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Research paper



Formulation of the Problem of Maximum Clique Determination in Non-Oriented Graphs

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Abstract

In the train traffic organization, large amount of information should be processed in real time. Thereby in complicated dispatching systems, rational use of resources is needed. To perform this work a model of a management system is constructed and it is represented as a sparse graph. Further optimization of the model requires solving the Maximum Clique Problem (MCP) for the less time than the exponential time. This article contains two procedures for solving the task for subexponential time. Procedure B accurately estimates the size of the maximum clique in the graph and performs the sorting of the vertices, in such a way that the vertices that are not exactly in the maximum clique can be rejected. Procedure A, in fact, finds cliques of the maximum size in a given graph, using the procedure B. The main advantage of this method is that using it is expedient in real time for sufficiently large graphs, which in turn is important for the construction of control systems.

Keywords: *cliques in non-oriented graph; cost optimization tools; maximum clique problem; railway infrastructure; safety of control systems*

1. Introduction

The use of ceramic cutting tools from nano-structured refractory compounds [1] partially improves the performance of railway transport. However, in the operation of industry management systems, the problems of forecasting the technical condition of railway automation equipment in the conditions of limited statistical data on the safety of information-control systems are acute [2]. Functional safety is defined in the paper [3] and imposes certain restrictions on the operation of new railway automation systems [4]. Control, signalling and communication systems received new requirements in the paper [5]. One of the ways to improve the safety and accuracy of determining technical parameters for determining the parameters of motion and, as a consequence, improving the prediction of the technical state - the use of video analysis methods [6]. The use of video analysis methods with increasing accuracy leads to a significant increase in information flows and the construction of distributed computer networks. This prompted specialists to search for opportunities to optimize resources, both in the technical and in the information management subsystem. Simulation of the work of a local scheduler on the basis of solving problems of nonlinear Boolean programming has already been considered by the authors in [7]. This will optimize computing resources and prevent premature investments in hardware infrastructure.

The model of control systems under consideration is equivalent to formal definitions of sparse graphs. It can be replaced by the Maximum Clique Problem (MCP), which is one of the known hard-to-solve problems in graph theory. Solutions exist, but they take time exponentially depending on the size of the graph. And so far no algorithms have been found that solve it in polynomial time with the possibility of parallel computations. Thus, the current direction of the MCP research is the development of new approaches for finding exact and approximate solutions taking into account the peculiarities of graphs arising in applications.

The objective of the work was to develop a method for determining maximal cliques in non-oriented graphs with a small time complexity.

To achieve this objective, there was a need for the following:

- enter procedure B, which allows determining the estimates on top of the sizes of cliques in the graphs;
- enter procedure A, which allows using procedure B to generate cliques on the basis of each vertex of the graph and then select the largest clique in the graph.

2. Determination the Problem

We construct the solution of the MCP problem on the basis of obtaining the upper estimates of the sizes of the cliques in the graph. The basis is the rather obvious assertion 1.

Assertion 1. Suppose that a clique of size k is presented in the graph G (V, E). Then the number of vertices i with degree $d_i \geq k-1$ must be at least *k*. Thus, we have an upper bound on the size of the maximal clique $\Delta_{max} = k$ in the graph G (V, E).

Let G (V, E) be the graph containing n vertices. We note at once that the graph is not complete. This is easily verified and the solution of the maximum clique problem, in this case, is trivial. Hence the size of the maximum clique is strictly less than n. But this is a rough estimate. For clarity, we construct n subgraphs corresponding to the vertices of the graph. Namely: choose the vertex i = 1, select all the vertices associated with it. Next, we connect all vertices by edges in accordance with the constraints in the original graph G (V, E). We obtain a subgraph G1 without a vertex I = 1, we similarly construct subgraphs G_2, G_3, ..., G_n.

