A dynamic programming approach to manage virtual machines allocation in cloud computing

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Abstract

As a result of the dynamic nature of Virtual Machine allocation in cloud computing, it is not easy to manage system resources or choose the best configuration based solely on human experience. In this work, we used stochastic modelling instead of comprehensive experiments to evaluate the best resource management of the system. In such complex systems, choosing the best decision is a challenge, for this reason we have designed a heuristic algorithm, specifically, dynamic programming as a resource management and programming tool that finds a way that attempts to satisfy the conflicting objectives of high performance and low power consumption. As a scenario for using this algorithm, we addressed the problem of virtual machine allocation, a subset of physical machines is designated as "reserve", and the reserves are actives when the number of jobs in the system is sufficiently high. The question is how to decide when to activate the reserves. The simulation results demonstrated the benefit of using our framework to identify the policy for consolidation or for a low energy consumption and in order to have a good quality of service in the system.

Keywords: Dynamic Programming, Cloud Computing, Management resources, Value Iteration, Stochastic Modeling.

1. Introduction

Cloud computing infrastructures are becoming more complex and difficult to configure or evaluate their performance. Scalability and multilocation are widely implemented in recent cloud platforms. For cloud computing management platforms, which are designed to manage multiple customers and a large number of resources, these platforms adopt a system design that takes into account different categories of customers with different characteristics [8],

In our model, we consider two customer categories (Type 1 and type 2), which have different priorities over their queues. Each client has different characteristics, such as performance and cost for each type of request. The system model contains a controller for routing allocation requests. The optimal query routing policy is obtained dynamically using the value iteration algorithm (VI) [9]. It will not be practical to perform repetitive experiments to evaluate the performance of such complicated systems, and it is difficult for an expert to predict the best system configuration (consolidation and assignment costs of each client) without using an algorithm such as the one we suggest in this work. As a use case of our algorithm, we studied the best controller decision in order to manage system performance and to optimize energy consumption.

The main contributions of this work are summarized as follows:

- Our algorithm focuses on evaluating cloud computing system management using a stochastic analytic model.
- Our algorithm finds the optimal routing policy for customers using dynamic programming (algorithm VI).

The rest of this paper is organized as follows. After presenting the system model in Sect. 2, we formulate the problem in Sect. 3 and we present the equivalent discrete-time time problem in Sect. 4, followed by the numerical results in Sect. 5, and finally, we conclude our paper in Sect. 6.

2. System Model

A server is considered to contain M physical machines, of which m are designated as reserves $0 \leq m \leq M$. Clients (Jobs/Requests) arrive according to a Poisson process of parameter λ_1 and find in front of them a controller that sends them directly to one of the main physical machines until the system receives n clients (Jobs). After that, the clients arrive according to another Poisson process of parameter λ_2 and in this case the controller decides to send them either to one of the Main Physical Machines (MPM) that already contains clients, or to one of the physical machines that we considered as reserves, depending on the state of the system.

Service times are independent and exponentially distributed with parameters $\mu_1, \mu_2, ..., \mu_M$, such as $\mu = \mu_i$ with

$i = 1, ..., M - m$ and $\mu' = \mu_j$ with $j = 1, ..., m$.

We notice the type 1 of clients, those that the controller sends them directly to one of the main physical machines, and the type 2 of clients, those that arrive and the controller has two choices either send them to one of the Main Physical Machines (MPM) and in this case the controller prefers to host several clients in a minimum number of physical machines in order to minimize the total energy consumption of the system, otherwise he chooses to acti-

vate a new physical machine among the reserves which leads to an increase in energy consumption. On the other hand, the goal of this choice and providing a good quality of service.

Type 1 of clients are always placed in front of type 2 of clients, i.e. type 1 of clients have priority over type 2 of clients. When a non-priority customer enters the system and finds a priority customer in front of him in service, he waits until he finishes. For each sending towards the set of reserves a cost C must be paid.

