International Journal of Engineering & Technology, 7 (4.10) (2018) 81-84

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

Non-Linear Mathematical Modelling for Phase Locked Loop

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Abstract

Design of Phase Locked Loop (PLL) plays a vital role in transceiver field. Phase Locked Loop comprises of three blocks, namely Phase and frequency detector, loop filter and voltage-controlled oscillator. The greater advancements in CMOS technology such as high frequency, high speed, low noise and phase error leads to low-cost PLL This work aims to develop higher order non-linear models of general Phase Locked Loop. The condition of stability and choice of loop filter is also determined. Based on the analysis, the transfer function for PLL is determined.

Keywords: *Phase Locked Loop; Nonlinear model; Stability*

1. Introduction

The Phase Locked Loop (PLL) based circuits are commonly used in applications such as wireless applications, digital circuits. PLL is a non-linear feedback system [1]. PLL is one of the important blocks in communication applications. It is used to generate a signal where it compares two signals (i) input reference signal (ii) feedback signal such that both the signal phase should be matched. The analysis of PLL can be done by two ways (i)linear, (ii) non-linear. The drawback of linear analysis is failed to detect many characteristics like latch-up, hysteresis jumps. This work aims to develop the non-linear model of PLL and the condition for stability.

2. Phase Locked Loop (PLL)

The Phase Locked Loop contains a phase detector (PD), loop filter (LF) and voltage-controlled oscillator (VCO) as shown in fig 1. The phase detector compares the two input signals such as input reference signal and a feedback signal to produce the error signal (i.e difference between two signals). The loop filter attenuates the noise and needless phase detector outputs. The loop filter transfer function is F(s). The voltage-controlled oscillator (VCO) give raise to a sinusoidal signal.
The phase detector evaluates the phase and frequency of the input reference signal s(t,ϕ) to the phase and frequency of the feedback signal r(t, ϕ) [2]. It produces an error signal $v_d(t)$. This $v_d(t)$ is given to the loop filter F(s) in order to attenuate the noise and needless component of the input spectrum. This block produces the output of $v_f(t)$. The addition of $v_f(t)$ and an external control voltage $v_c(t)$which controls the instantaneous VCO frequency. The phase detector must produce the zero-output voltage to get phase locked of two signals.
The PLL characteristics are (i) acquisition and tracking (ii) pull-in range (iii) lock-in range.

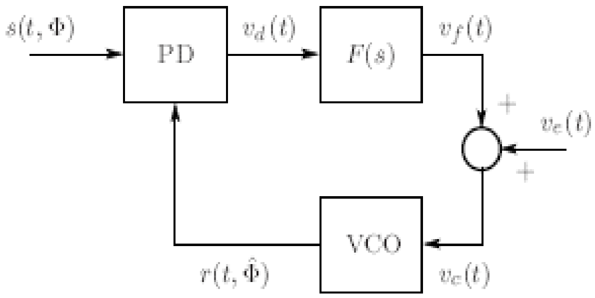

Fig. 1: Basic Diagram of PLL

2.1. Acquisition and Tracking

The phase of the input signal has been tracked by PLL. In general, the VCO operates at center frequency ω_θ which differs from the frequency of the input signal ω_i. The VCO has to be tuned to the input frequency by the PLL called frequency pull in. Then the VCO has to be varied accordingly to the input phase. This is called phase lock-in.
The combination of frequency pull in & phase lock is called as Acquisition. The acquisition is a non-linear process. After the process of acquisition, PLL achieves the phase locked condition, where both input reference signal and feedback signal are in phase.

2.2. Pull-in Range

$\Delta\omega_p = |\omega_i - \omega_o|$ is the maximum initial frequency difference between the input reference signal and VCO output (or) feedback signal. For this phase difference, PLL acquires the Phase Locked condition [3]

2.3. Lock-in Range

$\Delta\omega_L = |\omega_i - \omega_o|$ is the frequency range over which the PLL acquires the condition of phase locked [4].

3. PLL Model

Let the phase φ(t) of input reference signal s(t,φ) can be expressed with respect to the centre frequency ω_0 as

$$\Phi = \omega_0 t + \theta_i \quad (1)$$

Input signal of the PLL,

$$S(t, \Phi) = \sqrt{2}A \sin \phi \quad (2)$$

$$S(t, \Phi) = \sqrt{2}A \sin (\omega_0 t + \theta_i) \quad (3)$$

A(t) describes the modulation of the amplitude to the reference input signal, $\theta_i(t)$ is the modulation of input phase. Since the VCO phase φ(t) tracks the phase of the input signal φ(t). VCO centre frequency becomes,

$$\varphi = \omega_0 t + \theta_0 \quad (4)$$

$$r(t, \varphi) = \sqrt{2}V_0 \cos \varphi \quad (5)$$

$$r(t, \varphi) = \sqrt{2}V_0 \cos \omega_0 t + \theta_0 \quad (6)$$

θ_0(t) is the phase of VCO or feedback output V0 is the rms amplitude of the VCO output.