Summarizing, the formation of an arbitrary subgraph Gi on the basis of an arbitrary vertex i consists in isolating a subset of vertices connected with the vertex i. Next, we connect them by edges

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in accordance with the vertex connections that took place in the graph G. Obviously, the maximum size clique is presented in the original graph G (V, E) must be present in one of the subgraphs Gi. We construct procedure B to determine the upper estimates for the sizes of the cliques in the subgraphs G_i. Suppose that the graph G_i is given with its vertices sorted in decreasing order of powers d_v. That is, vertices having the same powers d_r are joined into subsets of $\{\alpha^r\}$ with cardinality $|\alpha^r| = p_r$ (where α^r are the numbers of the corresponding vertices in G_i), and p_r the number of vertices in G_i with the same powers of d_r. We denote the total number of such subsets γ. We sort the sequence of subsets of vertices $\{\alpha^r\}$ in such a way that they satisfy the inequalities:

$$d_{r=1} > d_{r=2} > \cdots > d_{r=\gamma}, \quad (1)$$

where d_r is the degree of the vertices of the subset$\{\alpha^r\}$. Then we form an ordered sequence:

$$\{\{\alpha^{r=1}\}, \{\alpha^{r=2}\}, \ldots \{\alpha^{r=\gamma}\}\}. \quad (2)$$

Ordered sequence (2) will be stored as a list S_γ. We propose the following procedure for determining the upper bound Δ_{max} on the list S_j on the basis of Assertion 1

Procedure B definitions Δ_{max}

Step 1. Sorting vertices and forming subsets $\{\alpha^r\}$.

Step 2. r: = 1.

Step 3. Check the inequality:

$$\sum_{i=1}^{r-1} P_i + P_r \geq d_r + 1, \quad (3)$$

if the inequality is satisfied, then $\Delta_{max} := d_r + 1$ and the procedure ends, otherwise, we perform the next step.

Step 4. $r := r + 1$ and proceed to step 3.

Note that in an inequality whose truth is verified in step 3 for r = 1, the sum is:

$$\sum_{i=1}^{r-1} P_i = 0. \quad (4)$$

An example of a definition Δ_{max} for the graph shown in Fig. 1.

When sorting vertices (Table 1), four subsets are formed: $\alpha^{r=1} = \{1\}$ with powers $d_r = 5$, $\alpha^{r=2} = \{3, 5, 6\}$ with powers $d_r = 4$, $\alpha^{r=3} = \{2, 7, 8\}$ with powers $d_r = 3$, and $\alpha^{r=4} = \{4\}$, with powers $d_r = 2$. Accordingly, the powers of the resulting subsets $P_{r=1}$ =1, $P_{r=2}$ = 3, $P_{r=3}$ = 3, $P_{r=4}$ =1. Assign r: = 1 to investigate the subset $\alpha^{r=3}$ and check the inequality (3), in this case it has the form $0 + 1 \geq 5 + 1$.

Inequality is not satisfied, therefore, further $r := r + 1 = 1 + 1 = 2$ and go to the set $\alpha^{r=2}$. We check the inequality $P_{r=1}+P_{r=2} \geq d_{r=2}+1$ and get 1+3 ≥ 4+1. This inequality is also not fulfilled. Then follows step 3 *r:*=r+1=2+1=3 and in step 3 go to the set $\alpha^{r=3}$. We check the inequality $P_{r=1}+P_{r=2}+P_{r=3} \geq d_{r=3}+1$, we get 1+3+3 ≥ 3+1. Since this inequality is satisfied, hence Δ_{max}=4. Thus, in the graph, the maximum clique cannot exceed the size equal to 4. If there was a larger clique in the graph, for example, size 5, then the number of vertices with a power of 4 should be greater than or equal to five, and for the size of the clique 4, the number of vertices with degrees 3 must be greater than or equal to four. The results of sorting the vertices of the graph are given in Table 1.

