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Research paper

Flow of a Bingham Fluid Through Circular Pipes with Variable Viscosity Coefficient Along the Pipe Length

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Abstract

Creation of conditions for optimal flow of Bingham media, such as ready-mixed concrete and mortar mixes, in the circular pipeline during delivery by various types of transporting equipment has not been sufficiently studied so far. Purpose: finding patterns of flow of concrete and mortar mixes in different sections of the pipeline, based on the variability of the viscosity coefficient when the medium is flowing through long circular pipes. In calculations of the flow capacity of mortar and concrete pipelines and the required power of the pumping equipment, the properties of concrete and mortar as Bingham media should be taken into account. Dependences for description of the operation process of flow of concrete and mortar mixes through circular pipelines have been found on the basis of the Buckingham equation in the laminar flow mode.

Dependences of determination of flow rate and required power with continuous delivery of concrete and mortar mixes in the pipeline, as well in case of alternate inlet pressure variation are presented. Law of variation of μ_p can be obtained only experimentally, which also results in errors in the computation model.

Keywords: *elastic limit; flow rate of ready-mixed concrete; pipeline; plastic viscosity coefficient; power shearing stress; tangential stress*.

1. Introduction

Creation of conditions for optimal flow of Bingham media, such as ready-mixed concrete and mortar mixes, in the circular pipeline during delivery by various types of transporting equipment has not been sufficiently studied so far.

The fact that the process of delivery of such viscoplastic media through pipelines is determined by their specific features and rheological properties is particularly noteworthy.

Choosing a flow model for viscoplastic media (fluids) enables a rational approach in future to choosing “pumping device – pipeline” characteristics.

2. Literature Data Analysis

When the processes occurring in non-Newtonian fluid are studied, concrete and mortar mixes are assumed as the Shvedov-Bingham media. The viscoplastic medium is a multicomponent medium that follows the rheological flow laws for Bingham fluids, and such a model is applicable within only a limited range of shear rates [1, 2, 3, 4, 5].

Such researchers as W.L. Wilkinson, A.V. Gnoyevoy, W. Prager, B.M. Smolsky, Z.P. Shulman, V.V. Gorislavets, S.S. Kutateladze, and others performed profound study of the processes and simulated the behaviour related to the flow of Bingham fluids.

The obtained models have been commonly used for rigid-viscoplastic media, such as concrete mixes, and generalized the majority of classical rheological equations of state of fluid media [6, 7, 8].

3. Purpose and Objectives of the Study

Purpose: finding patterns of flow of concrete and mortar mixes in different sections of the pipeline, based on the variability of the viscosity coefficient when the medium is flowing through long circular pipes.

The problems of theoretical studies include determining the productivity (flow rate) and necessary power consumption from the parameters of the concrete mix and the pipeline diameter.

An attempt is made to take these changes into account in solving the problems of flow of non-Newton power fluids through pipes of different cross sections, for example, a circular pipe is considered in this paper.

In calculations of the flow capacity of mortar and concrete pipelines and the required power of the pumping equipment, the properties of concrete and mortar as Bingham media should be taken into account.

The most important characteristics for the latter include the elastic limit τ_y and the plastic viscosity coefficient μ_p. The curve of one-dimensional flow of viscoplastic media is shown in Figure 1, where $\dot{\gamma}$ is shear rate of layers of fluid.

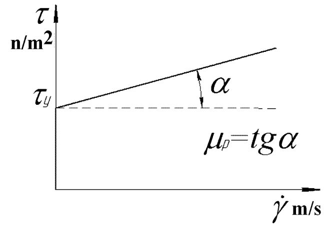

Fig. 1: Curve of one-dimensional flow of viscoplastic media.

4. Main Part of the Study

If the length of the pipeline is L= 100 m and the flow velocity is relatively slow (laminar), the properties of the fluid will vary along the pipeline. It refers primarily to the plastic viscosity μ_p.

