conclusion.

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2. The Model and Governing Equations

We consider unreliable server which is the primary server. The system has the provision of installing a second temporary server which is turned on when the number of customer with the first server threshold level. The various description studied in the model are discussed in the following subsections.

2.1 Model Description

The queuing model under consideration has the provision of two servers, out of which second temporary sever is activated only when the work load with the primary server crosses a prespecified level. The various type of assumptions are introducing are as follows:

Arrival Process

The customer in the system follows Poisson arrival with rate λ. There is a provision of two servers; the first is primary server and second one is temporary server. The second temporary server is installed only if K-policy customers are already queued up before the primary server including the one in the service. If an arriving customer finds less than K-policy customers with the primary server, then either customer wait for its turn in the queue with the primary server or customer may join the orbit. But if on arrival, the primary server's buffer is fully occupied with K-policy customers, then the new arrival has no other option rather than to join the buffer of secondary server.

Service process

The customers are served following exponential distribution with rate μ_i, if queued before i^{th} server (i=1 for primary server and i=2 for secondary server). The maximum number of customers joining the primary sever is K-policy i.e. a buffer of fixed capacity K is provided for the primary server. However the number of customer joining the secondary server is unlimited. Both the servers have their own independent queues but the formation of second queue takes place when the buffer of primary server is full. Queue shifting is not permitted to the customers once they join it.

Breakdown and Repair process

The primary server is unreliable and may breakdown while serving the customer; the broken down server is sent for repair immediately and after repair, it becomes as good as before failure. However, temporary second server is considered as reliable server. The inter failure time of the primary sever follows exponential distribution with rate α_1. The repair time of the primary sever follows exponential distribution with rate β_1.

The Markov process is $\{X(t) = (S(t),C_1(t),C_2(t)); t \geq 0\}$

To describe the state of the system at any instant, we consider the following three random variables that describe the system completely.

(i) where S(t) denote the server state at time t,

S(t) = 0 if the sever is idle,

S(t) =1 if the sever is busy,

S(t) = 2 if the sever is breakdown.

(ii) where $C_1(t)$ denote the number of customer with the first server, such that

$C_1(t)=i$, $(0\leq i\leq K)$.

(iii) where $C_2(t)$ denote the number of customer with the second server, such that $C_2(t)=j$, $j\geq 0$.

The state space of process is $\{0,1, 2\} \times \{0, 1, 2 ...K\} \times \{0, 1, 2 ...\}$.

2.2 Governing Equation

We frame the steady state equation the model by using appropriate transition rate of birth death process. The Chapman-Kolmogrov equations corresponding to different system states are formulated as:

Idle state

$$0=\lambda X_{0,0,0}+\mu_1 X_{1,0,0} \quad (1)$$

$$0=\lambda X_{0,i-1,0}+\mu_1 X_{1,i-1,0} \quad (2)$$

$$(\lambda+\mu_1)X_{0,K,0}=\lambda X_{0,K-1,0} \quad (3)$$

$$0=\lambda X_{0,K,0}+\mu_2 X_{1,K,0} \quad (4)$$

Busy state

$$(\lambda+\alpha_1)X_{1,0,0}=\mu_1 X_{1,1,0}+\beta_1 X_{2,0,0}+\mu_2 X_{1,0,1} \quad (5)$$

$$(\lambda+\alpha_1+\mu_1)X_{1,i,0}=\mu_1 X_{1,i+1,0}+\beta_1 X_{2,i,0}+\mu_2 X_{1,i,1}+\lambda X_{1,i-1,0} \quad (1\leq i\leq K\text{-}1) \quad (6)$$

$$(\lambda+\alpha_1+\mu_1)X_{1,K,0}=\mu_2 X_{1,K,1}+\beta_1 X_{2,K,0}+\lambda X_{1,K-1,0} \quad (7)$$

$$(\lambda+\alpha_1+\mu_2)X_{1,0,j}=\mu_2 X_{1,0,j+1}+\beta_1 X_{2,0,j}+\lambda X_{1,0,j-1}+\mu_1 X_{1,1,j}, \quad j\geq 1 \quad (8)$$

