



A New Two-Wing Chaotic System with Line Equilibrium, its Analysis, Adaptive Synchronization and Circuit Simulation

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Abstract

This work reports a new three-dimensional chaotic system with line equilibrium and two equilibrium points on the (x, y) plane. A qualitative analysis has been conducted on the system with the aid of bifurcation diagram, Lyapunov exponents spectrum, etc. It is shown that the new chaotic system is dissipative. Since the new chaotic system has infinitely many equilibrium points, it exhibits hidden attractor. Using adaptive control, global chaos synchronization of the new chaotic system with itself is established using Lyapunov stability theory. Finally, a circuit simulation of the new chaotic system with line equilibrium is carried out via MultiSim and the feasibility of implementing the new chaotic system is established.

Keywords: *Chaos, chaotic systems, line equilibrium, synchronization, circuit simulation.*

1. Introduction

Chaos theory ([1]-[2]) is a branch of Mathematics that deals with the modelling and analysis of nonlinear dynamical systems that are highly sensitive to small changes in the initial conditions. Chaotic systems find numerous applications such as weather models ([3]-[5]), biological models ([6]-[9]), jerk models ([10]-[13]), ecological models ([14]-[16]), finance systems ([17]-[20]), oscillations ([21]-[28]), neural networks ([29]-[31]), chemical reactions ([32]-[35]), steganography ([36]-[37]), cryptography ([38]-[41]), robotics ([42]-[43]), circuits ([44]-[55]), etc.

In the chaos literature, there is great interest shown in the finding of chaotic systems having an infinite number of equilibrium points such as line equilibrium ([56]-[59]), heart-shaped equilibrium [60], conch-shaped equilibrium [52], circle equilibrium [61], square equilibrium [62], rounded-square equilibrium [63], hyperbola equilibrium [64], cloud-shaped equilibrium [65], etc.

In the chaos literature, there is a good interest in finding chaotic systems exhibiting *two-wing* chaotic attractors ([66]-[72]). There are many classical examples of two-wing chaotic systems such as Lorenz system [66], Chen system [67], Lü system [68], Liu system [69], Tigan system [70], Cai system [71], Li-Wu system [72], etc.

In this research work, we report the finding of a new three-dimensional chaotic system exhibiting a two-wing attractor with line equilibrium and two equilibrium points on the (x, y) plane. We carry out a detailed dynamic analysis on the system with bifurcation diagram, Lyapunov exponents spectrum, etc. We show that the new

chaotic system is dissipative and has a rotation symmetry about the z -axis. Since the new chaotic system has infinitely many equilibrium points, it exhibits hidden attractor.

The synchronization of chaotic systems deals with the problem of asymptotically synchronizing a set of two chaotic systems named as master and slave systems ([73]-[75]). There are several methods available in the control literature for stabilizing and synchronizing chaotic systems such as active control ([76]-[78]), passive control ([79]-[80]), sliding mode control method ([81]-[83]), backstepping control method ([84]-[86]), adaptive control method ([87]-[89]), etc. In this work, we use adaptive control method for the global chaos synchronization of the new chaotic system with unknown parameters. Circuit realization of chaotic systems aids in implementing them for real-world applications [90]. Thus, it is important to build circuit designs for chaotic systems and verifying that the phase portraits match with the theoretic models ([91]-[93]). In this work, a circuit simulation of the new chaotic system with line equilibrium is carried out via MultiSim and the feasibility of implementing the new chaotic system is established.

This paper is arranged as follows. Section 2 describes the dynamics of the new chaotic system with line equilibrium and two equilibrium points on the (x, y) -plane. Section 3 describes the dynamic analysis of the new chaotic system. Section 4 contains new control results for the global chaos synchronization of the new chaotic system with itself. Section 5 describes the MultiSim circuit design of the new chaotic system and simulation results. Section 6 draws the main conclusions.

2. A new chaotic system with line equilibrium

In this research paper, we propose a new 3-D system given by the dynamics

$$\begin{cases} \dot{x} &= yz \\ \dot{y} &= x - y \\ \dot{z} &= a|x| - bx^4 \end{cases} \quad (1)$$

where x, y, z are the three states and a, b are system constants.

The system (1) has a total of 3 nonlinearities: an absolute function nonlinearity, a quadratic nonlinearity and a quartic nonlinearity.