4.1. The Case of Constant in Time Flow Rate through the Horizontal Pipeline Section

Let us consider a section of a horizontal pipeline of a length ΔL with the fluid, Figure 2. The equilibrium condition for the fluid in the section is represented as a volume.

$$\Delta P \cdot \pi \cdot R^2 = 2\pi \cdot R \cdot \tau_w \cdot \Delta L, \tag{1}$$

where τ_w – tangential stress on the pipe walls,
R – pipe radius, ΔP – pressure difference in ΔL.

Hence:

$$\tau_w = \frac{R \cdot \Delta P}{2 \cdot \Delta L}. \tag{2}$$

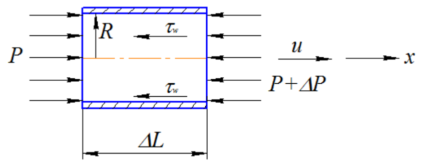

Fig. 2: Section of a horizontal pipe of a length ΔL.

If we cut mentally a cylindrical section of the fluid with a radius r, we will obtain:

$$\tau = \frac{r \cdot \Delta P}{2 \cdot \Delta L}, \tag{3}$$

where τ – shearing stress on radios r.

Thus, tangential stresses are distributed linearly on radius r:

$$\tau = \tau_w \cdot \frac{r}{R}.$$

Distribution of velocities on radius r_p can be found from the rheological equation for Bingham plastics (media) [9, 10]:

$$\tau - \tau_y = \mu_p \cdot \dot{\gamma}, \qquad (\tau > \tau_y),$$

where τ_y – static elastic limit of Bingham media.

Since $\dot{\gamma} = -\frac{du}{dr}$, then

$$\frac{du}{dr} = -\frac{\tau - \tau_y}{\mu_p(x)}. \tag{4}$$

On integrating this equation with respect to r, we will find:

$$u = -\frac{1}{\mu_p}\left(\frac{\tau_w}{2 \cdot R} \cdot r^2 - \tau_y \cdot r\right) + C. \tag{5}$$

The constant C can be found from the condition of fluid adhesion to the pipe wall: $r=R$, $u=0$.
Thus, we can find C:

$$C = \frac{1}{\mu_p}\left(\frac{\tau_w}{2 \cdot R} \cdot \frac{R^2}{2} - \tau_y \cdot R\right). \tag{6}$$

then

$$u(r) = \frac{(R-r) \cdot \tau_w}{\mu_p}\left(\frac{R+r}{2 \cdot R} - \frac{\tau_y}{\tau_w}\right). \tag{7}$$

On the boundary of the plastic flow

$$\left(r_p = \frac{\tau_y \cdot R}{\tau_w}\right):$$

$$u_p = \frac{R \cdot \tau_w}{2 \cdot \mu_p}(1-\tau_0)^2, \tag{8}$$

where u_p – flow velocity on the boundary of the plastic flow,

where we express as $\tau_0 = \frac{\tau_y}{\tau_w}$.

Let us find the expression for the rate:

$$Q_p = \pi \cdot r_p^2 \cdot u_p + \int_{r_p}^{R} 2\pi \cdot r \cdot u(r) \cdot dr = \\ = \frac{\pi \cdot R^3 \cdot \tau_w}{\mu_p}\left(\frac{1}{4} - \frac{\tau_0}{3} + \frac{\tau_0^4}{12}\right). \tag{9}$$

The last expression coincides with the known Buckingham formula [1]
The non-dimensional value τ_0 has the order 0.2 [9].

Therefore the last summand in the parentheses can be ignored as opposed to the first two ones, and then the rate is defined using the formula:

$$Q_p = \frac{\pi \cdot R^3 \cdot \tau_w}{\mu_p}\left(\frac{1}{4} - \frac{\tau_y}{3\tau_w}\right). \tag{10}$$

Average velocity of the fluid in the section is:

$$u_{cp} = \frac{Q_p}{\pi \cdot R^2} = \frac{R \cdot \tau_w}{\mu_p}\left(\frac{1}{4} - \frac{\tau_y}{3\tau_w}\right). \tag{11}$$

The rate is determined by the piston stroke of the pumping unit which creates pressure P_0 at the pipeline inlet.

Then the required capacity is $W_p = \pi \cdot R^2 \cdot P_0 \cdot u_{cp}$.