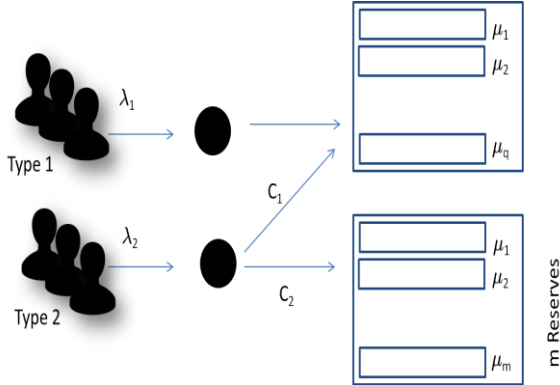The model considered in this problem is illustrated in the figure

Fig. 1: Illustration of the proposed model

3. Problem Formulation

The state of the system at an instant $t, t > 0$ is described by a stochastic process $(x_t)_{t \geq 0}$ with $x_t = (x_t^a, x_t^b)$ where $x_t^a = (x_t^{a1}, x_t^{a2})$, With $x_t^{a1} = (x_t^{a11}, x_t^{a12}, ..., x_t^{a1q})$, $x_t^{a2} = (x_t^{a21}, x_t^{a22}, ..., x_t^{a2q})$ and $x_t^b = x_t^{b2} = (x_t^{b21}, x_t^{b22}, ..., x_t^{b2m})$ and x_t^{aik} is the number of type i clients in the PM k among the Main Physical Machines (MPM) at the moment t, $i = 1.2$ and $k = 1.2, ..., q$.

x_t^{b2j} is the number of clients type 2 in the PM j among the Main Physical Machines (MPM) at the moment t, $j = 1, 2, ..., m$.

If at the moment t "after all MPM contains a certain number of clients", a client arrives in the system, then an action a_t^1 (respectively a_t^2) is taken and it is defined by $a_t^1 = 1$ if the controller sends the arriving client to a MPM (respectively by $a_t^2 = 2$) if the controller decides to activate a new PM.

Let t_n be the moment of the nth transition, x_n the state of the system after this transition $\tau_n = t_n - t_{n-1}$ the nth residence time and a_n the decision taken at the moment t_n.

For a given policy π, we note x_t and a_t the state and the action during the last transition before the moment t.

$$(x_t, a_t) = (x_n, a_n) \quad si \quad t_n \leq t < t_{n+1}$$

At each moment t, an instant cost $c(x_t, a_t)$ is incurred and it is given by:

$$c(x_t, a_t) = c_1 x_t^a + c_2 x_t^b$$

With $c_2 > c_1$, where all these coefficients are positive real numbers. $x_t = (x_t^a, x_t^b)$ is a Markovian process of space decision $E = \square^{2q+m}$ and action $A = \{1, 2\}$.

Our objective is to find the optimal policy π that minimizes the following discounted total cost function.

$$V^{\delta}(x, \pi) = E_{\pi} \left[\int_0^{\infty} e^{-\delta t} c(x_t, a_t) dt / x_0 = x \right] \tag{1}$$

Where E_{π} is the mathematical expectation under the π policy, $\delta > 0$ and x is an initial system state. Since in the problem we have the instant cost function $c(.,.)$ positive and the action space A is finite, then the optimal policy exists.

4. Discretization of the problem

We will turn the original problem (1) into an equivalent PMD problem formulated in discrete time by the uniformization procedure [2, 3, 7].

We write $\nu = \lambda_1 + \lambda_2 + q\mu_1 + m\mu'$.

As already defined $0 < t_0 < t_1 < ... < t_n < ...$ are the system state transition moments, by suitably introducing dummy departures as in [4, 7]. The time intervals are seen to be i.i.d random variables with distribution:

$$P(t_{l+1} - t_l > t) = e^{-t\nu} \quad l = 0, 1, 2, ...$$

Proposition For any policy π and any initial state x, the cost $V^{\delta}(x, \pi)$ is:

$$E_{\pi} \left(\int_0^{+\infty} e^{-\delta t} c(x_t, a_t) dt \right) = \frac{1}{\delta + \nu} \sum_{n=0}^{+\infty} \left(\frac{\nu}{\delta + \nu} \right)^n E_{\pi} \left(c(x_n, a_n) \right) \tag{2}$$

with $x_n = x_{t_n}$ and $a_n = a_{t_n}$.

