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Research paper



Sum of Vague Ideals of Gamma Near-Ring

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Abstract

In this paper, we introduce the concept of sum and direct sum of vague ideals of gamma near-ring and also discuss some related properties of vague ideals of gamma near-ring.

Keywords: *near-ring, gamma near-ring, vague set, vague ideals.*

1. Introduction

The Γ-ring theory has been investigated in many directions and has evoked great interest among mathematicians working in different fields of mathematics. Research in the near-rings has had a long history. The concept of the generalization of a ring (or Γ -ring) has been studied by Nobusawa [5]. Further, Barnes [1] was analyzed the generalized form of Γ-ring. In 1984, Satyanarayana [7] has been investigated generalization concepts of both the near-ring and the Γ-ring. The notion of fuzzy sets and fuzzy subsets were first introduced by Zadeh [10] in 1965. Gau and Buehrer [4] introduced the concept of vague theory. Accordingly a vague set F of a given universes X can be characterized by means of a pair of functions (t_F, f_F) where t_F and f_F are functions from X to $[0,1]$ such that $t_F(x)+ f_F(x)\le 1, \forall x \in X$. Then t_F is called the membership function and $t_F(x)$ gives evidence of how much an element x belongs to F and f_F is called non-membership function and $f_F(x)$ gives evidence of how much an element x does not belongs to F. Thus the theory of vague set is a generalization theory of fuzzy set. These concepts are being applied in several areas like decision making, fuzzy control system, knowledge discovery and fault diagnosis etc. and vague group concept was introduced by Biswas [2]. The notion of intuitionistic fuzzy sub near-ring, intuitionistic fuzzy N-subgroup and intuitionistic fuzzy ideal of a near-ring can be founded in [8,9,11]. Narasimha Swamy and et.al [6] studied sum of fuzzy ideal of gamma near-ring. L. Bhaskar and T. Nagaiah [3] studied the notion of vague ideals of a gamma near-ring. In this direction, we study the concept of sum and direct sum of vague ideals of a gamma near-ring. Throughout this paper we will use the word ''near-ring'' to mean ''left near-ring''.

2. Preliminaries

Definition 2.1. **[4]** A vague set A in the universe of discourse U is a pair (t_A, f_A) where $t_A : U \to [0,1]$ and $f_A : U \to [0,1]$ are mappings such that $t_A(u)+ f_A(u)\le 1$ for all $u \in U$. The functions t_A and f_A are called true membership function and false membership function in [0,1] respectively. And the interval $[t_A(u),1- f_A(u)]$ is called vague value of u in A and it is denoted by $V_A(u)$, that is $V_A(u)=[t_A(u),1- f_A(u)]$.

Definition 2.2. **[3]**
A vague set u of a Γ-near-ring N is called a vague ideal of N, if

$(i)\ V_\mu(p-q) \ge \min(V_\mu(p), V_\mu(q)),$

$(ii)\ V_\mu(q+p-q) \ge V_\mu(p),$

$(iii)\ V_\mu((p+i)\alpha q - p\alpha q) \ge V_\mu(i),$

$(or\ equivalently, V_\mu(i\alpha q - r\alpha q) \ge V_\mu(i-r)$

$(iv)\ V_\mu(p\alpha q) \ge V_\mu(q).$

That is

$(i)\ t_\mu(p-q) \ge \min(t_\mu(p), t_\mu(q))\ and$

$1- f_\mu(p-q) \ge \min(1- f_\mu(p), 1- f_\mu(q)),$

$(ii)\ t_\mu(q+p-q) \ge t_\mu(p)\ and$

$1- f_\mu(q+p-q) \ge 1- f_\mu(p)$

For all $p,q,r,i \in N$ and $\alpha \in \Gamma$.

$(iii)\ t_\mu((p+i)\alpha q) - p\alpha q) \ge t_\mu(i)\ and$

$1- f_\mu((p+i)\alpha q) - p\alpha q) \ge 1- f_\mu(i),$

$(or\ equivalently, t_\mu(i\alpha q - r\alpha q) \ge t_\mu(i-r)\ and$

$1- f_\mu(i\alpha q - r\alpha q) \ge 1- f_\mu(i-r)$

$(iv)\ t_\mu(p\alpha q) \ge t_\mu(q)\ and$

$1- f_\mu(p\alpha q) \ge 1- f_\mu(q).$

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3. Sum and Direct Sum of Vague Ideals

Definition 3.1.

