



# Analysis of Non Constant Viscosity on Avertical Oscillating Cylinder with Mhd Effects

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## Abstract

A numerical analysis was carried out on convective flow past a vertical oscillating cylinder with non-constant viscosity under the influence of magnetic field effects. The governing equations related to the flow problem are unsteady, nonlinear coupled partial differential equations. The same was transformed into a dimensionless form and the final form of equations was solved numerically. The approximate values of velocity, temperature, skin friction and Nusselt number are obtained. The viscosity considered as a function of temperature.

**Keywords:** Magnetic field, Oscillating, Skin friction, Velocity, Viscosity.

## 1. Introduction

The first exact solution of Navier-Stokes equation was given by Stokes [1] which is concerned with flow of viscous incompressible past an horizontal plate oscillating in its own plane. When the vertical isothermal plate is oscillating in its own plane, in a viscous fluid, how the fluid is affected by free convection currents? it was first examined by Soundalgekar [2]. Revankar [3] analysed the same problem for an impulsively started or oscillating plate. Turbatu *et al.* [4] investigated the flow of an incompressible viscous fluid past an infinite oscillating with increasing or decreasing velocity amplitude of oscillation. Soundalgekar [5] gave an exact solution for magnetic free convection flow past an oscillating plate..

All the above research were restricted to constant viscosity. However the non-constant viscosity is important in the case of high temperature (Schlichting [6], Kakak *et al.* [7], Cengel [8]). Kafoussius and Rees [9] the mixed convection boundary layer flow over a vertical plate with temperature dependent viscosity. Elbashbeshy and Bazid [10] considered the variable viscosity and shown the importance for considering the variable viscosity, *i.e.*, the flow fields are effected significantly compared to the constant viscosity. Cengel [8] and Touloukain *et al.* [11] derived an empirical relationship between the viscosity and temperature.

However, the influence of thermal radiation by natural convection flow past a semi-infinite vertical oscillating cylinder subjected to a constant heat flux has not received the attention of any researcher. It is now proposed to study the effects of oscillating vertical cylinder with thermal radiation and free convection currents on the flow of a viscous fluid which is stationary and extending to infinity. As the leading edge effects are important in such a problem. It is interesting to know the effects of oscillating cylinder and the free convection currents on the leading edge effects. The present

study will be found useful in the change of space ships.

## 2. Basic Equations and Mathematical Analysis

A numerical analysis is made of the flow generated in a viscous incompressible fluid past a semi-infinite vertical oscillating cylinder of radius  $r_0$ . We choose the  $x$ - axis is taken along the cylinder in the upward direction and  $r$  to be chosen along the radial direction *i.e.*, normal to the vertical cylinder. Further we assume that the fluid and the surrounding temperature are the same  $T_\infty'$ . As  $t' > 0$  the cylinder starts oscillating in the vertical direction with velocity  $u_0 \cos \omega t'$  and the temperature of the cylinder is

maintained at  $T_w'$  as a result it raise in the buoyancy force. Also, we are not considered the effect of viscous dissipation in the energy equation. With the Boussinesq approximation, the boundary layer equations governing the natural convection flow can be governed by the following set of equations:

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial r} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t'} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = g\beta(T' - T_\infty') + \frac{1}{\rho r} \frac{\partial}{\partial r} \left( \mu r \frac{\partial u}{\partial r} \right) - \frac{\sigma B_0^2}{\rho} u \quad (2)$$

$$\frac{\partial T'}{\partial t'} + u \frac{\partial T'}{\partial x} + v \frac{\partial T'}{\partial r} = \frac{\alpha}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T'}{\partial r} \right) \quad (3)$$

$$\begin{aligned}
 t' \leq 0: u = 0, \quad v = 0, \quad T' = T_\infty' \quad \text{for all } x \text{ and } r \geq r_0 \\
 t' > 0: u = u_0 \cos \omega t', \quad v = 0, \quad T' = T_w' \quad \text{at } r = r_0 \\
 u = 0 \quad T' = T_\infty' \quad \text{at } x = 0 \text{ and } r \geq r_0 \quad (4) \\
 u \rightarrow 0, \quad T' \rightarrow T_\infty', \quad \text{as } r \rightarrow \infty
 \end{aligned}$$

The initial and boundary conditions are where  $u$  and  $v$  are the velocity components along  $x$  and  $r$  directions respectively,  $T'$  the temperature of the fluid in the boundary layer,  $T_\infty'$  is the temperature far away from the surface,  $t'$  is the time,  $g$  is the acceleration due to gravity,  $\beta$  is the volumetric coefficient of thermal expansion, and  $\rho$  is the density.