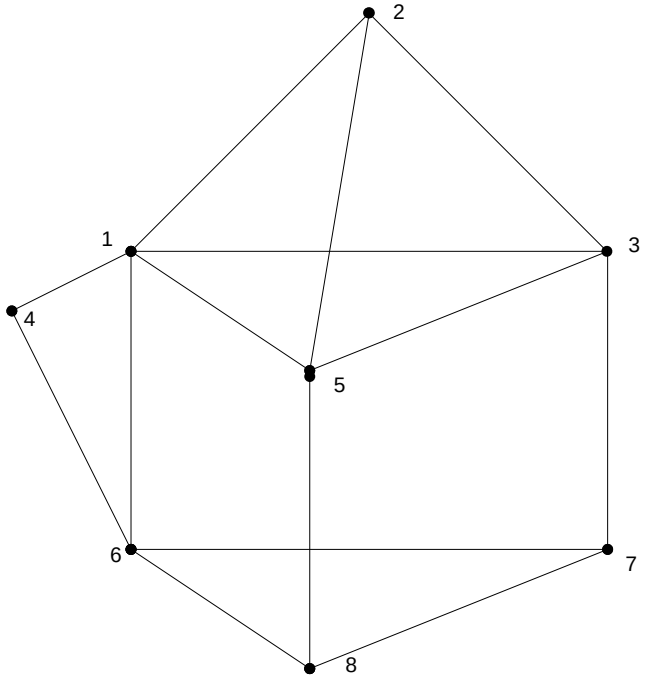


Fig. 1: The graph G.

Table 1: Results of sorting the vertices of a graph

Sets$\{\alpha^r\}$	Vertices *i*	Number of edges d_r	Cardinality P_r of the set$\{\alpha'\}$
Set $\alpha^{r=1}$	1	5	$P_{r=1}$=*1*
Set $\alpha^{r=1}$	3	4	$P_{r=2}$=3
	5	4	
	6	4	
Set $\alpha^{r=1}$	2	3	$P_{r=3}$=3
	7	3	
	8	3	
Set $\alpha^{r=1}$	4	2	$P_{r=4}$=1

Let the graph G consist of vertices {i}, $i = (\overline{1,n})$. We define the upper bounds $H = \{\Delta_{maxi}\}$ of the sizes of the maximal clique sizes that can be formed on the basis of vertices {i}, $i = (\overline{1,n})$. We introduce the notation, let there be k vertices in the set $H = \{\Delta_{maxi}\}$ with estimates$\{\Delta^*_{maxi} = k\}$, then we denote the subset of these vertices as $h_{max} = \{\Delta^*_{maxi} = k\}$. Let's say that a subset $h_{max} = \{\Delta^*_{maxi} = k\}$ has a property ν, if an inequality holds $\Delta^*_{maxi} > \Delta_{maxi}$, where Δ_{maxi} is the estimate of the vertices $\epsilon\, h_{max}$, and Δ_{maxi} is the estimate of the vertices belongs H. Then the following assertion holds.

Assertion 2. If in the set $H = \{\Delta_{maxi}\}$ obtained for some graph G there is a subset $h_{max} = \{\Delta^*_{maxi} = k\}$ satisfying the property ν, then the size of the maximal clique in the graph is equal to $|h_{max}| = \{\Delta^*_{maxi} = k\}$ and the subset of vertices {i} ϵh_{max}, that is, the corresponding $\{\Delta^*_{maxi} = k\}$ form a given clique.

Assertions 2 are true, since on the basis of all vertices not belonging to $h_{max} = \{\Delta^*_{maxi} = k\}$ it is impossible to build cliques large in size than cliques that can be constructed on the basis of vertices belonging to $h_{max} = \{\Delta^*_{maxi} = k\}$ (which follows from the property ν). The estimates in $h_{max} = \{\Delta^*_{maxi} = k\}$ are the same by definition, ($|h_{max}| = \Delta^*_{maxi}$) therefore it is impossible to construct a larger clique than the set of vertices {i}, corresponding to $\{\Delta_{maxi}\}$ in the graph G.

If in the set $H = \{\Delta_{maxi}\}$ obtained for the graph G, there is no subset $h_{max} = \{\Delta^*_{maxi}\}$ satisfying the property ν, then we sort the vertices of this graph in the decreasing order of the estimates Δ_{maxi}. We combine vertices with equal estimates Δ_{maxi} into subsets $\{\lambda^r\}$ with cardinality $|\lambda^r| = p_r$, the total number of such subsets denote as β.