Let $\mu = (t_\mu, f_\mu)$ and $\nu = (t_\nu, f_\nu)$ be two vague ideals of a zero symmetric Γ-near-ring N. Then the sum $\mu + \nu$ is a vague set of N defined as follows;

$$V_{\mu+\nu}(p) = \begin{cases} Sup\{\min\{V_\mu(q), V_\nu(r)\}\} / \ p = q + r, \\ 0, p \neq q + r, where \ q, r \in N. \end{cases}$$

That is

$$t_{\mu+\nu}(p) = \begin{cases} Sup\{\min\{t_\mu(q), t_\nu(r)\}\} / \ p = q + r, \\ 0, p \neq q + r, where \ q, r \in N. \end{cases}$$

$$1 - f_{\mu+\nu}(p) = \begin{cases} Sup\{\min\{1 - f_\mu(q), 1 - f_\nu(r)\}\} / \ p = q + r, \\ 0, p \neq q + r, where \ q, r \in N. \end{cases}$$

4. Main Results

Theorem 4.1

If μ and ν are two vague ideals of a zero symmetric Γ-near-ring N, then $\mu + \nu$ is also a vague ideal of N.

Proof: Let $p, q, u \in N$ and $\alpha \in \Gamma$.

(i) Put $p = p_1 + p_2, \ q = q_1 + q_2; \ p_1, p_2, q_1, q_2 \in N.$ Then

$$p - q = p_1 + p_2 - (q_1 + q_2)$$
$$= p_1 - q_1 + q_1 + p_2 - (q_1 + q_2)$$

$$V_{u+v}(p - q) = V_{u+v}(p_1 - q_1 + q_1 + p_2 - q_1 - q_2)$$
$$= \sup(V_\mu(p_1 - q_1) \wedge V_\nu(q_1 + p_2 - q_1 - q_2))$$
$$\geq \mathbf{V}((V_\mu(p_1) \wedge V_\mu(q_1)) \wedge (V_\nu(q_1 + p_2 - q_1) \wedge V_\nu(q_2)))$$
$$\geq \mathbf{V}((V_\mu(p_1) \wedge V_\mu(q_1)) \wedge (V_\nu(p_2) \wedge V_\nu(q_2)))$$
$$\geq (\mathbf{V}(V_\mu(p_1) \wedge V_\mu(p_2))) \wedge (\mathbf{V}(V_\nu(q_1) \wedge V_\nu(q_2)))$$
$$= V_{u+v}(p) \wedge V_{u+v}(q).$$

(ii) Put $p = p_1 + p_2; \ p_1, p_2 \in N.$ Then

$$q + p - q = q + p_1 + p_2 - q$$
$$= q + p_1 - q + q + p_2 - q.$$

$$V_{u+v}(q + p - q) = V_{u+v}(q + p_1 - q + q + p_2 - q)$$
$$= \mathbf{V}((V_\mu(q + p_1 - q) \wedge (V_\nu(q + p_2 - q))$$
$$\geq \mathbf{V}(V_\mu(p_1) \wedge V_\nu(p_2))$$
$$= V_{u+v}(p).$$

(iii) Let $u - p = s_1 + s_2; \ s_1, s_2 \in N.$ Which implies that $u = s_1 + s_2 + p.$ Then

$$u\alpha q - p\alpha q = (s_1 + s_2 + p)\alpha q - p\alpha q$$
$$= (s_1 + s_2 + p)\alpha q - (s_2 + p)\alpha q + (s_2 + p)\alpha q - p\alpha q.$$

$$V_{u+v}(u\alpha q - p\alpha q) = V_{u+v}((s_1 + s_2 + p)\alpha q - (s_2 + p)\alpha q + (s_2 + p)\alpha q - p\alpha q)$$
$$= \mathbf{V}(V_u((s_1 + s_2 + p)\alpha q - (s_2 + p)\alpha q) \wedge (V_u(s_2 + p)\alpha q - p\alpha q)$$
$$\geq \mathbf{V}(V_u(s_1) \wedge V_v(s_2))$$
$$= V_{u+v}(u - p).$$