The variation of viscosity can be taken in the form  $\mu(T) = \mu_\infty(1 + \gamma T)$  which was proposed by Cengel [8] and Toulokian *et al.*[11].

The physical quantities of interest are the local skin friction,  $\tau_x$  and the local Nusselt number  $Nu_x$  which are given respectively by

$$\tau_x = -\mu \left( \frac{\partial u}{\partial r} \right)_{r=r_0}, \quad Nu_x = \frac{x}{T_w' - T_\infty'} \left( -\frac{\partial T'}{\partial r} \right)_{r=r_0} \quad (5)$$

Also the average skin friction  $\bar{\tau}_L$  and the average Nusselt number is given by

$$\begin{aligned}
 \bar{\tau}_L &= \frac{1}{L} \int_0^L \mu \left( \frac{\partial u}{\partial r} \right)_{r=r_0} dx, \\
 \overline{Nu}_L &= \int_0^L \frac{-1}{T_w' - T_\infty'} \left( \frac{\partial T'}{\partial r} \right)_{r=r_0} dx \quad (6)
 \end{aligned}$$

On introducing the following non-dimensional quantities:

$$\begin{aligned}
 X &= \frac{xv}{u_0 r_0^2}, \quad R = \frac{r}{r_0}, \quad U = \frac{u}{u_0 v}, \quad V = \frac{vr_0}{v}, \quad t = \frac{vt'}{r_0^2}, \\
 T &= \frac{T' - T_\infty'}{T_w' - T_\infty'}, \quad Gr = \frac{g\beta(T_w' - T_\infty')r_0^2}{\nu u_0} \quad (7) \\
 Pr &= \frac{\nu}{\alpha}, \quad M = \frac{\sigma B_o^2 r_0^2}{\rho \nu}, \quad \omega = \frac{r_0^2 \omega'}{\nu}
 \end{aligned}$$

where  $Pr$  is the Prandtl number of the fluid and  $Gr$  is the Grashof number.

Equations (1), (2) and (3) reduce to the following dimensionless form:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial R} + \frac{V}{R} = 0 \quad (8)$$

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial R} = GrT + (1 + \gamma T) \left( \frac{\partial^2 U}{\partial R^2} + \frac{1}{R} \frac{\partial U}{\partial R} \right) + \gamma \frac{\partial T}{\partial R} \frac{\partial U}{\partial R} \quad (9)$$

$$\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial R} = \frac{1}{Pr} \left( \frac{\partial^2 T}{\partial R^2} + \frac{1}{R} \frac{\partial T}{\partial R} \right) \quad (10)$$

The corresponding initial and boundary conditions in a dimensionless form are given by

$$\begin{aligned}
 t \leq 0: \quad U = 0, \quad V = 0, \quad T = 0 \quad \text{for all } X \text{ and } R \\
 t > 0: \quad U = \cos \omega t, \quad V = 0, \quad T = 1, \quad \text{at } R = 1 \\
 U = 0 \quad T = 0 \quad \text{at } X = 0 \\
 U \rightarrow 0, \quad T \rightarrow 0 \quad \text{as } R \rightarrow \infty \quad (12)
 \end{aligned}$$