According to this proposition, we can write:

$$V^{\delta}(x, \pi) = \frac{1}{\delta + \nu} E_{\pi} \left(\sum_{n=0}^{+\infty} \left(\frac{\nu}{\delta + \nu} \right)^n c(x_n, a_n) / x_0 = x \right) \tag{3}$$

Which is the expression of the discounted cost of a PMD in discrete time under the π policy, with discount factor $\beta = \dfrac{\nu}{\delta + \nu}$, $(0 < \beta < 1)$ and instant cost function $\dfrac{c(x_n, a_n)}{\delta + \nu}$.

we put: $\delta + \nu = 1$, to simplify the calculations.

The cost β - discounted in N steps is defined by:

$$V_N^{\beta}(x, \pi) = E_{\pi} \left(\sum_{n=0}^{N-1} \beta^n c(x_n, a_n) / x_0 = x \right) \tag{4}$$

And the cost β - discounted for infinite horizon is defined [3, 5] as:

$$V^{\beta}(x, \pi) = E_{\pi} \left(\sum_{n=0}^{+\infty} \beta^n c(x_n, a_n) / x_0 = x \right) \tag{5}$$

We notice:

$$V_N^\beta(x) = \min_\pi V_N^\beta(x, \pi) \tag{6}$$

And

$$V^\beta(x) = \min_\pi V^\beta(x, \pi) \tag{7}$$

By posing

$$V_\infty^\beta(x) = \lim_{N \to \infty} V_N^\beta(x) \tag{8}$$

we have

$$V_\infty^\beta(x) = V^\beta(x) \tag{9}$$

for any initial state x.

4.1 Existence of optimal policy

We have the following proposition
Proposition

     i.   for all $x \in E$, we have:

$$\begin{cases} V_N^\beta(x) = \min_{a \in A(x)} \{c(x,a) + \beta \sum_{x' \in E} p(x'/x,a) V_{N-1}^\beta(x'), \quad \forall N \geq 1 \\ avec \ \ V_0(.) = 0. \end{cases} \tag{10}$$

Where $A(x)$ is the set of actions available to the controller when the system is in the state x , and $p(x'/x,a)$ is the conditional probability that the system will displaces to the state x' at the time t_{n+1} When the action a has been applied at the instant t_n to the state t_n .

     ii.   If $c(.,.) \geq 0$ and A is finite, then:

$$\lim_{N \to +\infty} V_N^\beta(x) = V^\beta(x) \tag{11}$$

and V^β is the only bounded solution to the following dynamic programming equation:

$$V^\beta(x) = \min_{a \in A(x)} \{c(x,a) + \beta \sum_{x' \in E} P(x'/x,a) V^\beta(x)\} \tag{12}$$

Since the $c(.,.)$ instant cost function is positive and the A action space is finite, then the optimal policy exists based on the assertion (ii) of the above proposition.

4.2 Function of transition probabilities

For each control value from $A = (1,2)$, we define the transition probability function $p(./.,a)$ on $E \times E$ by:

$$P(y\,x,a) = \lambda_1 \sum_{i=1}^q \mathbb{1}_{\{y = A_{a1i}\}}$$

$$+ \lambda_2 \left[\sum_{l=1}^q \mathbb{1}_{\{y = A_{a2l}\}} \mathbb{1}_{\{a=1\}} + \sum_{j=1}^m \mathbb{1}_{\{y = A_{b2i}\}} \mathbb{1}_{\{a=2\}} \right]$$

$$+ \mu \left[\sum_{l=1}^q \mathbb{1}_{\{y = D_{a1i}\}} + \sum_{l=1}^q \mathbb{1}_{\{y = D_{a2l}, x^{a1l}=0\}} \right]$$