3.1 Transfer Function of VCO

The instantaneous of VCO frequency referred as ω_0 By changing the control voltage, the frequency of VCO changes from ω_0.

$$\frac{d\varphi}{dt} - \omega_0 = \frac{d}{dt}[\omega_0 t - \theta_0 t] - \omega_0 \quad (7)$$

$$\frac{d\varphi}{dt} - \omega_0 = \frac{d\theta_0}{dt} \quad (8)$$

$$\frac{d\varphi}{dt} - \omega_0 = K_v v_c \quad (9)$$

K_v is the VCO gain in the rad/s. If the control voltage $V_c(t)$=0, it equals to the center frequency.

3.2 Transfer Frequency of Phase Detector

The Phase detector produces the difference between the input reference signal $S(t, \Phi) = \sqrt{2}A \sin (\omega_0 t + \theta_i)$ and feedback signal $r(t, \phi) = \sqrt{2}V_0 cos\omega_0 t + \theta_0$ and produces the difference signal.

$$v_d = k_d A \sin\theta_e \quad (10)$$

Phase error is defined as,

$$\theta_e(t) = \theta_i(t) - \theta_o(t) \quad (11)$$

From above equation, the output depends on the difference of two input signals. It also depends on the A(t) i.e., on the AM of the input signal.

3.3 Transfer Function of Loop Filter

The addition of loop filter output $v_f(t)$ and external control voltage $v_e(t)$ gives VCO control voltage $v_c(t)$.

$$v_c(t) = v_f(t) + v_e(t) \quad (12)$$

To write the differential equation in simple form, the d/dt in the time domain will be expressed as multiplication of Headviside operator P.

$$v_c(t) = F(p)v_d(t) + v_e(t) \quad (13)$$

If a PLL operates in steady state (t=∞), the PLL is operating in the Quiescent point.

$$\theta_e = \theta_i - \theta_0 \quad (14)$$

$$\theta_e = \theta_i - \frac{kF(p)}{p} A\sin \theta_e - \frac{k_v}{p} v_e \quad (15)$$

$$p\theta_e = p\theta_i - kF(p)A\sin \theta_e - k_v v_e \quad (16)$$

Under steady state condition, all signals are constant but phase error may present.

$$\theta_e(t) = \theta_{ss} \quad (17)$$

$$v_e(t) = v_e o \quad (18)$$

$$\theta_i(t) = (\omega_i - \omega_0)t + \theta_{i0} \quad (19)$$

$$\theta_i(t) = \Delta\omega_i t + \theta_{i0} \quad (20)$$

Deriving θ_e,

$$\theta = \Delta\omega_i - KF(\theta)A\sin\theta_{ss} - k_v v_e \theta \quad (21)$$

Quiescent point $\theta_{ss}$of PLL is obtained as,

$$\theta_{ss} = \sin^{-1}\frac{\Delta\omega_i - k_v v_e}{k(0)A} \quad (22)$$

F(0) is the dc gain of loop filter. To get the excellent performance of PLL, the Q value of phase error has to be zero, θ_{ss}=0. To achieve θ_{ss}=0, by making the dc gain of loop filter F(0) tends to ∞. This can be achieved by using loop filter. To make the non-linear model to linear model, Taylor series approximation has been used.

$$y = g(\theta) = g(\theta_{ss}) + \frac{1}{1!}\frac{dg(\theta)}{d\theta}\bigg|_{(\theta_{ss})(\theta-\theta_{ss})} + \cdots + \frac{1}{n!}\frac{dg^n(\theta)}{d\theta^n}\bigg|_{(\theta_{ss})(\theta-\theta_{ss})^n} \quad (23)$$

For small signal model, the only linear term is considered,

$$\Delta y = y - g(\theta_{ss}) = \frac{1}{1!}\frac{dg(\theta)}{d\theta}\bigg|_{(\theta_{ss})(\theta-\theta_{ss})} \quad (24)$$

$$\Delta y = y - g(\theta_{ss}) = \frac{dg(\theta)}{d\theta}\bigg|_{(\theta_{ss})} \Delta_\theta \quad (25)$$