The system (1) is *chaotic* when the parameters take the following values:

$$a = 5, \quad b = 2 \quad (2)$$

To simplify our notation, we set $X = (x, y, z)$.

For numerical simulations, we take $X(0) = (0.2, 0.2, 0.2)$.

Using Wolf's algorithm [95], the Lyapunov exponents of the system (1) for $(a, b) = (5, 2)$ and $X(0) = (0.2, 0.2, 0.2)$ are determined as

$$LE_1 = 0.1786, \quad LE_2 = 0, \quad LE_3 = -1.1786 \quad (3)$$

From the spectrum of Lyapunov exponents in (3), it is noted that the system (1) is chaotic as $LE_1 > 0$ and dissipative as the sum of Lyapunov exponents in (3) is found to be negative.

Furthermore, we calculate the Kaplan-Yorke dimension of the 3-D chaotic system (1) as

$$D_{KY} = 2 + \frac{LE_1 + LE_2}{|LE_3|} = 2.1515 \quad (4)$$

Thus, the chaotic system (1) has a fractional chaotic dimension.

The equilibrium points of the system (1) are calculated by solving the system of equations

$$yz = 0 \quad (5a)$$

$$x - y = 0 \quad (5b)$$

$$a|x| - bx^4 = 0 \quad (5c)$$

From Eq. (5b), it is immediate that $x = y$ for all equilibrium points. Thus, finding the equilibrium points of (1) amounts to solving the following equations

$$xz = 0 \quad (6a)$$

$$a|x| - bx^4 = 0 \quad (6b)$$

and noting that $y = x$ for the equilibrium points of (1).

We have two cases to consider, *viz.* (I) $x = 0$ and (II) $x \neq 0$.

In Case (I), we take $x = 0$. Since $y = x$ for equilibrium points, it is immediate that $y = 0$. The values $x = y = 0$ satisfy (6b) for all values of $z \in \mathbf{R}$.

Thus, we conclude that the z -axis in \mathbf{R}^3 consists of equilibrium points of the system (1).

In Case (II), we take $x \neq 0$. From (6a), we must have $z = 0$. Solving the equation $a|x| - bx^4 = 0$ for $a = 5$ and $b = 2$, we get the two roots $x = \pm 1.3572$. For the equilibrium points, $y = x$. Thus, correspondingly, we have $y = \pm 1.3572$. Hence, $E_1 = (1.3572, 1.3572, 0)$ and $E_2 = (-1.3572, -1.3572, 0)$ are also equilibrium points of the system (1).

Combining Cases (I) and (II), we conclude that the equilibrium points of the system (1) consist of the z -axis in \mathbf{R}^3 and the two points E_1 and E_2 in the (x, y) -plane.

Since the chaotic system (1) has a line equilibrium, it follows in particular that it has infinitely many equilibrium points. Thus, we conclude that this system exhibits hidden attractor.

It is easy to verify that the chaotic system (1) is invariant under the change of coordinates

$$(x, y, z) \rightarrow (-x, -y, z) \quad (7)$$

for all values of the parameters a and b . This shows that the chaotic system (1) has rotation symmetry about the z -axis. Every non-trivial trajectory $(x(t), y(t), z(t))$ of the chaotic system (1) has a twin trajectory in view of this rotation symmetry about the z -axis.

For $(a, b) = (5, 2)$ and $X(0) = (0.2, 0.2, 0.2)$, the phase plots of the chaotic system (1) with hidden attractor are shown in Figures 1 to 3. The Lyapunov exponents of the chaotic system (1) are exhibited in Figure 4. The phase portraits of the new chaotic system (1) show that it exhibits a *two-wing attractor*.

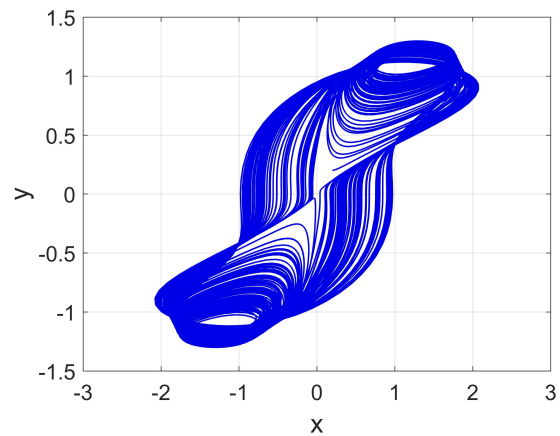


Figure 1: 2-D phase plot of the new two-wing chaotic system (1) in the (x, y) plane for $X(0) = (0.2, 0.2, 0.2)$ and $(a, b) = (5, 2)$