$$+ \mu' \left[\sum_{j=1}^m \mathbb{1}_{\{y = D_{b2j}\}} \right]$$

$$\tag{13}$$

with $x = (x^a, x^b) = (x^{a1}, x^{a2}, x^{b2})$, where
$$x = (x^a, x^b) = (x^{a11}, x^{a12}, ..., x^{a1q}, x^{a21}, x^{a22},$$
$$..., x^{a2q}, x^{b21}, x^{b22}, ..., x^{b2m})$$

$$A_{a1i}x = (x^{a11}, ..., (x^{a1i}+1), ..., x^{a1q}, x^{a21}, x^{a22}$$
$$, ..., x^{a2q}, x^{b21}, x^{b22}, ..., x^{b2m})$$
$$A_{a2i}x = (x^{a11}, x^{a12}, ..., x^{a1q}, x^{a21}, ..., (x^{a2i}+1)$$
$$..., x^{a2q}, x^{b21}, x^{b22}, ..., x^{b2m})$$
$$A_{b2j}x = (x^{a11}, x^{a12}, ..., x^{a1q}, x^{a21}, x^{a22}$$
$$, ..., x^{a2q}, x^{b21}, ..., (x^{b2j}+1), ..., x^{b2m})$$
$$D_{a1i}x = (x^{a11}, ..., (x^{a1i}-1)^+, ..., x^{a1q}, x^{a21}, x^{a22}$$
$$, ..., x^{a2q}, x^{b21}, x^{b22}, ..., x^{b2m})$$
$$D_{a2i}x = (0, x^{a12}, ..., x^{a1q}, x^{a21}, ..., (x^{a2i}-1)^+$$
$$, ..., x^{a2q}, x^{b21}, x^{b22}, ..., x^{b2m})$$
$$D_{b2j}x = (x^{a11}, x^{a12}, ..., x^{a1q}, x^{a21}, x^{a22}$$
$$, ..., x^{a2q}, x^{b21}, ..., (x^{b2j}-1)^+, ..., x^{b2m})$$

We define the function value in N step by $V_N^\beta = T V_{N-1}^\beta$ with T is the operator defined on $F(E, \Box)$ by:

$$V_N^\beta = T V_{N-1}^\beta = c_1(x^{a1} + x^{a2}) + c_2 x^{b2}$$

$$+ \beta\mu \left(\sum_{i=1}^q V_{N-1}^\beta(D_{a1i}x) + V_{N-1}^\beta(D_{a2i}x) \mathbb{1}_{\{x^{a1i}=1\}} \right)$$

$$+ \beta\mu' \sum_{j=1}^m V_{N-1}^\beta(D_{b2j}x) + \beta\lambda_1 \sum_{i=1}^q V_{N-1}^\beta(A_{a1i}x)$$

$$+ \beta\lambda_2 \min_a \left(\sum_{i=1}^q V_{N-1}^\beta(A_{a2i}x) \mathbb{1}_{\{a=1\}}, \sum_{j=1}^m V_{N-1}^\beta(A_{b2j}x) \mathbb{1}_{\{a=2\}} \right)$$

4.3 Structural properties

We will show that the optimal cost function $V_N^\beta(x)$ defined by 6, satisfies the following properties:

- $V_N(x)$ is increasing in $x^{a1i}, x^{a2i}, x^{b2j}$ with $i = 1, 2, ..., q$ and $j = 1, 2, ..., m.$

- $V_N(A_{a2i}x) - V_N(A_{b2j}x)$ is increasing in x^{a1i}, and x^{a2i} is decreasing in x^{b2j}.

- $V_N(x+y) - V_N(x)$ is increasing in x^a, x^b.