(iv) Put $q = q_1 + q_2; \ q_1, q_2 \in N.$ Then

$$V_{u+v}(p\alpha q) = V_{u+v}(p\alpha q_1 + p\alpha q_2)$$
$$\geq \mathbf{V}(V_u(p\alpha q_1) \wedge V_v(p\alpha q_2))$$
$$= \mathbf{V}(V_u(q_1) \wedge V_v(q_2))$$
$$= V_{u+v}(q).$$

Hence $\mu + \nu$ is a vague ideal of N.

Example 4.2

Let $N = \{0, x, y, z\}$ and $\Gamma = \{\alpha, \beta\}$. Define a binary operation "+" on N and a mapping $N \times \Gamma \times N \to N$ by the following tables;

+	0	x	y	z
0	0	x	y	z
x	x	0	z	y
y	yy	z	0	x
z	z	y	x	0

α	0	x	y	z
0	0	0	0	0
x	0	0	0	0
y	0	0	0	0
z	0	0	0	0

β	0	x	y	z
0	0	0	0	0
x	0	x	x	x
y	0	y	y	y
z	0	z	z	z

Clearly, (N, +) is a group and N is a Γ-near-ring.
it is clear that, N is a zero symmetric Γ-near-ring .
Now a vague set $\mu = (t_\mu, f_\mu)$ of N defined as

$t_\mu : N \to [0,1]$ and $f_\mu : N \to [0,1]$ by

$$t_\mu(p) = \begin{cases} 0.8 \ \text{if} \ p = 0, \\ 0.5 \ ; otherwise \end{cases}$$

$$f_\mu(p) = \begin{cases} 0.2 \ \text{if} \ p = 0, \\ 0.5 \ ; otherwise. \end{cases}$$

Another vague set $\nu = (t_\nu, f_\nu)$ of N defined as $t_\nu : N \to [0,1]$ and $f_\nu : N \to [0,1]$ by

$$t_\nu(p) = \begin{cases} 0.9 \ \text{if} \ p = 0, \\ 0.2 \ ; otherwise \end{cases}$$

$$f_\nu(p) = \begin{cases} 0.1 \ \text{if} \ p = 0, \\ 0.8 \ ; otherwise. \end{cases}$$

The routine calculation shows that, u and v are vague ideal of N.

Now for any $q, r \in N.$

$$V_{u+v}(0) = \underset{0=q+r}{\mathbf{V}} \{\min(V_u(q), V_v(r))\}, where \ q, r \in N ....(i)$$

Consider $\underset{0=q+r}{\mathbf{V}} \{\min(V_u(q), V_v(r))\}$

$$= \mathbf{V}\{\min(V_\mu(0), V_\nu(0)), \min(V_\mu(x), V_\nu(x)),$$
$$\min(V_\mu(y), V_\nu(y)), \min(V_\mu(z), V_\nu(z))\}$$
$$= \mathbf{V}\{\min([t_\mu(0), 1 - f_\mu(0)], [t_\nu(0), 1 - f_\nu(0)]),$$
$$\min([t_\mu(x), 1 - f_u(x)][t_\nu(x), 1 - f_\nu(x)]),$$
$$\min([t_\mu(y), 1 - f_\mu(y)], [t_\nu(y), 1 - f_\nu(y)]),$$
$$\min([t_\mu(z), 1 - f_u(z)][t_\nu(z), 1 - f_\nu(z)]\}$$
$$= \mathbf{V}\{\min([0.8, 1 - 0.2], [0.9, 1 - 0.1]),$$
$$\min([0.5, 1 - 0.5][0.2, 1 - 0.8]),$$
$$\min([0.5, 1 - 0.5], [0.2, 1 - 0.8]),$$
$$\min([0.5, 1 - .05][0.2, 1 - 0.8]\}$$
$$= [0.8, 0.8].$$