$$(\lambda+\alpha_1+\mu_1+\mu_2)X_{1,i,j}=\mu_1 X_{1,i+1,j}+\beta_1 X_{2,i,j}+\mu_2 X_{1,i,j+1}+\lambda X_{1,i-1,j}+\lambda X_{1,i,j-1}, \quad (1\leq i\leq K\text{-}1),\ j\geq 1 \quad (9)$$

$$(\lambda+\alpha_1+\mu_1+\mu_2)X_{1,K,j}=\mu_2 X_{1,K,j+1}+\beta_1 X_{2,K,j}+\lambda X_{1,K-1,j}+\lambda X_{1,K,j-1}, j\geq 1 \quad (10)$$

Repair state

$$(\lambda+\beta_1)X_{2,0,j}=\alpha_1 X_{1,0,j}, \quad j\geq 1 \quad (11)$$

$$(\lambda+\beta_1)X_{2,i,j}=\alpha_1 X_{1,i,j}+\lambda X_{2,i-1,j}, \quad (1\leq I\leq K\text{-}1), \quad j\geq 1 \quad (12)$$

$$(\lambda+\beta_1)X_{2,K,j}=\alpha_1 X_{1,K,j}+\lambda X_{2,K,j-1}+\lambda X_{2,K-1,j}, \quad j\geq 1 \quad (13)$$

In order to determine the solution of equations (1) to (13), we shall employ matrix geometric method as explained in the section 3.

3. The Analysis

The matrix geometric method (cf. Neuts (1981)) can be used to solve the stationary state probabilities for the vector space markov process with repetitive structure. Therefore, in order to find the solution for the system of equations constructed in section 2.2, we consider this technique to determine the associated state probability vectors.

3.1 Matrix Geometric Method

The matrix geometric method to determine the probability vector is applicable for the system of equations whose transition matrices have special block structure with repetition of elements of sub matrices. The concerned model can be structured as a square matrix of infinite dimension that converges to finite dimension matrix using the minimal matrix to get recursive relation of probability vectors. The above set of equations (1)-(13) can be written in matrix form as $\pi Q=0$, where Q is the infinitesimal generator of the continuous time Markov chain and '0' is a zero column vector of suitable dimension. Also, let $\pi = (\pi_0, \pi_1, \pi_2, \pi_3,....)$ be the vector defining the steady state probabilities of all the governing states of the system under consideration.

The matrix Q can be given in partition form as

$$Q=\begin{bmatrix} F_0 & F_1 & 0 & 0 & 0 & 0 & 0 & ... \\ F_2 & F_3 & F_4 & 0 & 0 & 0 & 0 & ... \\ 0 & F_5 & F_3 & F_4 & 0 & 0 & 0 & ... \\ 0 & 0 & F_5 & F_3 & F_4 & 0 & 0 & ... \\ 0 & 0 & 0 & F_5 & F_3 & F_4 & 0 & ... \\ M & M & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (14)$$

$F_0=A_1$; $F_1=\begin{bmatrix} B_1 & C_1 \end{bmatrix}$; $F_2=\begin{bmatrix} D_1 \\ G_1 \end{bmatrix}$;

$$F_3 = \begin{bmatrix} E_1 & 0 \\ 0 & H_1 \end{bmatrix}_{(2n+2)\times(2n+2)} \quad F_4 = \begin{bmatrix} C_1 \\ B_1 \end{bmatrix} \; ; \quad F_5 = \begin{bmatrix} D_1 \\ G_1 \end{bmatrix}$$

$$B_1 = \text{diag.}\left[\alpha_1\right]_{(n+1)\times(n+1)} \; ; \quad D_1 = \text{diag.}\left[\beta_1\right]_{(n+1)\times(n+1)} \; ;$$