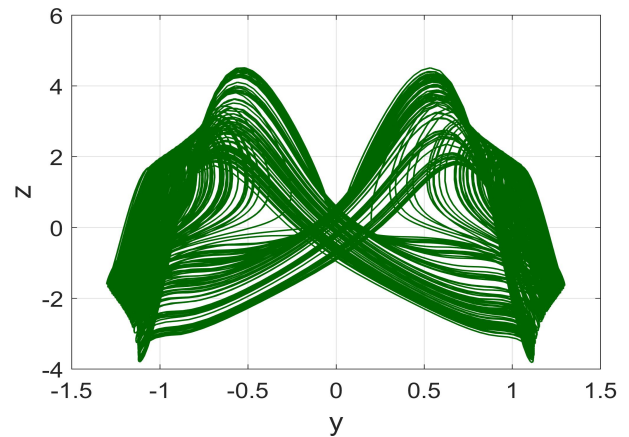


Figure 2: 2-D phase plot of the new two-wing chaotic system (1) in the (y, z) plane for $X(0) = (0.2, 0.2, 0.2)$ and $(a, b) = (5, 2)$

3. Dynamic analysis of the new chaotic system with line equilibrium

Bifurcation diagram is a dynamical tool, which can exhibit different dynamical behaviors of a system. The bifurcation diagram and the Lyapunov exponents of system (1) are reported in Figures 5 and 6. The parameter a is varied from 1 to 5 and the local maxima of z is plotted. For $a \geq 2$, system (1) displays only periodic state. For $a < 2$, new chaotic system with line equilibrium (1) exhibits chaotic behavior. In addition, the bifurcation diagram and Lyapunov exponent spectrum of system (1) are presented in Figures 7 and 8,

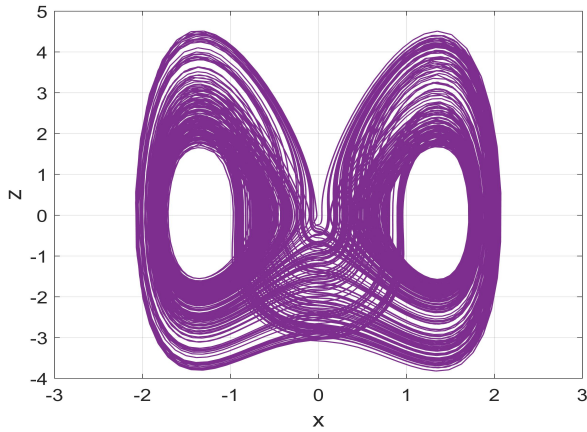


Figure 3: 2-D phase plot of the new two-wing chaotic system (1) in the (x, z) plane for $X(0) = (0.2, 0.2, 0.2)$ and $(a, b) = (5, 2)$

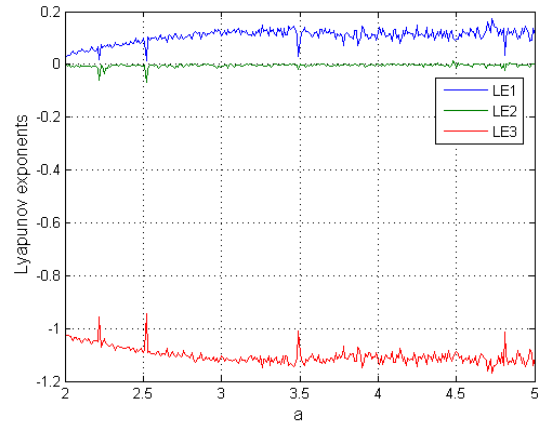


Figure 6: The diagram of Lyapunov exponents when varying a for $b = 2$

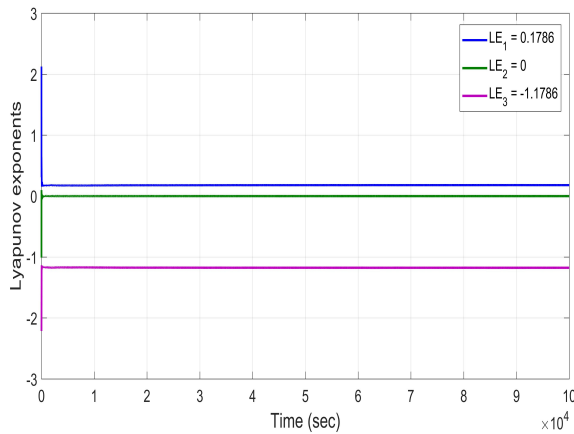


Figure 4: Lyapunov exponents of the new chaotic system (1) for $X(0) = (0.2, 0.2, 0.2)$ and $(a, b) = (5, 2)$