We sort the sequence of subsets of vertices $\{\lambda^r\}$ into a sequence in such a way that the estimates corresponding to Δ_{maxi} satisfy inequalities:

$$\Delta_{maxi=1} > \Delta_{maxi=2} > \cdots > \Delta_{maxi=\beta}. \quad (5)$$

That is, we will compose an ordered sequence:

$$\{\{\lambda^{r=1}\}, \{\lambda^{r=2}\}, \ldots \{\lambda^{r=\beta}\}\}. \quad (6)$$

Sequence (6) will be stored as a list S_β. Using procedure B, we define a subset of vertices r^* for which inequality:

$$\sum_{i=1}^{r-1} P_i + P_r \geq \Delta_{\max r} + 1. \quad (7)$$

After determining the subset of vertices r^*, remove from the list S_β all the vertices of the graph *G* that lie after r^* with the edges incident to them.

Assertion 3. If the subgraph G' is constructed on the basis of the list S_β obtained after deleting the vertices for which $r>r^*$, then it contains a clique of the maximum size.

The validity of Assertion 3 is fairly obvious, because on the basis of the vertex in the list S_β after r^*, it is impossible to build cliques of a size larger than cliques based on the vertices in the list up to r^*.

3. Method of Solving the Problem

Using procedure *B*, we construct the procedure *A* of forming the maximal clique in an arbitrary graph *G*. The initial data for the operation of procedure *A* is the graph *G* itself and the list of vertices of this graph *S*.

Procedure *A*:

Step 1. We check whether there are vertices *Y = {i}* in the graph *G* associated with all vertices of *G*. The set of vertices of *Y* and the edges incident to them is deleted from the graph and put into the set *U*. After this, we check whether all vertices are removed from the graph *G*. If so, then the procedure ends, since the original graph represents the maximum size clique, otherwise we perform the next step.

Step 2. We check all the vertices that have the same degree in the graph. If so, we check if there is a vertex j connected to all vertices in U among the remaining vertices. If so, the procedure ends, since the set of vertices $\{j \cup U\}$ form the maximum clique in the graph, otherwise we perform the next step.

Step 3. Select the vertex i from the list S and form the subgraph Gi (Vi, Ei) on the basis of vertices adjacent to i.

We verify that in the subgraph $G_i(V_i, E_i)$ there are vertices D = {i} connected with all vertices of the subgraph $G_i(V_i, E_i)$, and remove the set of vertices D and the edges incident to them from the graph and place them in the set U_i, after this verifies whether all vertices are removed from the subgraph G. If so, then $\Delta_{maxi}=|V_I|$ otherwise we perform the next step.

Step 4. For the current subgraph $G_i^{/}(V_i, E_i)$, we define $\Delta^{|}_{maxi}$ on the basis of procedure B and then define $\Delta_{maxi}= \Delta^{|}_{maxi} + |U_i| + |U|$ and place Δ_{maxi} in the set H, and the vertices j included in U, if they exist in this set, we assign estimates equal to the maximal estimate in the set H, and assign them to H.

Step 5. Check the list of S for the presence of elements. If there are no items, go to step 3, otherwise, we perform the next step.

Step 6. Select in $H = \{\Delta_{maxi}\}$ a subset of $h_{max} = \{\Delta^*_{maxi}\}$ upper bounds for vertices having maximal same values. Check if equality holds$|h_{max}| = \{\Delta^*_{maxi}\}$. If this is true, then the set of vertices for which the $\{\Delta^*_{maxi}\}$ bounds are obtained form the maximum size clique and the procedure finishes, otherwise we perform the next step.

Step 7. We sort out the vertices of this graph in decreasing order of the estimations Δ_{maxi}, and we combine vertices with the same estimates Δ_{maxr} into subsets $\{\lambda^r\}$ with cardinality $|\lambda^r| = p_r$, the total number of such subsets will be denoted by β. As a result of sorting, we obtain a sequence of vertices satisfying the inequality:

$$\{\Delta_{maxr=1}\} \in \lambda^{r=1} > \{\Delta_{maxr=2}\} \in \lambda^{r=2} > \cdots > \{\Delta_{r=\beta}\} \in \lambda^{r=\beta}, \quad (8)$$

store it as a list S_r and proceed to the next step.

Step 8. Using procedure B, we define a subset of vertices r^*, starting with which inequality to (7).

In this case, all the vertices in the list S_r after r^* are deleted from the graph G with edges incident to them and delete these vertices from the list S_r. Next, *S*:= S_r and proceed to step 1.