$$G_1 = \text{diag.}\left[\mu_2\right]_{(n+1)\times(n+1)}$$

$$A_1 = \begin{bmatrix} -(\lambda+\alpha_1) & \lambda & 0 & 0 & \dots & 0 \\ \mu_1 & -(\lambda+\alpha_1+\mu_1) & \lambda & 0 & \dots & 0 \\ 0 & \mu_1 & -(\lambda+\alpha_1+\mu_1) & \lambda & \dots & 0 \\ M & 0 & \mu_1 & -(\lambda+\alpha_1+\mu_1) & \lambda & 0 \\ 0 & 0 & 0 & \mu_1 & -(\lambda+\alpha_1+\mu_1) & \lambda \\ 0 & 0 & 0 & 0 & \mu_1 & -(\lambda+\alpha_1+\mu_1) \end{bmatrix}_{(n+1)\times(n+1)}$$

$$E_1 = \begin{bmatrix} -(\lambda+\beta_1) & \lambda & 0 & 0 & \dots & 0 \\ 0 & -(\lambda+\beta_1) & \lambda & 0 & \dots & 0 \\ 0 & 0 & -(\lambda+\beta_1) & \lambda & \dots & 0 \\ M & 0 & 0 & -(\lambda+\beta_1) & \lambda & 0 \\ 0 & 0 & 0 & 0 & -(\lambda+\beta_1) & \lambda \\ 0 & 0 & 0 & 0 & 0 & -(\lambda+\beta_1) \end{bmatrix}_{(n+1)\times(n+1)}$$

$$C_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ M & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix}_{(n+1)\times(n+1)} ; B_1 = \begin{bmatrix} \alpha_1 & 0 & 0 & 0 & \dots & 0 \\ 0 & \alpha_1 & 0 & 0 & \dots & 0 \\ 0 & 0 & \alpha_1 & 0 & \dots & 0 \\ M & 0 & 0 & \alpha_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha_1 \end{bmatrix}_{(n+1)\times(n+1)}$$

$$D_1 = \begin{bmatrix} \beta_1 & 0 & 0 & 0 & \dots & 0 \\ 0 & \beta_1 & 0 & 0 & \dots & 0 \\ 0 & 0 & \beta_1 & 0 & \dots & 0 \\ M & 0 & 0 & \beta_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_1 \end{bmatrix}_{(n+1)\times(n+1)} ; G_1 = \begin{bmatrix} \mu_2 & 0 & 0 & 0 & \dots & 0 \\ 0 & \mu_2 & 0 & 0 & \dots & 0 \\ 0 & 0 & \mu_2 & 0 & \dots & 0 \\ M & 0 & 0 & \mu_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu_2 \end{bmatrix}_{(n+1)\times(n+1)}$$

The normalizing condition is represented by Π e=1, where 'e' is a column vector of suitable dimension with all its entries as 1.

In order to determine the probability vector, we partition vector π conformably with the block of matrix Q as

$$\pi_0 = (X_{0,0,0}, X_{1,0,0}; X_{0,1,0}, X_{1,1,0}; \dots; X_{0,k,0}, X_{1,k,0}); \; X_{0,0,0} = 0 \qquad (15)$$

$$\pi_1 = (X_{2,0,0}, X_{1,0,1}; X_{2,1,0}, X_{1,1,1}; \dots; X_{2,k,0}, X_{1,k,1})$$

$$\pi_2 = (X_{2,0,1}, X_{1,0,2}; X_{2,1,1}, X_{1,1,2}; \dots; X_{2,k,1}, X_{1,k,2})$$

$$\vdots$$

$$\pi_j = (X_{2,0,j-1}, X_{1,0,j}; X_{2,1,j-1}, X_{1,1,j}; \dots; X_{2,k,j-1}, X_{1,k,j}); \quad j \geq 1 \qquad (16)$$

Using matrix geometric approach (cf.Neuts, 1981), we have

$$\pi_j = \pi_1 R^{j-1}, (j \geq 2) \qquad (17)$$

where, R is the minimal non-negative matrix known as rate matrix. The balance equation for the repeating states is

$$\pi_{j-1}F_4 + \pi_j F_3 + \pi_{j+1}F_5 = 0; \quad j = 2, 3, 4, \qquad (18)$$

The balance equations for the boundary states are

$$\pi_0 F_0 + \pi_1 F_2 = 0 \qquad (19)$$

$$\pi_0 F_1 + \pi_1 F_3 + \pi_2 F_5 = 0 \qquad (20)$$

The value π_j, $(j \geq 2)$ is a probability function of the transition between the states with j-1 queued customer and states with j queued customers using (17) and (18), We have