Let H be the set of functions $V_N^\beta(x)$ defined by 6, and which verify the three properties P1), P2) and P3) above, containing the function identically zero, such that:

$$V_N^\beta \in H \to V_{N+1}^\beta := TV_N^\beta \in H$$

Where T is the operator defines in the Function value in N steps. Assuming that H is closed for point limits (i.e., the point limit of any f_n function sequence of H is a H function), then $V_\beta \in H.$

The property P2 characterizes the sending regions of clients of type 2 arriving at *a and b*.

If the optimality is achieved when a client of type 2 is sent to the MPM (respectively, to the RPM) then any decision to add after that, one or more clients of type 2 in the RPM (respectively, in the MPM) is optimal. If it is optimal to do consolidation when the system state is x (i.e., $V(A_{a2i}x) \leq V(A_{b2j}x)$) then the same decision remains optimal when there is moreover a customer in PM reserves (i.e., to the states A_{b2j}).

Indeed, property P2) implies that:

$$V(A_{a2i}A_{b2j}x) - V(A_{b2j}A_{b2j}x) \leq V(A_{a2i}x) - lV(A_{b2j}x) < 0$$
$$\Rightarrow V(A_{a2i}A_{b2j}x) \leq V(A_{b2j}A_{b2j}x)$$

Similarly, using the same property P2), we show that if it is optimal to use a new PM for a client of type 2 when the system state is x (i.e., $V(A_{b2j}x) \leq V(A_{a2i}x)$)), then the same decision remains optimal when there is more than one client in all MPM (i.e., in the states $A_{a1i}x, A_{a2i}x$).

5. Numerical Results

We will execute the value-iteration algorithm on MATLAB for different cases. Since the set of states E is of dimension 3, and the optimal policy for the control of type 2 arrivals depends on all 3 variables of the state of the system, then to properly observe the difference between the illustrations of the optimal policies for different cases, we limit ourselves to the presentation of these illustrations at the intervals [0, 10] on the axis (OX) (10 RMP) and [0, 25] on the axis (OY) (25 MPM) for the optimal policy for the control of type 2 arrivals.

For the illustration of the optimal policy for routing control of type 2 arrivals, the sending region to the MPM is characterized by $a = 1$, and the sending region to the RPM by $a = 2.$
In all the examples below we take,

$$\lambda_1 = 20, \lambda_2 = 10, \mu = 55, \mu' = 45, \beta = 0.99.$$

5.1. Transition probabilities

We define the transition probabilities for each action $a = (1, 2)$
We notice

$$\upsilon_a = \lambda_1 + \lambda_2 + \mu 1\!1_{\{x^{a1i} \geq 1\} \cup \{x^{a1i} = 0, x^{a2i} \geq 1\}} + \mu' 1\!1_{\{x^{b2j} \geq 1\}}$$

for action $a = 1$ at the moment $x = (x^a, x^b)$

-

$$P(y/x,1) = \begin{cases} \dfrac{\lambda_1}{\upsilon_a} & \text{si } y = A_{a1i}x \\[2mm] \dfrac{\lambda_2}{\upsilon_a} & \text{si } y = A_{a2i}x \\[2mm] \dfrac{\lambda_1}{\upsilon_a} & \text{si } y = D_{a1i}x 1\!1_{\{x^{a1i} \geq 1\}} + D_{a2i}x 1\!1_{\{x^{a1i}=0, x^{a2i} \geq 1\}} \\[2mm] \dfrac{\lambda_1}{\upsilon_a} & \text{si } y = D_{b2j}x 1\!1_{\{x^{b2j} \geq 1\}} \end{cases}$$

- for action $a = 2$ at the moment $x = (x^a, x^b)$

-

$$P(y/x,2) = \begin{cases} \dfrac{\lambda_1}{\upsilon_a} & \text{si } y = A_{a1i}x \\[2mm] \dfrac{\lambda_2}{\upsilon_a} & \text{si } y = A_{b2j}x \\[2mm] \dfrac{\lambda_1}{\upsilon_a} & \text{si } y = D_{a1i}x 1\!1_{\{x^{a1i} \geq 1\}} + D_{a2i}x 1\!1_{\{x^{a1i}=0, x^{a2i} \geq 1\}} \\[2mm] \dfrac{\lambda_1}{\upsilon_a} & \text{si } y = D_{b2j}x 1\!1_{\{x^{b2j} \geq 1\}} \end{cases}$$