Now $V_{u+v}(0)=[t_{u+v}(0),1-f_{u+v}(0)]....(ii)$

Consider

$t_{u+v}(0)=\mathbf{V}\{\min(t_\mu(0),t_v(0)),\min(t_\mu(x),t_v(x)),$

$\min(t_\mu(y),t_v(y)),\min(t_\mu(z),t_v(z))\}$

$=\mathbf{V}\{\min(0.8,0.9),\min(0.5,0.2),\min(0.5,0.2),\min(0.5,02)\}$

$=\mathbf{V}\{0.8,0.2,0.2,0.2\}$

$=0.8$ *and*

$1-f_{u+v}(0)=\mathbf{V}\{\min(1-f_\mu(0),1-f_v(0)),\min(1-f_\mu(x),1-f_v(x)),$

$\min(1-f_\mu(y),1-f_v(y)),\min(1-f_\mu(z),1-f_v(z))\}$

$=\mathbf{V}\{\min(1-0.2,1-0.1),\min(1-0.5,1-0.8),$

$\min(1-0.5,1-08),\min(1-0.5,1-0.8)\}$

$=\mathbf{V}\{0.8,0.2,0.2,0.2\}$

$=0.8$

From (ii) $V_{u+v}(0)=[t_{u+v}(0),1-f_{u+v}(0)]=[0.8,0.8].$

L.H.S of equation (i) = R.H.S of equation (i) for p=0

Similarly, continue this processes for p=x, y, z also. We can easily verify that, $u+v$ is a vague ideal of N.

Definition 4.3(Addition of finite number of vague ideals)

Let N be a zero symmetric Γ-near-ring and let $u_1,u_2,u_3.....u_n$ be the vague ideals of a Γ-near-ring N. Forany $p\in N,$

Put $S(p)=\{V_{u_1}(p_1)\wedge V_{u_2}(p_2)\wedge....\wedge V_{u_n}(p_n):p=p_1+p_2+....+p_n$

Where $p_i\in N,i=1\ to\ n\}.$ Define

$V_{u_1+u_2+.....u_n}(p)=Sup S(p)$

$=Sup\{V_{u_1}(p_1)\wedge V_{u_2}(p_2)\wedge....\wedge V_{u_n}(p_n):p=p_1+p_2+....+p_n$

Remark:

Let $p=p_1+p_2+....+p_n.$

Consider a transposition of the indices $(1,k),k\geq 1,$ then

$p=p_1+p_2+....+p_{k-1}+p_k+p_{k+1}+....+p_n$

$=q+p_k-q+p_1+p_2+.....+p_{k-1}+p_{k+1}+....+p_n$

$Where\ \ q=p_1+p_2+....+p_{k-1}$

$=(q+p_k-q)+r-r+p_1+r+p_{k+1}+...+p_n$

$Where\ \ r=p_2+....+p_{k-1}$

$=p_k'+r+p_1'+p_{k+1}+....+p_n$

$Where\ \ p_k'=q+p_k-q\ ,p_1'=-r+p_1+r$

$=p_k'+p_2+...+p_{k-1}+p_1'+p_{k+1}+...p_n.$

Thus

$V_{u_1}(p_k')\wedge V_{u_2}(p_2)\wedge....\wedge V_{u_{k-1}}(p_{k-1})\wedge V_{u_k}(p_1')\wedge V_{u_{k+1}}(p_{k+1})\wedge....\wedge V_{u_n}(p_n)$

$=V_{u_1}(p_k)\wedge V_{u_2}(p_2)\wedge....\wedge V_{u_{k-1}}(p_{k-1})\wedge V_{u_k}(p_1)\wedge V_{u_{k+1}}(p_{k+1})\wedge....\wedge V_{u_n}(p_n)$

belongs to $S(p)$.

This is true for every transposition (i, j) of the indices.