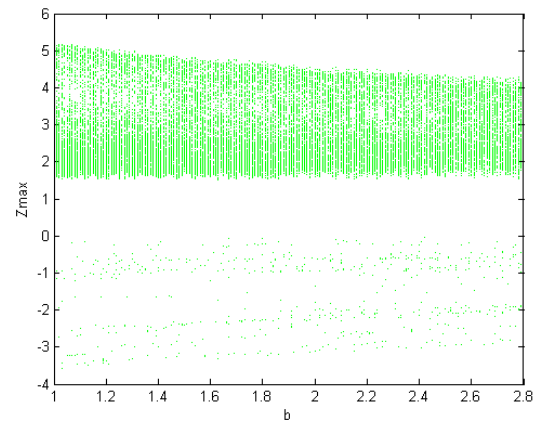


Figure 7: Bifurcation diagram of Z_{max} when varying b for $a = 5$

respectively. It is clear to see that the proposed system can generate chaotic behavior in the range 1 to 2.8 for parameter b .

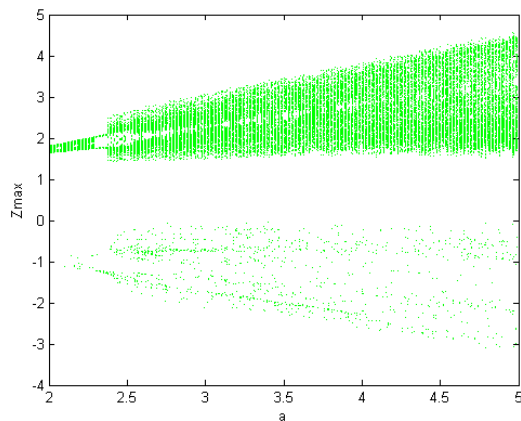


Figure 5: Bifurcation diagram of Z_{max} when varying a for $b = 2$

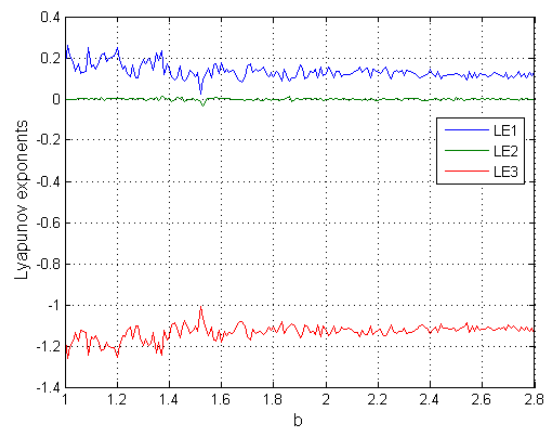


Figure 8: The diagram of Lyapunov exponents when varying b for $a = 5$

4. Adaptive synchronization of the new chaotic system with line equilibrium

In this section, we use adaptive control method for the design of synchronizing the new chaotic system with itself having unknown

system parameters.

As the master system, we take the new chaotic system

$$\begin{cases} \dot{x}_1 = y_1 z_1 \\ \dot{y}_1 = x_1 - y_1 \\ \dot{z}_1 = a|x_1| - bx_1^4 \end{cases} \quad (8)$$

where x_1, y_1, z_1 are the three states and a, b are unknown, system constants.

As the slave system, we take the new chaotic system with controls

given by

$$\begin{cases} \dot{x}_2 = y_2 z_2 + u_x \\ \dot{y}_2 = x_2 - y_2 + u_y \\ \dot{z}_2 = a|x_2| - bx_2^4 + u_z \end{cases} \quad (9)$$

where x_2, y_2, z_2 are the three states and u_x, u_y, u_z are the adaptive controls.