Example 1:
We take $c_1 = 50, c_2 = 60,:$

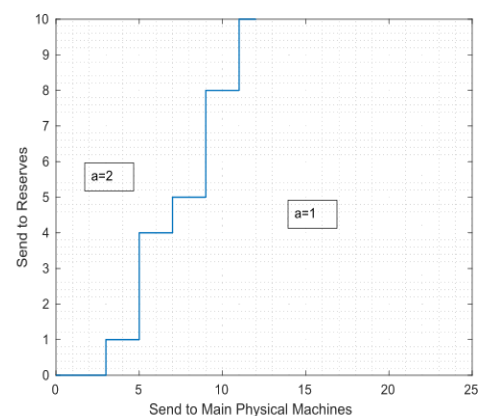Number of iterations done is: $n = 3261$

Fig. 2: Illustration of the optimal policy for routing control

of type 2 arrivals Ex.1

Example 2
$c_1 = 50, c_2 = 80,:$
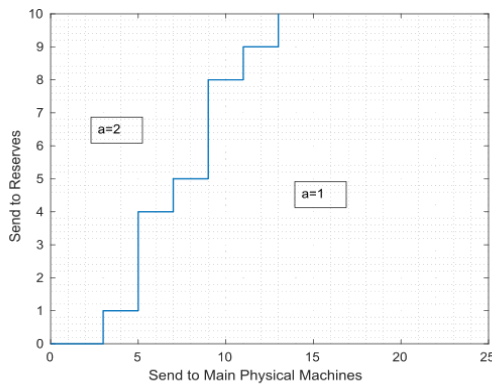
Number of iterations done is: $n = 3293$

Fig. 3: Illustration of the optimal policy for routing control

of type 2 arrivals Ex.2

Example 3:

$$c_1 = 50, c_2 = 2 * c_1,:$$

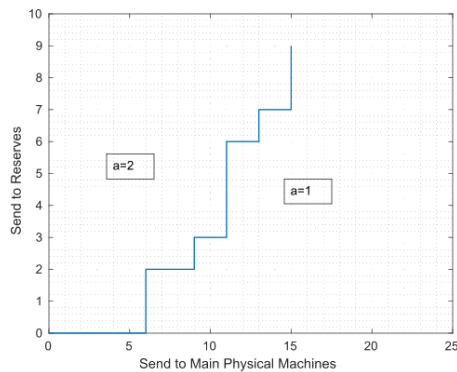Number of iterations done is: $n = 3274$.

Fig. 4: Illustration of the optimal policy for routing control of type 2arrivals Ex.3

Figures (2), (3) and (4) clearly illustrate that there is a switching curve between the sending regions of the type 2 arriving customers towards the MPM and RPM, and that this curve is monotonic. Comparing the 3 figures, we see that the sending region of type 2 arrivals to the RPM increases when the cost c_2 increases.

6. Conclusion

Using dynamic programming theory, the existence of the optimal policy for the control of type 2 arrivals has been well demonstrated. Structural properties have also been obtained for this optimal policy, and it has been shown that this optimal policy is monotonous, i.e:

- There is a monotonic switching curve between the sending regions of Type 2 arrivals to MPM and RPM.
- When the cost of sending a client of type 2 increase, then the sending region of the type 2 clients decreases.

In addition, we made a numerical study of our problem under MATLAB by programming the value-iteration algorithm for our model. The execution of this code gave us illustrations of the optimal policy.

As a perspective for this work, we propose the same model in which we add the breakdown to the servers, and we can add the rejection option either for type 2 arrivals, or type 1 arrivals, or for both. Also as a perspective of this work we will study the performance and energy consumption for the system using the machine allocation policy presented in [1, 6].

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