$$\pi_1 R^{j-2} F_4 + \pi_1 R^{j-1} F_3 + \pi_1 R^j F_5 = 0; \quad j = 2,3,4,....$$

$$\Rightarrow F_4 + RF_3 + R^2 F_5 = 0 \qquad (21)$$

On solving equation (21), we get the rate matrix R, which can be further
Used to compute steady state probability for repeating states.
Now,
Using equation(17) in equation(20) for j = 2, the balance equations for
Boundary states reduces to- $\pi_0 F_0 + \pi_1 F_2 = 0$

$\pi_0 F_1 + \pi_1 (F_3 + RF_5) = 0$ and can be further written in matrix form as

$$\begin{pmatrix} \pi_0 & \pi_1 \end{pmatrix} \begin{bmatrix} F_0 & F_1 \\ F_2 & F_3 + RF_5 \end{bmatrix} = 0$$

In order to find π_0, we use normalizing

$$\pi_0 e + \pi_1 e + \sum_{j=2}^{\infty} \pi_j e = 1 \qquad (22)$$

this, further gives $\pi_0 e + \pi_1 e + \left(\sum_{j=0}^{\infty} R_j \right) e = 1 \qquad (23)$

The Eigen values of R lie inside the unit circle which means that (I-R) is

non-singular We have $\left(\sum_{j=0}^{\infty} R_j \right) = (I - R)^{-1}. \qquad (24)$

We first obtain the rate matrix R by iterative procedure as R(0)=0, and
move further with successive approximation as:

$$R(n+1) = -\left[F_4 + R^2 F_5\right] F_3^{-1}, \quad n \geq 0 \qquad (25)$$

Since, $-F_3^{-1}$ is non-negative matrix, therefore it can be conclude that
the sequence $\{R(n)\}_n$ is a non-decreasing sequence which converges
monotonically to a non-negative matrix R. The stage when $\| R(n+1) - R(n) \| < \varepsilon$ (ε is a constant) is satisfied ,we terminate the
solution process and obtain R which further also helps in determining
the steady state probabilities numerically.

4. Numerical Study

We now present numerical results related to the model discussed in the above section. We take the parameters are
$\lambda = 0.01$, $\mu_1 = 2$, $\mu_2 = 4$, $\alpha_1 = 0.1$, $\beta_1 = 1$, K = 5. Various sub matrices and rate matrix R are computed as:

$$F_0 = \begin{bmatrix} -0.11 & 0.01 & 0 & 0 & 0 \\ 2 & -2.11 & 0.01 & 0 & 0 \\ 0 & 2 & -2.11 & 0.01 & 0 \\ 0 & 0 & 2 & -2.11 & 0.01 \\ 0 & 0 & 0 & 2 & -2.11 \end{bmatrix}$$

$$F_1 = \begin{bmatrix} 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1 & 0 & 0 & 0 & 0 & 0.3 \end{bmatrix}$$

$$F_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

$$F_3 = \begin{bmatrix} -1.01 & 0.01 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1.01 & 0.01 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1.01 & 0.01 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1.01 & 0.01 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1.01 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -4.4 & 0.3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -6.4 & 0.3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -6.4 & 0.3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & -6.4 & 0.3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & -6.4 \end{bmatrix}$$

$$F_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.01 \\ 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0.1 \end{bmatrix}; \quad F_5 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

Using different sub-matrices as obtained, the rate matrix R is computed as-

$$R = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.028 & 0.082 & 0.451 & 0.751 & 0.811 & 0.212 & 0.178 & 0.002 & 0.711 & 0.824 \\ 0.141 & 0.241 & 0.714 & 0.841 & 0.942 & 0.412 & 0.072 & 0.085 & 0.164 & 0.182 \\ 0.756 & 0.721 & 0.624 & 0.682 & 0.448 & 0.521 & 0.211 & 0.241 & 0.208 & 0.192 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

By using the above R matrix, the probability vectors are calculated from equation (17) and equation (25) and the normalizing condition. Our objective is to demonstrate the effect of the parameters on the Probabilities by varying λ_i's, are given below in the Figure 1: We can take the λ vary from the value 0.01 to 0.10

5. Conclusion

In this paper, we have considered unreliable server. We have obtained the steady state probability vector by applying matrix geometric method. Furthermore, we have performed numerical analysis by assuming particular values to the parameter. It is verified that the total probability is ≈1.