The local non-dimensional skin friction and the local Nusselt number are given by:

$$\tau_x = -\left( \frac{\partial U}{\partial R} \right)_{R=1}, \quad Nu_x = \frac{-X \left( \frac{\partial T}{\partial R} \right)_{R=1}}{T_{R=1}}, \quad (12)$$

$$\bar{\tau} = -\int_0^1 X \left( \frac{\partial U}{\partial R} \right)_{R=1} dX, \quad \overline{Nu} = -\int_0^1 \frac{\left( \frac{\partial T}{\partial R} \right)_{R=1}}{T_{R=1}} dX, \quad (13)$$

### 3. Numerical Techniques

The governing partial differential equation in non-dimensional form is unsteady, non-linear and coupled, so it is quite difficult to solve the problem. So, we proposed to solve the governing non-dimensional form equations by an efficient and also more accurate finite difference scheme Crank-Nicolson. For this purpose, we consider the rectangular region with appropriate mesh sides  $X_{\max}$  ( $=1$ ) and  $R_{\max}$  ( $=22$ ) where  $R_{\max}$  corresponds to  $R = \infty$ , which lies very well outside the momentum and thermal boundary layers. In order to get the reliable numerical results, we choose the mesh sizes  $\Delta X = 0.02$ ,  $\Delta R = 0.2$  and  $\Delta t = 0.01$ . Computational procedures carried out till we get the steady state solution with the tolerance limit  $10^{-5}$ . The proposed scheme is more comfortable to handle the finite difference scheme.

### 4. Results and Discussion

Figure 1 represents the flow field with respect to variation of frequency parameter  $\omega t$  and magnetic field parameter  $M$ . When there is no applied magnetic field *i.e.*,  $M = 0$ , more time is needed to reach the steady state solution. The velocity of fluid increases rapidly near the surface of the cylinder and decreases gradually far away from the surface. The velocity of fluid gets reduced by the applying more magnetic field. Similarly, the behaviour of temperature of the fluid was also analysed and shown graphically in Figure 2. The temperature of the air raised due to increase in the magnetic field. Also, the temperature increases with the increasing oscillating frequency parameter.

The approximate values of  $U$  and  $T$  are calculated for different values of  $Pr$  and  $Gr$  during the transient period and are shown graphically in the Figure 3 and 4. From the approximate value, we conclude that the velocity of air increases drastically compare than the velocity of water. The velocity of fluid increases rapidly near the surface  $R=2$  and then gradually reduced to zero. More calculation time is required for higher values of Prandtl number of the fluid. Similarly, the computation procedure takes more time to get the steady state in the case of higher values of Grashof number in compare with the lower values of Grashof number. Due to increase in the value of Grashof number, the velocity of fluid get increases. The temperature of the water is very low, in comparison with the temperature of air, this is quite expected.

The effect of variation of viscosity are considered and calculated

the numerical values of velocity and temperature are shown graphical form in the Figure 5 and 6. When we consider the viscosity as constant ( $\gamma=0$ ), the velocity increases drastically, but when we consider the variation of viscosity the velocity get decreases. Also, the calculating time will be very low in the case of constant viscosity, but the viscosity parameter increases the computational time will increases gradually. The velocity increases as  $\gamma$  decreases near the surface and the reverse trend is noticed far away from the surface of the cylinder. The temperature get reduced by increasing the viscosity parameter.

The study of local as well as average shearing stress and rate of heat transfer is quite interesting and has various physical applications. Due to this, we studied the local as well as average skin friction and Nusselt number with the effect of various parameter which involved in the flow governing equations.

The numerical values of skin friction and Nusselt number are calculated from the equations (12) and (13) with respect to variation of different parameters involved in the flow equations and are depicted graphically in Figures 7-12. From the figures, we see that the local and average skin friction increases as the viscosity parameter increases. Also, we see that the values of local skin friction is more in the case of higher values of oscillating parameter and similarly the same trend is notice for magnetic field parameter. From figure 9, we notice that the heat transfer coefficient get reduced in the case of higher values of  $M$ , but the reverse trend is noticed in the case of variation of oscillating parameter.