As every permutation is a product of transposition, then for any permutation $\begin{pmatrix}1 & 2.....n\\ i_1 & i_2.....i_n\end{pmatrix}$

We have $V_{u_1}(p_{i_1})\wedge V_{u_2}(p_{i_2})\wedge....\wedge V_{u_n}(p_{i_n})$ belongs to S(p) for any $p=p_1+p_2+....+p_n.$

Hence $V_{u_1+u_2+.....+u_n}=V_{u_{i_1}+u_{i_2}+.....+u_{i_n}}.$

Theorem 4.4

Let N be a zero symmetric Γ-near-ring. If $u_1,u_2,..,u_n$ are the vague ideals of N, then $u_1+u_2+....+u_n$ is also vague ideal of N.

Proof: Put $u=u_1+u_2+....+u_n.$

(i) Let $p=p_1+p_2+....+p_n,\ q=q_1+q_2+....+q_n;$ $p_i,q_i\in N,i=1,2,....,n.$ Then $p-q=p_1+p_2+....+p_n-q_1-q_2-....-q_n.$ This can be expressed as $p-q=p_1'-q_1'+p_2'-q_2'+....+p_n'-q_n',$ Where p_i' is a conjugate of p_i and q_i' is a conjugate of $q_i.$

$\therefore\ p-q=(p_1'-q_1')+(p_2'-q_2')+....+(p_n'-q_n').$

$V_u(p-q)=V_u((p_1'-q_1')+(p_2'-q_2')+....+(p_n'-q_n'))$

$=\mathbf{V}(V_{u_1}(p_1'-q_1')\wedge V_{u_2}(p_2'-q_2')\wedge....\wedge V_{u_n}(p_n'-q_n'))$

$\geq\mathbf{V}(V_{u_1}(p_1')\wedge V_{u_1}(q_1')\wedge V_{u_2}(p_2')\wedge V_{u_2}(q_2')\wedge....\wedge V_{u_n}(p_n')\wedge V_{u_n}(q_n'))$

$=(\mathbf{V}(V_{u_1}(p_1')\wedge V_{u_2}(p_2')\wedge...\wedge V_{u_n}(p_n')))\wedge(\mathbf{V}(V_{u_1}(q_1')\wedge V_{u_2}(q_2')\wedge)\wedge...\wedge V_{u_n}(q_n')))$

$=SupS(p)\wedge SupS(q)$

$=V_u(p)\wedge V_u(q).$

(ii) Let $p,q\in N$ and $p=p_1+p_2+...+p_n;p_i\in N,\ i=1,2,....,n.$ Then

$q+p-q=q+p_1+p_2+....+p_n-q$

$=q+p_1-q+q+p_2-q+q+p_3-q+...+q+p_n-q$

$V_u(q+p-q)=V_u(q+p_1-q+q+p_2-q+...+q+p_n-q)$

$=\mathbf{V}(V_{u_1}(q+p_1-q)\wedge V_{u_2}(q+p_2-q)\wedge....\wedge V_{u_n}(q+p_n-q))$

$\geq\mathbf{V}(V_{u_1}(p_1)\wedge V_{u_2}(p_2)\wedge....\wedge V_{u_n}(p_n))$

$=SupS(p)$

$=V_u(p).$

(iii) Let $p,q,u\in N$ and $\alpha\in N$.

Let $u-p=s_1+s_2+...+s_n;s_i\in N,i=1,2,...,n.$

$\Rightarrow u=s_1+s_2+...+s_n+p.$

$u\alpha q-p\alpha q=(s_1+s_2+...+s_n+p)\alpha q-p\alpha q$

$=(s_1+s_2+...+s_n+p)\alpha q-(s_2+s_3+...+s_n+p)\alpha q$

$+(s_2+s_3+...+s_n+p)\alpha q-(s_3+s_4+...+s_n+p)\alpha q$

$+(s_3+s_4+...+s_n+p)\alpha q-.....+(s_n+p)\alpha q-p\alpha q.$

$V_u(u\alpha q-p\alpha q)=V_u\{s_1+s_2+...s_n+p)\alpha q-(s_2+s_3+...+s_n+p)\alpha q$