4. Example of the Problem Solution

As an example, we find the maximum cliques in the graph G shown in Figure 1. We apply the procedure A to the list of vertices of the graph S = {1, 2, 3, 4, 5, 6, 7, 8}. None of the vertices of the graph is connected with all other vertices, therefore the set *U* = Ø. Now we proceed to the formation of subgraphs G_i corresponding vertices {i} for i=1, …, 8 of the original graph *G*. For each G_i , calculate Δ_{maxi}, using *B*. We start from the vertex i=1, which is connected with the vertices 2, 3, 4, 5, 6. Based on these vertices, we construct the subgraph G1 in Fig. 2.

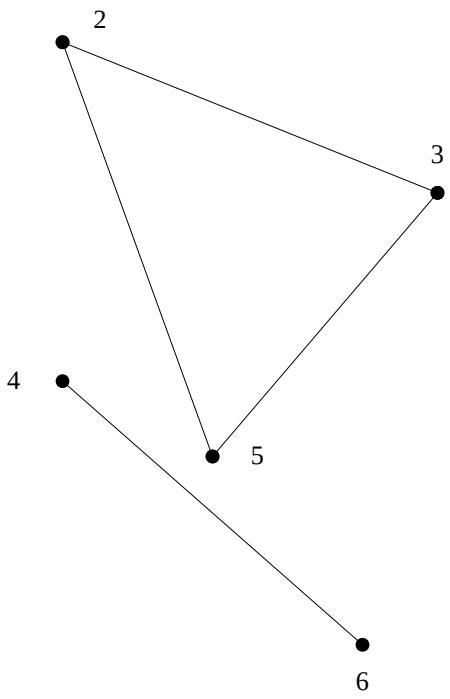


Fig. 2: Subgraph G_1.

We note that in the subgraph G_1 there are no vertices associated with all vertices of G_1. Let us specify that the subgraph G_1 is doubly-connected. Therefore, the set *U*1 = Ø. We sort the vertices of the graph G_1 according to formula (1). The result obtained will be placed in Table 2.

Assign *r:*=1, investigate the subset $\alpha^{r=1}$ and check the inequality (3). In this case it has the form $0 + 3 \geq 2 + 1$. The inequality is satisfied and, consequently, according to formula $\Delta_{max} := d_r + 1$ for a subgraph G_1 $\Delta_{max1} := 3$.

Obviously, the calculation for the subgraphs $G_2, \ldots, G_8$ is carried out in a similar manner. We obtain the set $H = \{\Delta_{maxi}\}$: $\Delta_{max2} := 3$, $\Delta_{max3} := 3$, $\Delta_{max4} := 2$, $\Delta_{max5} := 3$, $\Delta_{max6} := 2$, $\Delta_{max7} := 2$, $\Delta_{max8} := 8$. In this set we select the subset $h_{max} = \{\Delta^*_{maxi}\}$ upper bounds of vertices having maximal same values, that is $\Delta_{max1} := 3$, $\Delta_{max2} := 3$, $\Delta_{max3} := 3$, and $\Delta_{max5} := 3$.

Table 2: Results of sorting the vertices of a graph G_1.

Sets$\{\alpha^r\}$	Vertices i	Number of edges d_r	Cardinality P_r of a set $\{\alpha^r\}$
Set $\alpha^{r=1}$	2	2	$P_{r=1}$=3
	3	2	
	5	2	
Set $\alpha^{r=2}$	4	1	$P_{r=2}$=2
	6	1	

Based on this, we construct the sequence S_β. on Δ_{maxi} and placed in Table 3.

Table 3: Dependence of the sequence S_β on Δ_{maxi}

Sets$\{\beta^\gamma\}$	Vertices i	The upper bound on the size of the maximal clique Δ_{maxi} in the graph G_i.	Cardinality P_r of a set $\{\beta^\gamma\}$
Set $\beta^{\gamma=1}$	1	3	$P_{r=1}$=4
	2	3	
	3	3	
	5	3	
Set $\beta^{\gamma=2}$	4	2	$P_{r=2}$=4
	6	2	
	7	2	
	8	2	

It can be seen from the table that the inequality (7) starts to be fulfilled for r^*=1 and, therefore, the vertices *{4, 6, 7, 8}*, corresponding to the values r>1, are removed from the initial graph G and from the list. We obtain a new list S_β *={1, 2, 3, 5}*, and the graph G is transformed into the graph G' (Figure 3) with a list of vertices S= S_β.