### 5. Concluding Remarks

A numerical analysis is performed to study the non-constant viscosity with the magnetic field along the free convection flow past a semi-infinite vertical oscillating cylinder with uniform surface. The effect of viscous dissipation in the temperature equation are not considered. The group of partial differential equations in non-dimensional form are solved numerically using well know method. The approximate values of velocity, temperature, skin friction and Nusselt number are obtained for different values of parameters involved in the flow problem. The following conclusions are made during the analysis of the problem:

1. More calculation time is needed to reach the steady state solution in the case of constant viscosity.
2. The variation of viscosity is more effect near the surface of the cylinder.
3. The variation of viscosity shows that it introduce a substantial error, which means that we cannot neglect the variation of viscosity.
4. The local sand average skin friction increases as  $\gamma$  increases.
5. The local Nusselt number decreases with the increasing value of  $\gamma$ .

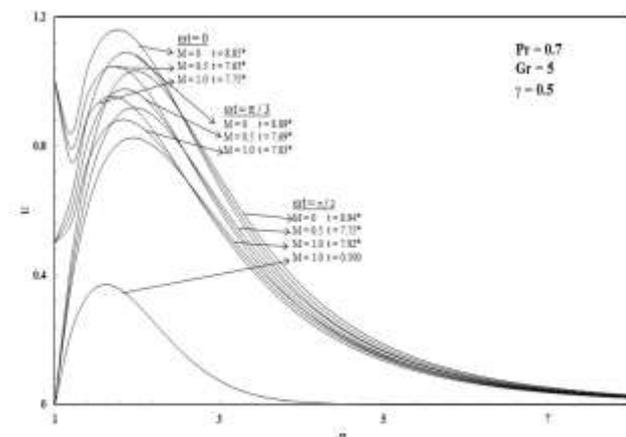


Fig.1 Transient velocity profiles at X = 1.0 for different values of  $\omega$  and M (\*-steady state)

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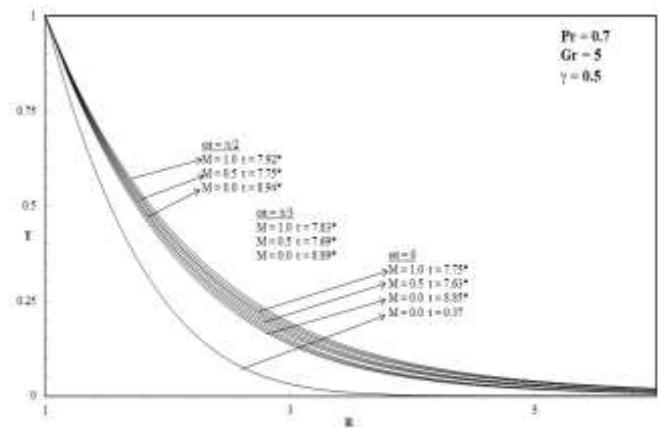


Fig.2 Transient temperature profiles at X = 1.0 for different values of  $\omega$  and M (\*-steady state)

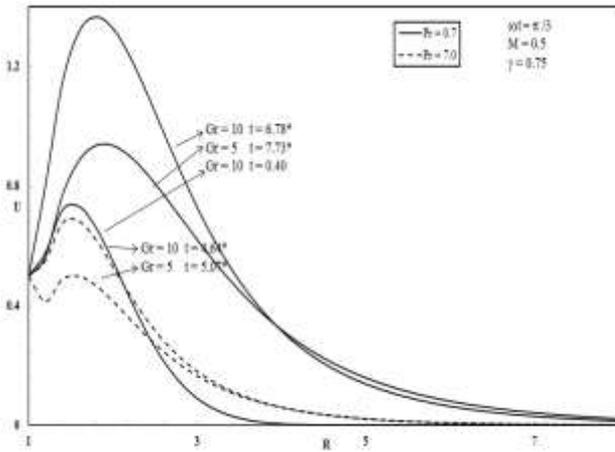


Fig.3 Transient velocity profiles at  $X = 1.0$  for different values of  $Pr$  and  $Gr$  (\*-steady state)