If flow rate Q_p is constant, then average velocity $u_{cp} = \frac{R \cdot \tau_w}{\mu_p}\left(\frac{1}{4} - \frac{\tau_y}{3 \cdot \tau_w}\right)$, should be constant in any section.

Hence, we can obtain the expression for τ_w, bound to μ_p (provided τ_y is fixed):

$$\tau_w = 4\left(\frac{u_{cp} \cdot \mu_p}{R} + \frac{\tau_y}{3}\right). \tag{12}$$

Pressure difference in the pipe section of a length ΔL with the variable in length near-wall stress $\tau_w = \tau_u(\mu_p(x))$ can be found from the condition:

$$\Delta P \cdot \pi \cdot R = \int_0^L 2 \cdot \pi \cdot R \cdot \tau_w(x) dx,$$

or

$$\Delta P_{(0-L)} = \frac{2}{R}\int_0^L \tau_w(x) dx. \tag{13}$$

If we assume that the viscosity coefficient μ_p varies linearly along the pipeline length, i.e. $\mu_p(x) = \mu_0 \pm kx$, then the corresponding variation of the near-wall stress τ_w will be:

$$\tau_w(x) = 4\left[\frac{u_{cp}}{R} \cdot (\mu_0 \pm k \cdot x) + \frac{\tau_y}{3}\right] = \tag{14}$$
$$= 4\left(\frac{u_{cp} \cdot \mu_0}{R} + \frac{\tau_y}{3} \pm \frac{u_{cp}}{R} \cdot k \cdot x\right).$$

Pressure difference in the section ΔL will be:

$$\Delta P_{(0-L)} = \frac{8}{R}\left[\begin{array}{l}\left(\frac{u_{cp} \cdot \mu_0}{R} + \frac{\tau_y}{3}\right) \times \\ \times L \pm \frac{u_{cp} \cdot k}{R} \cdot \frac{L^2}{2}\end{array}\right]. \tag{15}$$

In general terms, the effective power required to outdo friction on the pipe walls will be:

$$W_{(0-L)} = P_{(0-L)} \cdot \pi \cdot R^2 \cdot u_{cp}. \tag{16}$$

Alternatively, the power can be represented like this:

$$W_{(0-L)} = \frac{8}{R}\left[\begin{array}{l}\left(\frac{u_{cp} \cdot \mu_0}{R} + \frac{\tau_y}{3}\right) \times \\ \times L \pm \frac{u_{cp} \cdot k}{R} \cdot \frac{L^2}{2}\end{array}\right] \cdot \pi \cdot R^2 \cdot \frac{R \cdot \tau_w}{\mu_p}\left(\frac{1}{4} - \frac{\tau_y}{3 \cdot \tau_w}\right). \tag{17}$$

4.2. Flow of the Bingham Fluid through the Inclined Circular Pipe at Varying in Length Plastic Viscosity M_p and Varying in Length Pressure at the Pipeline Inlet

Figure 3 shows a section of an inclined pipeline of a length ΔL with the fluid.

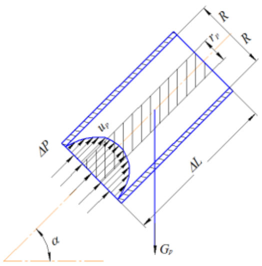

Fig. 3: Section of an inclined pipeline.

Internal radius of the inclined pipeline section is R.
r_p – radius of the plastic flow area;
ΔP – pressure difference in the section;
G – weight of the section element;
γ – specific gravity of the fluid;
α – angle of inclination of the section.

Let us write down condition of static equilibrium of the fluid volume in this section:

$$\Delta P \cdot \pi \cdot R^2 = 2\pi \cdot R \cdot \Delta L \cdot \tau_w + G \sin\alpha.$$

Here τ_w is near-wall shearing stress.

Since $G = \pi \cdot R^2 \cdot \Delta L \cdot \gamma$, then

$$\tau_w = \frac{\Delta P \cdot R}{2 \cdot \Delta L} - \frac{R_\gamma}{2}\sin\alpha. \tag{18}$$

where γ is specific gravity of the fluid.