Where $\frac{dg(\theta)}{d\theta}\Big|_{(\theta_{ss})}$ is the tangent of the non linear function f(θ) at the q point θ_{ss}. Δy and Δ_θ are called perturbations. If θ_{ss}=0, g(0)=0,

$$y = \frac{dg(\theta)}{d\theta}\Big|_{(\theta_{ss})}\theta \qquad (26)$$

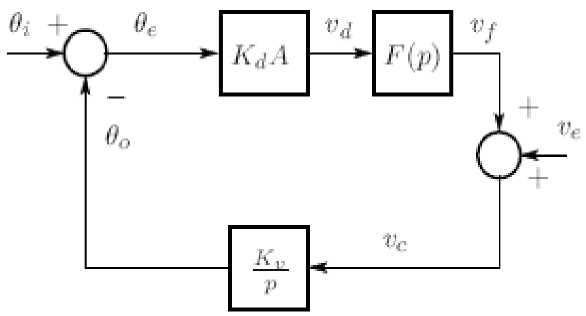

Figure 2: Linear baseband model of PLL

4. Linearization of Non-Linear Baseband Model

The phase detector is the only non-linear component $v_d = k_dA\ sin\theta_e$. Since θ_{ss}=0, $v_d(0) = 0$, $\Delta v_d = v_d$, $\Delta\theta_e = \theta_e$. The phase detector can be linearized,

$$v_d = k_dA\ sin\theta_e = k_dA\theta_e \qquad (27)$$

$$\theta_0 = \frac{kF(p)}{p}A\theta_e + \frac{k_v}{p}v_e \qquad (28)$$

$$\theta_0 = \frac{k_v}{p}[F(p)k_dA\theta_e + v_e] \qquad (29)$$

The last equation defines the linearized PLL equation in time domain. The next step is to convert the signal to s domain by help of Laplace transform.

$$\phi_i(s) = L\theta_i(t) \qquad (30)$$

$$\phi_o(s) = \frac{kF(s)}{S}A\phi_e(s) + \frac{k_v}{s}v_e(s) \qquad (31)$$

$$v_c(s) = 0,$$
$$\phi_e(s) = \phi_i(s) - \phi_0(s) \qquad (32)$$

$$\phi_o(s) = \frac{(AkF(s))}{S}[\phi_i(s) - \phi_0(s)] \qquad (33)$$

$$\left[1 + \frac{(AkF(s))}{S}\right]\phi_o(s) = \frac{(AkF(s))}{S}\phi_i(s) \qquad (34)$$

$$\frac{(\phi_o(s))}{(\phi_i(s))} = \frac{(AkF(s))}{S + AkF(s)} = H(s)\ \phi_i(s) \qquad (35)$$

H(s) is called closed loop transfer function.

$$\phi_e(s) = [1 - H(s)][\phi_i(s) - \frac{k_v}{s}v_e(s) \qquad (36)$$

$$\phi_o(s) = H(s)\phi_i(s) + [1 - H(s)]\frac{k_v}{s}v_e(s) \qquad (37)$$

[1-H(s)] is the error function. The transfer function of PLL is given as

$$H(s) = \frac{AkF(s)}{(s + AkF(s))} \qquad (38)$$

The error function of PLL is given as

$$1 - H(s) = \frac{s}{s + AkF(s)} \qquad (39)$$

5. Implementation of PLL with Active Loop Filter

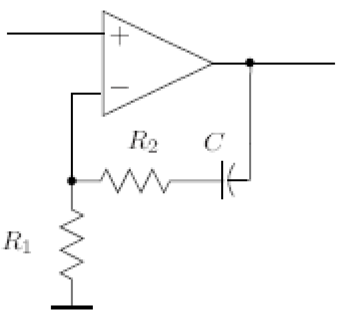

Figure 3: Active Loop Filter

In the loop filter, if the ideal operational amplifier used, then the transfer function of loop filter is,

$$F(s) = \frac{(1 + s\tau_1)}{\tau_2} \qquad (40)$$

Where, $\tau_1 = R_1C$,$\tau_2 = (R_1 + R_2)C$. The DC gain of ideal op amp is infinite, consequently the steady state phase error is θ_{ss}=0. The closed loop transfer function of PLL is

$$H(s) = \frac{(2\varepsilon\omega_n s + \omega_n{}^2)}{(S^2 + 2\varepsilon\omega_n s + \omega_n{}^2)} \qquad (41)$$

Where ω_n is the natural frequency, ε is the damping factor. The error function of PLL with active loop filter is

$$1 - H(s) = \frac{S^2}{(S^2 + 2\varepsilon\omega_n s + \omega_n{}^2)} \qquad (42)$$

A system is called stable when the all poles of transfer function lies in the left side of the S plane. From equation number 41, The characteristic equation is

$$(S^2 + 2\varepsilon\omega_n s + \omega_n{}^2) = 0 \qquad (43)$$

From the equation number 43, the two poles of PLL with active filter is lies on the left half plane. Si it leads to the circuit becomes more stable.

6. Conclusion

This paper presented the Non-linear mathematical modelling of Phase Locked Loop, Findings of the modelling are: (i) From the Characteristic equation, the two poles of PLL with active filter is lies on the left half plane. So, it leads to the circuit becomes more stable. (ii) Even though the amplitude of input reference signal varies continuously and changes the variables of closed loop, PLL is stable.

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