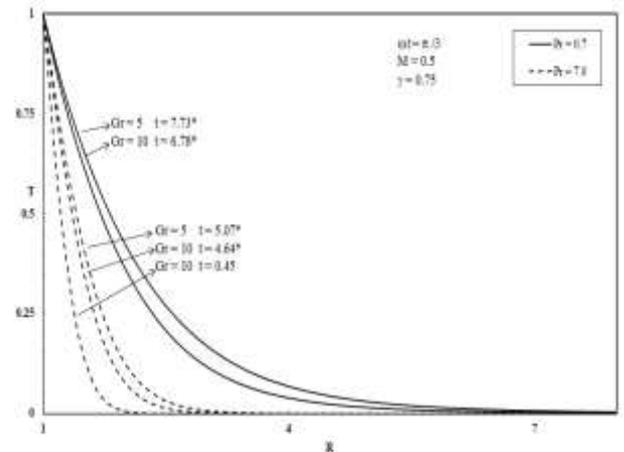


Fig.4 Transient temperature profiles at  $X = 1.0$  for different values of  $Pr$  and  $Gr$  (\*-steady state)

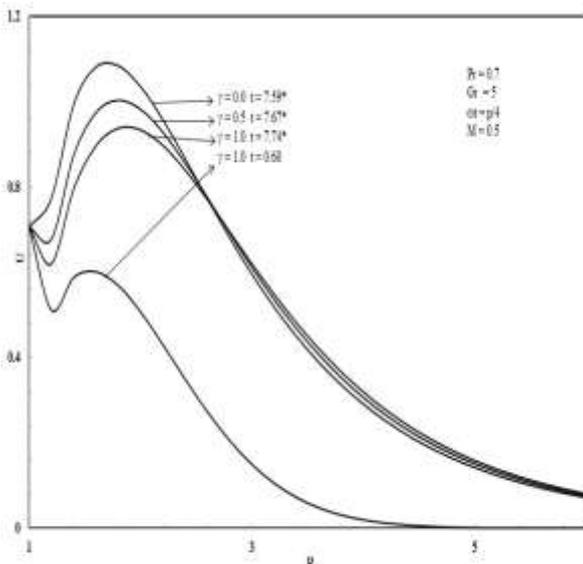


Fig.5 Transient velocity profiles at  $X = 1.0$  for different  $\gamma$  (\*-steady state)

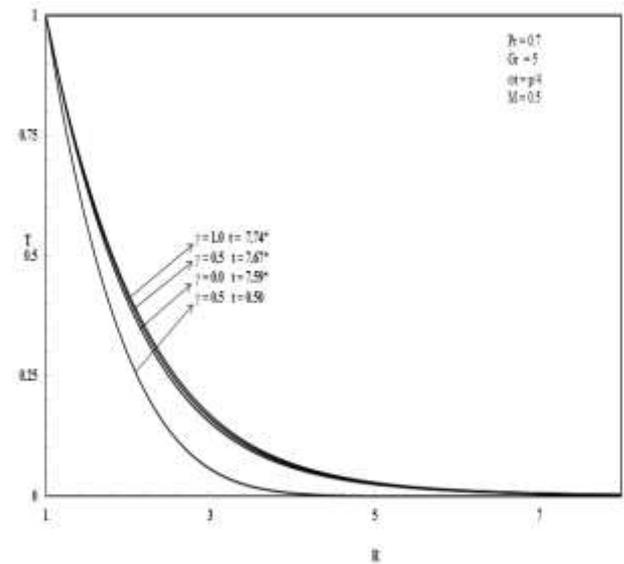


Fig.6 Transient temperature profiles at  $X = 1.0$  for different  $\gamma$  (\*-steady state)