If we represent:

$$C_0 = \frac{\Delta P}{2 \cdot \Delta L} - \frac{\gamma \cdot \sin\alpha}{2}. \tag{19}$$

then

$$\tau_w = R \cdot C_0. \tag{20}$$

For a cylindrical liquid element of a length ΔL and radius r, the expression for the shearing stress is written as:

$$\tau = C_0 \cdot r \cdot \tag{21}$$

4.3. Flow Rates

According to the rheological equation for Bingham fluids

$$\tau - \tau_y = -\mu_p \frac{du}{dr}.$$

If $r \le r \le R.$

then

$$\mu_p \frac{du}{dr} = \tau_y - C_0 \cdot r.$$

On integrating this equation from r to R, we obtain:

$$-\mu_p \cdot (u_{(R)} - u) = C_0 \cdot \left(\frac{R^2 - r^2}{2}\right) - \tau_y \cdot (R - r). \tag{22}$$

Since on the pipe wall $u_{(R)} = 0$,

$$\mu_p \cdot u = \frac{C_0}{2} \cdot (R^2 - r^2) - \tau_y \cdot (R - r) \tag{23}$$

On the boundary of the plastic flow $r = r_p$, $u = u_p$

$$\mu_p \cdot u_p = \frac{C_0}{2} \cdot (R^2 - r_p^2) - \tau_y \cdot (R - r_p). \tag{24}$$

From (20) we obtain: $C_0 = \frac{\tau_w}{R}$, an also

$$\frac{r_p}{R} = \frac{\tau_y}{\tau_w} = \tau_0.$$

Let us denote the parameter ratio q_0:

$$q_0 = \frac{u_p \cdot \mu_p}{R \cdot \tau_y}. \qquad (25)$$

Thus, the equation (2.7) can be reconstructed as:

$$\tau_0^2 - 2 \cdot (1 + q_0) \cdot \tau_0 + 1 = 0. \qquad (26)$$

Solution of this quadratic equation:

$$\tau_{01,2} = 1 + q_0 \pm \sqrt{(q_0^2 + 2 \cdot q_0)}. \qquad (27)$$

4.4. With the Solution (27) the Near-Wall Stress T_w Can be Related to the Viscosity Coefficient M_p.

In Table 1 below, dependence of $\tau_{01,2}$ on q_0 is shown. The diagram is constructed for small values q_0 (Figure 4).

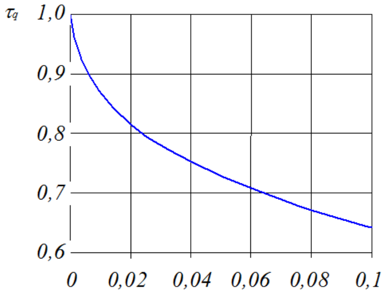

Fig. 4: Dependence of $\tau_{01,2}$ on q_0.

Table 1: Dependence of $\tau_{01,2}$ on q_0 for small values of q_0

q_0	0.05	0.1	0.5	0.8
$\tau_{01,2}^{(+)}$	1.37	1.56	2.62	3.30
$\tau_{01,2}^{(-)}$	0.73	0.64	0.38	0.30
q_0	1	1.5	2	3
$\tau_{01,2}^{(+)}$	3.73	4.79	5.83	7.87
$\tau_{01,2}^{(-)}$	0.27	0.21	0.17	0.13
q_0	4	5	8	10
$\tau_{01,2}^{(+)}$	9.90	11.92	17.94	21.9
$\tau_{01,2}^{(-)}$	0.10	0.08	0.06	0.05

Thus, the algorithm to determine τ_w depending on μ_p is as follows:
- on the basis of the problem setting we calculate

$$q_0 = \frac{u_p \cdot \mu_p}{R \cdot \tau_y}.$$

(here coefficient μ_p is assumed to be given, τ_y – elastic limit of the fluid, which also should be known, and velocity u_p is determined by the piston stroke of the pumping unit).
- in accordance with q_0 we can either calculate τ_0 using formula (27) or find it in the table (only the second radical shall be taken $\tau_{01,2}^{(-)}$, since in the physical sense $\tau_0 = \frac{\tau_y}{\tau_w} < 1$,
- knowing τ_0, we can determine $\tau_w = \frac{\tau_y}{\tau_0}$.