The synchronization error between the master and slave systems is defined via the equations

$$e_x = x_2 - x_1, \quad e_y = y_2 - y_1, \quad e_z = z_2 - z_1 \quad (10)$$

We calculate the synchronization error dynamics as follows.

$$\begin{cases} \dot{e}_x = y_2 z_2 - y_1 z_1 + u_x \\ \dot{e}_y = e_x - e_y + u_y \\ \dot{e}_z = a(|x_2| - |x_1|) - b(x_2^4 - x_1^4) + u_z \end{cases} \quad (11)$$

Since the parameters a, b are unknowns, we estimate them with $A(t), B(t)$, which are to be determined in a suitable manner.

We consider the adaptive controller given as follows:

$$\begin{cases} u_x = -y_2 z_2 + y_1 z_1 - k_x e_x \\ u_y = e_x - e_y - k_y e_y \\ u_z = a(|x_2| - |x_1|) - b(x_2^4 - x_1^4) - k_z e_z \end{cases} \quad (12)$$

where $k_x > 0, k_y > 0, k_z > 0$ are gain constants.

Substituting the adaptive control (12) into (11), we obtain the closed-loop error equation given by

$$\begin{cases} \dot{e}_x = -k_x e_x \\ \dot{e}_y = -k_y e_y \\ \dot{e}_z = e_a(|x_2| - |x_1|) - e_b(x_2^4 - x_1^4) - k_z e_z \end{cases} \quad (13)$$

where the parameter estimation errors are defined by

$$e_a = a - A(t), \quad e_b = b - B(t) \quad (14)$$

A simple calculation shows

$$\dot{e}_a = -\dot{A}(t), \quad \dot{e}_b = -\dot{B}(t) \quad (15)$$

Next, we consider the candidate Lyapunov function given by

$$V(e_x, e_y, e_z, e_a, e_b) = \frac{1}{2} (e_x^2 + e_y^2 + e_z^2 + e_a^2 + e_b^2) \quad (16)$$

The time-derivative of V is calculated as follows:

$$\dot{V} = \left. \begin{aligned} & -k_x e_x^2 - k_y e_y^2 - k_z e_z^2 + e_a [e_z(|x_2| - |x_1|) - \dot{A}] \\ & + e_b [-e_z(x_2^4 - x_1^4) - \dot{B}] \end{aligned} \right\} \quad (17)$$

In view of (17), we take the parameter update law as follows:

$$\begin{cases} \dot{A} = e_z(|x_2| - |x_1|) \\ \dot{B} = -e_z(x_2^4 - x_1^4) \end{cases} \quad (18)$$

Next, we state the main result of this section.

Theorem 1. *The new chaotic systems (8) and (9) are globally and exponentially synchronized for all initial states in \mathbf{R}^3 under the adaptive feedback control law (12) and the parameter update law (18), where $A(t), B(t)$ are estimates of the unknown constants a, b , respectively and k_x, k_y are positive gain constants.*

Proof. Clearly, V defined in (16) is a quadratic and positive definite function on \mathbf{R}^5 .

Substituting the parameter update law (18) into the time-derivative of the Lyapunov function V given by the equation (17), we obtain

$$\dot{V} = -k_x e_x^2 - k_y e_y^2 - k_z e_z^2 \quad (19)$$

which is negative semi-definite in \mathbf{R}^5 . Thus, by Barbalat's lemma in Lyapunov stability theory [94], the synchronization error $(e_x(t), e_y(t), e_z(t))$ converges to zero exponentially as $t \rightarrow \infty$.

This completes the proof. ■ □

For numerical simulations, we take the parameter values as in the chaotic case, viz. $(a, b) = (5, 2)$. We take the gain constants as $k_x = k_y = k_z = 10$.

The initial conditions of the master system (8) are taken as

$$x_1(0) = 2.1, \quad y_1(0) = -1.8, \quad z_1(0) = 1.2 \quad (20)$$

The initial conditions of the slave system (9) are taken as

$$x_1(0) = 3.1, \quad y_1(0) = 5.3, \quad z_1(0) = -2.4 \quad (21)$$

The initial conditions of the parameter estimates are taken as

$$A(0) = 2.7, \quad B(0) = 5.4 \quad (22)$$

Figure 9 shows the synchronization of the new chaotic systems (8) and (9), while Figure 10 shows the time-history of the synchronization errors e_x, e_y, e_z .