We apply the procedure A to the obtained graph. In Step 1 we find out that all the vertices of the graph G' are connected with all vertices of this graph G' and go to the set U from the list S= S_β. Thus, they form a clique of the maximum size, both in the graph G' , and in the graph G. So, the problem is solved: the maximal clique from the vertices {1, 2, 3, 5} in the graph G. Other examples of the solution of Maximum Clique Problem are considered in [8].

Application of the described method to optimize the computational components of information and control systems of railway transport will allow to optimize the computing and telecommunication resources and to refrain from premature investments in the hardware infrastructure. Reducing the number of control components of technical systems increases operational safety and extends the additional, service indicators and characteristics of railway infrastructure management systems. Methodological aspect of the logistics technologies formation in reforming processes on the railways is described in paper [9].

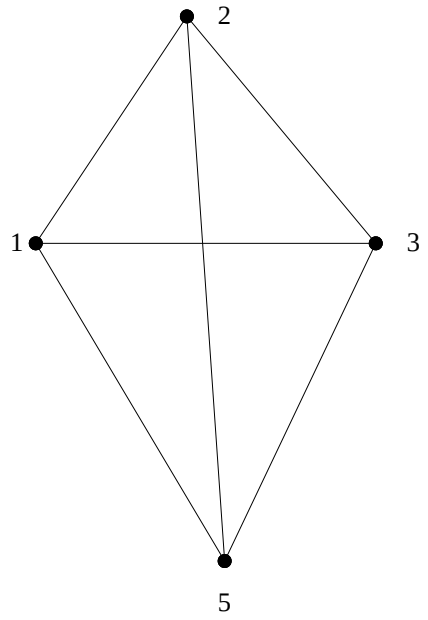


Fig. 3: Graph G' with list of vertices S= S_β.

The problem of finding the maximum clique in an undirected graph is the key method of optimization of computing resources for distributed, parallel computations in railway transport information systems. The proposed method is advisable to use to improve a qualimetric criterion for the generalized level of vehicle [10]. In the future, it is possible to combine the examples presented in paper with the use of the method with fuzzy support system for decision-making on safety of rolling stock [11].

A model of optimization and improvement of algorithms for air fleet planning was proposed in [12], and an application of an intelligent algorithm of fuzzy regression in the modeling of freight traffic was proposed in [13]. Improvement of the technology of rolling stock management in railway transport, in work [14] is realized on the basis of an artificial neural network. The proposed mathematical model requires substantial computing resources for its implementation. These computing resources can be redistributed by optimizing the already available computer equipment in the method proposed in this article.

An interesting aspect of the application of the method can be the transport infrastructure. The influence of structural characteristics on the electrical and physical properties of electrically conductive compositions based on mineral binders is proved in [15]. At the same time, continuous improvement, modernization and cheapening of sensors for monitoring soil moisture, especially capillary type, allows continuous monitoring of microprocessor and microcontroller systems by many parameters of building objects of critical transport infrastructure [16]. In the mass application of such monitoring, it will be advisable to use network technologies to consolidate data and distribute computing resources in the network. The quantitative proportions of the technical and economic indicators of the functioning of railway management systems, in particular and the safety of train traffic control, will be reported in subsequent publications. The expected effect of the innovations presented is several percent.

5. Conclusion

The procedure A, developed and presented, has substantive differences from the existing approaches, which use the solution of the problems of graph coloring and have exponential complexity. The proposed procedure has a small temporal complexity $O(2n^3 log_2 n)$, which allows one to solve such problems as determining the maximum clique in non-oriented graphs, determining the maximum independent sets and the minimum vertex coatings in the graphs, determining the graph isomorphism and isomorphic

embedding. This procedure can be effectively parallelized, which will reduce the temporal complexity of its work to $O(2n^3 log_2 n)$, and as a result it will solve the complex set of problems in real time.

The introduction of procedure A for making decisions on the safety of rolling stock will reduce losses by several percent.

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