$+(s_2+s_3+...+s_n+p)\alpha q-(s_3+s_4+...+s_n+p)\alpha q$

$+(s_3+s_4+...+s_n+p)\alpha q-.....+(s_n+p)\alpha q-p\alpha q\}$

$= \mathbf{V}\{V_{u_1}(s_1 + s_2 + ...s_n + p)\alpha q - (s_2 + s_3 + ... + s_n + p)\alpha q)$
$\wedge V_{u_2}((s_2 + s_3 + ... + s_n + p)\alpha q - (s_3 + s_4 + ... + s_n + p)\alpha q)$
$\wedge (s_3 + s_4 + ... + s_n + p)\alpha q \wedge ..... \wedge V_{u_n}((s_n + p)\alpha q - p\alpha q)\}$
$= \mathbf{V}(V_{u_1}(s_1) \wedge V_{u_2}(s_2) \wedge ..... \wedge V_{u_2}(s_n))$
$= \sup S(u - p)$
$= V_u(u - p)$

(iv) Let $p, q \in N$ and $\alpha \in \Gamma.$

Put $q = q_1 + q_2 + ... + q_n;$
$q_i \in N, i = 1, 2, ..., n.$ Then

$V_u(p\alpha q) = V_u(p\alpha(q_1 + q_2 + ... + q_n))$
$= V_\mu(p\alpha q_1 + p\alpha q_2 + ... + p\alpha q_n)$
$= \mathbf{V}(V_{\mu_1}(p\alpha q_1) \wedge V_{\mu_2}(p\alpha q_2)) \wedge ..... \wedge V_{u_n}(p\alpha q_n))$
$\geq \mathbf{V}(V_{\mu_1}(q_1) \wedge V_{\mu_2}(q_2)) \wedge ..... \wedge V_{u_n}(q_n))$
$= \sup S(p)$
$= V_u(q).$

Hence u is a vague ideal of N

Definition 4.5

Let N be a zero symmetric Γ-near-ring and $u_1, u_2, ..., u_n$ be the vague ideal of N. Then a sum $V_u = V_{u_1 + u_2 + ... + u_n}$ is said to be direct if $(V_{u_1 + u_2 + ... + u_{i-1} + u_{i+1} ..... + u_n}) \wedge V_{u_i} = 0.$

Theorem 4.6

Let $N = N_1 \oplus N_2 \oplus ..... \oplus N_n$ be the direct sum of Γ-near-rings $N_1, N_2, ..., N_n$ with the left or right identity $e = (e_1, e_2, ..., e_n)$ and u be a vague ideal of N. Then there exists vague ideals $u_1, u_2, ..., u_n$ of N such that $V_u = V_{u_1 \oplus u_2 \oplus ..... \oplus u_n}.$

Proof: Let $p_i = (o, o, o, ..., o, p_i, o, ..., o)$ and $e_i = (o, o, .., o, e_i, o, .., o)$ $\alpha \in \Gamma$

Then for $p = (p_1, p_2, ..., p_n) = p_1 + p_2 + ... + p_n$, we have

$V_u(p) = V_u(p_1 + p_2 + ... + p_n)$
$= V_u(p_1) \wedge V_u(p_2) \wedge .... \wedge V_u(p_n)$

But $V_u(p_i) = V_u(e_i \alpha p)$ for $i = 1, 2, 3, ..., n$
$= V_u(p),$

That is $V_u(p_1) \wedge V_u(p_2) \wedge .... \wedge V_u(p_n) \geq V_u(p)$

Thus $V_u(p) = V_u(p_1) \wedge V_u(p_2) \wedge .... \wedge V_u(p_n).$

Define V_{u_i} on N by $V_{u_i}(p) = \begin{cases} V_u(p) & : \ p \in N_i \\ 0 & otherwise. \end{cases}$

Then $V_u = V_{u_1 \oplus u_2 \oplus ..... \oplus u_n}.$

5. Conclusion

In this study, we introduced and studied the notion of sum of vague ideals of gamma near-ring. We proved that sum of two vague ideal is again a vague ideal of a gamma near-ring. Also, we proved that sum of finite number of vague ideal is again a vague ideal of a gamma near-ring.

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