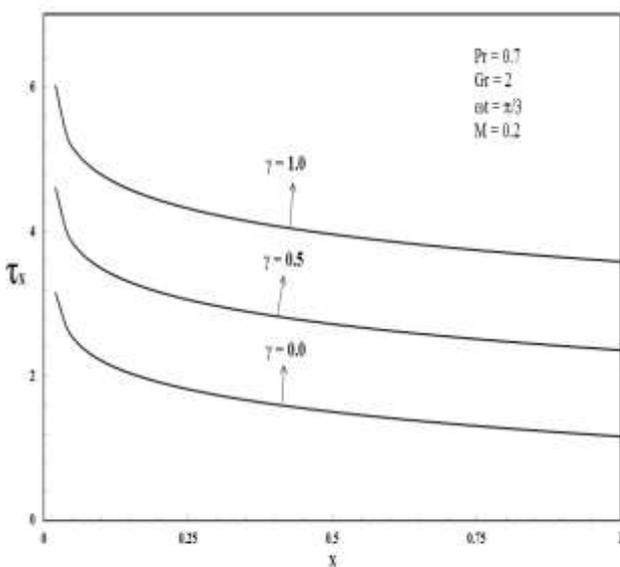


Fig.7 Local skin friction

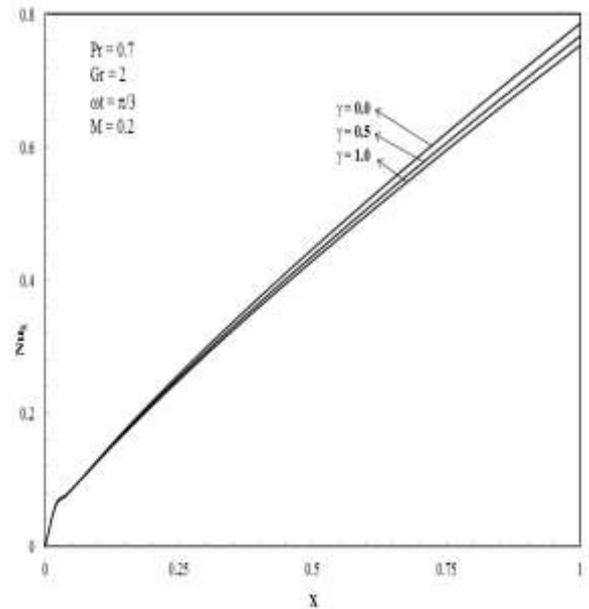


Fig.8 Local Nusselt number

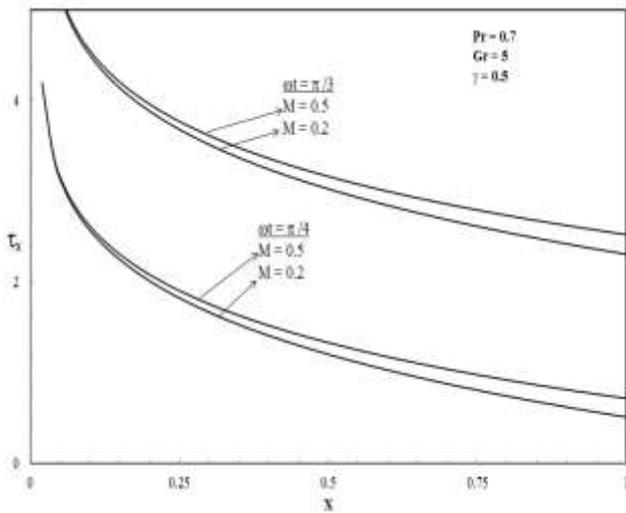


Fig.9 Local skin friction

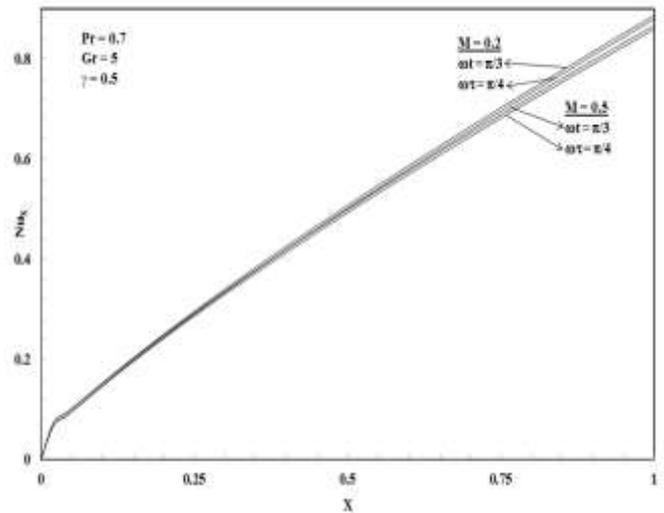


Fig.10 Local Nusselt number