4.5. Flow of the Bingham Fluid through the Inclined Pipe at the Varying Pressure at the Pipeline Inlet.

If a distributor of piston mortar and concrete pumps is used, the diagram of dependency of inlet pressure on time is as shown in Figure 5.
If small pressure drops ΔF in the end of each stroke of the delivery of fluid to the pipeline are ignored (Figure 5), then power impulses in each stroke ($I_1, I_2, I_3 \ldots$) will be equal to the areas under the power diagram in this diagram.
Fluid is delivered to the pipeline in portions. Mass m is delivered with each stroke.

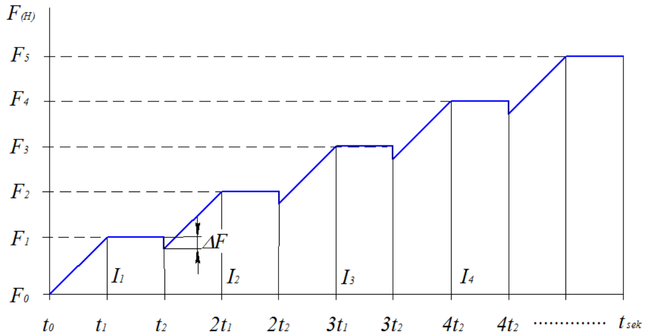

Fig. 5: Diagram of dependency of inlet pressure on time.

Centre of mass of this volume is situated on the axle of the pipe. Centre-of-mass velocity is velocity u_p on the boundary of the plastic flow, since this area move as a solid body.
Length l of the pipeline section taken by mass m of one stroke will be:

$$l = \frac{m \cdot g}{\gamma \cdot \pi \cdot R^2}. \qquad (28)$$

Let us write down the impulse theorem successively for each stroke shown in Figure 6:

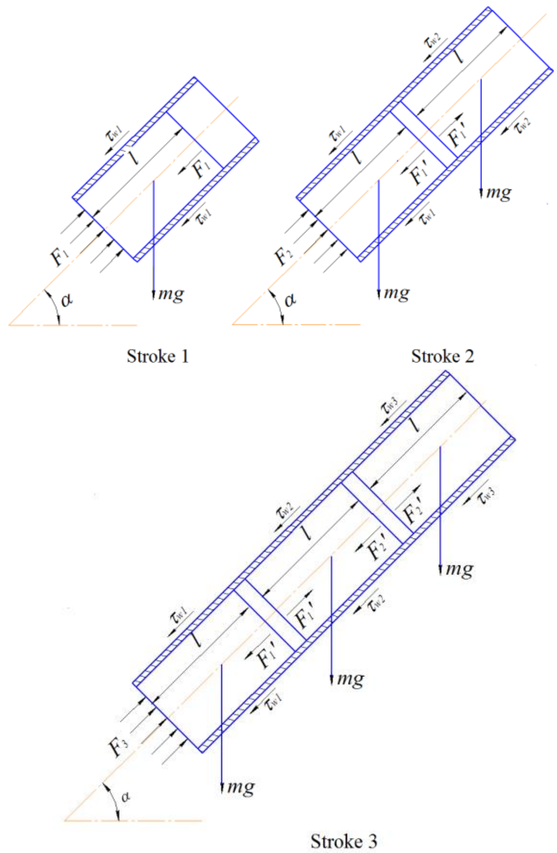

Fig. 6: Diagram of the fluid delivery in strokes.