Λ	0.01	0.02	0.03	0.04	0.06	0.07	0.08	0.09	0.10
X_{00}	0.32	0.31	0.32	0.32	0.32	0.32	0.32	0.31	0.31
$_0$	1	9	5	4	8	7	0	1	8
X_{01}	0.12	0.12	0.12	0.11	0.12	0.11	0.11	0.12	0.12
$_0$	2	4	6	9	0	6	4	5	0
X_{02}	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.00	0.00
$_0$	5	3	1	8	2	1	3	5	4
X_{11}	0.12	0.12	0.11	0.12	0.11	0.11	0.12	0.11	0.11
$_0$	3	5	9	0	5	7	5	3	3
X_{12}	0.05	0.05	0.05	0.05	0.06	0.06	0.06	0.04	0.05
$_0$	6	4	0	9	3	2	4	6	5
X_{13}	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00
$_0$	2	4	8	9	5	6	4	2	3
X_{11}	0.09	0.09	0.10	0.10	0.09	0.10	0.09	0.09	0.10
$_1$	8	6	2	1	1	4	0	0	7
X_{11}	0.00	0.01	0.01	0.00	0.00	0.00	0.01	0.00	0.00
$_2$	9	1	3	6	2	3	0	0	0
X_{11}	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.00
$_3$	6	4	0	9	3	2	4	2	9
X_{12}	0.02	0.02	0.02	0.02	0.01	0.02	0.01	0.01	0.01
$_1$	6	8	4	3	9	0	8	6	7
X_{12}	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01
$_2$	7	5	9	0	4	3	5	7	6
X_{12}	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
$_3$	6	8	2	3	0	0	4	3	1
X_{13}	0.01	0.01	0.01	0.02	0.02	0.01	0.01	0.02	0.02
$_1$	7	5	1	0	4	1	5	2	6
X_{13}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$_2$	2	4	8	0	2	8	6	4	1
X_{13}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.01
$_3$	1	0	2	4	8	3	9	1	0
X_{15}	0.01	0.01	0.01	0.00	0.00	0.01	0.00	0.00	0.00
$_1$	2	4	6	9	5	8	4	2	3
X_{15}	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.01
$_2$	5	3	7	8	2	4	3	3	4
X_{15}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$_3$	3	5	1	6	2	9	0	1	7
X_{20}	0.02	0.02	0.02	0.02	0.03	0.01	0.02	0.03	0.03
$_0$	3	1	5	6	0	7	9	0	2
X_{20}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
$_1$	2	4	8	5	6	8	9	5	0
X_{20}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00
$_2$	0	2	6	3	7	1	8	0	9
X_{21}	0.02	0.02	0.01	0.01	0.01	0.02	0.00	0.02	0.01
$_0$	1	3	9	8	4	7	3	1	2
X_{21}	0.02	0.01	0.01	0.03	0.02	0.02	0.02	0.02	0.02
$_1$	8	6	0	1	5	2	6	8	7
X_{21}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00
$_2$	3	5	9	6	2	9	4	1	1
X_{22}	0.02	0.02	0.02	0.02	0.03	0.01	0.02	0.01	0.03
$_0$	5	3	7	8	0	9	1	5	4
X_{22}	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00
$_1$	6	8	4	3	3	2	2	4	9
X_{22}	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.01
$_2$	2	1	5	5	9	4	0	2	1
X_{24}	0.01	0.02	0.02	0.01	0.01	0.02	0.00	0.00	0.00
$_0$	8	0	5	5	1	4	8	8	9
X_{24}	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.01	0.02
$_1$	6	4	8	9	2	0	7	6	5
X_{24}	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.00
$_2$	4	6	2	1	9	0	8	5	3

Figure 1: (Probability Vectors)

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