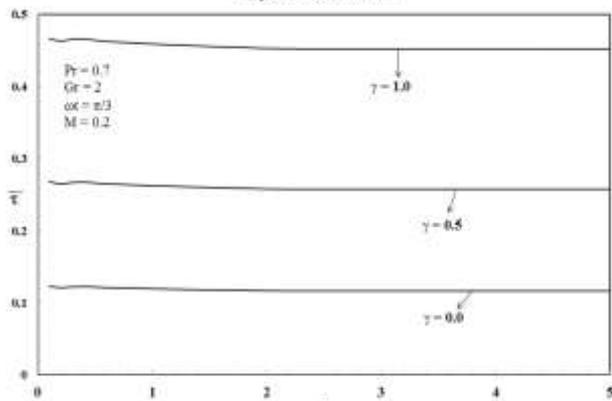


Fig.11 Average skin friction

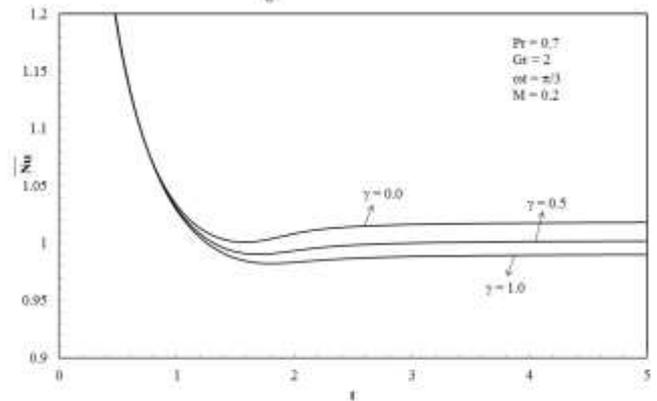


Fig.12 Average Nusselt number

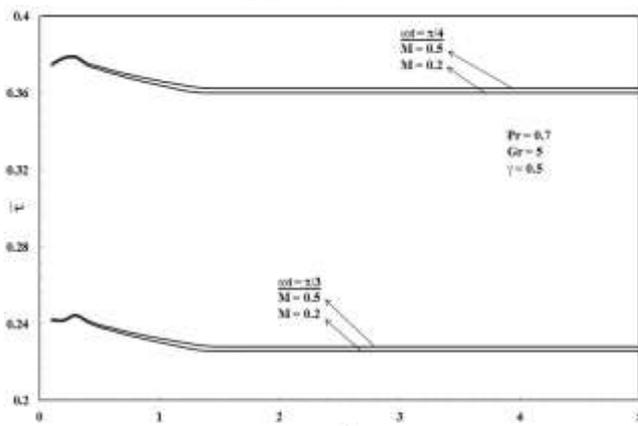


Fig.13 Average skin friction

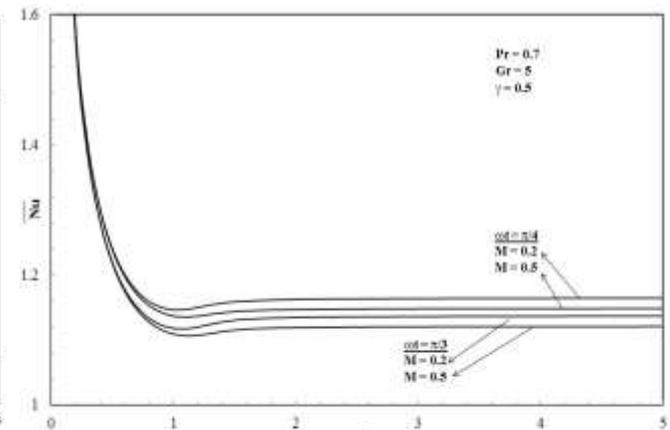


Fig.14 Average Nusselt number