Stroke 1:

$$m \cdot u_p = I_1 = (F_1 - m \cdot g \cdot \sin\alpha - 2\pi \cdot R \cdot l \cdot \tau_{w1}) \cdot (t_2 - \frac{t_1}{2}),$$

Stroke 2:

$$2 \cdot m \cdot u_p = I_2 =$$
$$= (F_2 - 2 \cdot m \cdot g \cdot \sin\alpha - 2\pi \cdot R \cdot l \cdot (\tau_{w1} + \tau_{w2})) \cdot (t_2 - \frac{t_1}{2}),$$

Stroke 3:

$$3 \cdot m \cdot u_p = I_3 =$$
$$= (F_3 - 3 \cdot m \cdot g \cdot \sin\alpha - 2\pi \cdot R \cdot l \cdot (\tau_{w1} + \tau_{w2} + \tau_{w3})) \cdot (t_2 - \frac{t_1}{2}),$$

Stroke *n*:

$$n \cdot m \cdot u_p = I_n = (F_n - n \cdot m \cdot g \cdot \sin\alpha - 2\pi \cdot R \times$$
$$\times l \cdot (\tau_{w1} + \tau_{w2} + \tau_{w3} + ... + \tau_{wn})) \cdot (t_2 - \frac{t_1}{2}), \quad (29)$$

Here it is assumed that the right (upper) end of the pipeline is open and the pressure in it is equal to atmospheric pressure.
Number of strokes *n* depends on the length of the pipeline and is

$$n \approx \frac{L}{l},$$

where *L* – total length of the pipeline.

Hence we can obtain the inlet pressure force required for fluid delivery through the pipeline of a length $L = n \cdot l$ to a height

$$H \cdot \sin\alpha = \frac{H^2}{L}:$$

$$F_n = \frac{n \cdot m \cdot u_p}{t_2 - \frac{t_1}{2}} + n \cdot m \cdot g \frac{H}{L} + 2\pi \cdot R \cdot l \cdot \sum_{i=1}^{n} \tau_{wi}. \quad (30)$$

Rate u_p on the boundary of the plastic flow is used in the formulas (29) and (30) to determine the centre-of-mass velocity of each portion of fluid.
If we know the flow rate *Q* we can calculate only average in the section velocity u_{cp}.
The relation between u_p and u_{cp} can be approximately established as follows. We use the formulas (8) and (11):

$$u_p = \frac{R \cdot \tau_w}{2 \cdot \mu_p}(1 - \tau_0)^2, \qquad u_{cp} = \frac{R \cdot \tau_w}{2 \cdot \mu_p}(\frac{1}{4} - \frac{\tau_0}{3}).$$

It follows that:

$$\frac{R \cdot \tau_w}{2\mu_p} = \frac{2 \cdot u_p}{(1 - \tau_0)^2}.$$

Then $u_{cp} = \frac{2 \cdot u_p}{(1 - \tau_0)^2} \cdot \left(\frac{1}{4} - \frac{\tau_0}{3}\right).$

As it was mentioned above, τ_0 has the order 0.2. With such τ_0, $u_{cp} = 0{,}573 \cdot u_p$ [9, 10].

$$u_p = 1{,}57 \cdot u_{cp}. \quad (31)$$

τ_0 can be determined more precisely in each stroke as it is shown above in par. 4.1. Of course, the formula (30) will be more complicated, since velocity u_p will be different in each stroke.

The required capacity of the pumping unit (piston mortar and concrete pump) is:

$$W_{рбн} = \frac{n \cdot m \cdot u_p}{t_2 - \frac{t_1}{2}} + n \cdot m \cdot g \frac{H}{L} +$$
$$+ 2\pi \cdot R \cdot l \cdot \sum_{i=1}^{n} \tau_{wi} \cdot \frac{2 \cdot u_p}{(1 - \tau_0)^2}\left(\frac{1}{4} - \frac{\tau_0}{3}\right). \quad (32)$$

Or

$$W_{рбн} = F_n \cdot (0.573 \cdot u_p). \quad (33)$$

5. Conclusion

The proposed method of calculation of the required power of a pumping unit is approximate primarily because the viscosity coefficient μ_p varies along the length of the pipeline (concrete line) continuously, and not discretely, as it is assumed in the calculation.
Law of variation of μ_p can be obtained only experimentally, which also results in errors in the computation model.
The last conclusion is true if concrete and mortar mixes are considered as Bingham fluids, where the elastic limit value τ_y is established approximately.

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