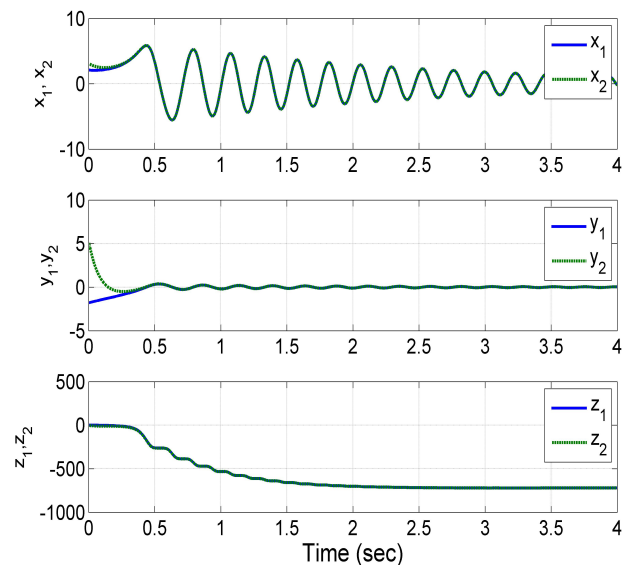


Figure 9: Global chaos synchronization of the new chaotic systems (8) and (9)

5. Circuit simulation of the new chaotic system with line equilibrium

In this work, circuit design and implementation of the new chaotic system (1) are described to validate the chaotic behavior of system (1). We utilize MultiSIM program in designing the electronic circuit. The circuit design of system (1) with $a = 5, b = 2$, is shown in Figure 11. In this study, a linear scaling is considered as follows:

$$\begin{cases} \dot{x} = \frac{1}{2}yz \\ \dot{y} = x - y \\ \dot{z} = a|x| - \frac{b}{8}x^4 \end{cases} \quad (23)$$

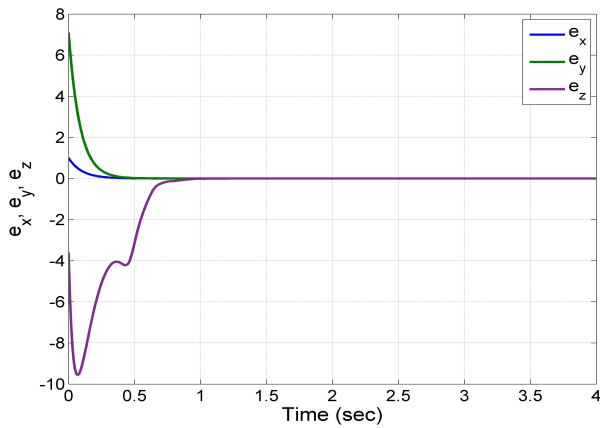


Figure 10: Time-history of the synchronization error between the new chaotic systems (8) and (9)

The state-space equations of the electronic circuit are as follows:

$$\begin{cases} \dot{x} = \frac{1}{C_1 R_1} yz \\ \dot{y} = \frac{1}{C_2 R_2} x - \frac{1}{C_2 R_3} y \\ \dot{z} = \frac{1}{C_3 R_4} |x| - \frac{1}{C_3 R_5} x^4 \end{cases} \quad (24)$$

We have implemented the circuit in MultiSIM by selecting the components $R_1 = 800 \text{ k}\Omega$, $R_2 = R_3 = 400 \text{ k}\Omega$, $R_4 = 80 \text{ k}\Omega$, $R_5 = 1.6 \text{ M}\Omega$, $R_6 = R_7 = R_8 = R_9 = R_{10} = R_{11} = R_{12} = R_{13} = R_{14} = R_{15} = R_{16} = 100 \text{ k}\Omega$, and $C_1 = C_2 = C_3 = 1 \text{ nF}$. Figure 12 presents the phase portraits of the electronic circuit. It is seen from Figure 12 that attractors plot obtained using MultiSIM simulation matches with the results obtained using MATLAB simulation (see Figures 1-3).

6. Conclusion

This work reported a new 3-D chaotic system with line equilibrium and two equilibrium points on the (x,y) plane. We carried out a detailed dynamic analysis of the new dissipative chaotic system with hidden attractors via bifurcation diagram, Lyapunov exponents spectrum, etc. As an engineering application, global chaos synchronization of the new chaotic system with itself was established using adaptive control method and Lyapunov stability theory. Finally, a circuit simulation of the new chaotic system with line equilibrium was performed via MultiSIM and the feasibility of implementing the new chaotic system was successfully established.

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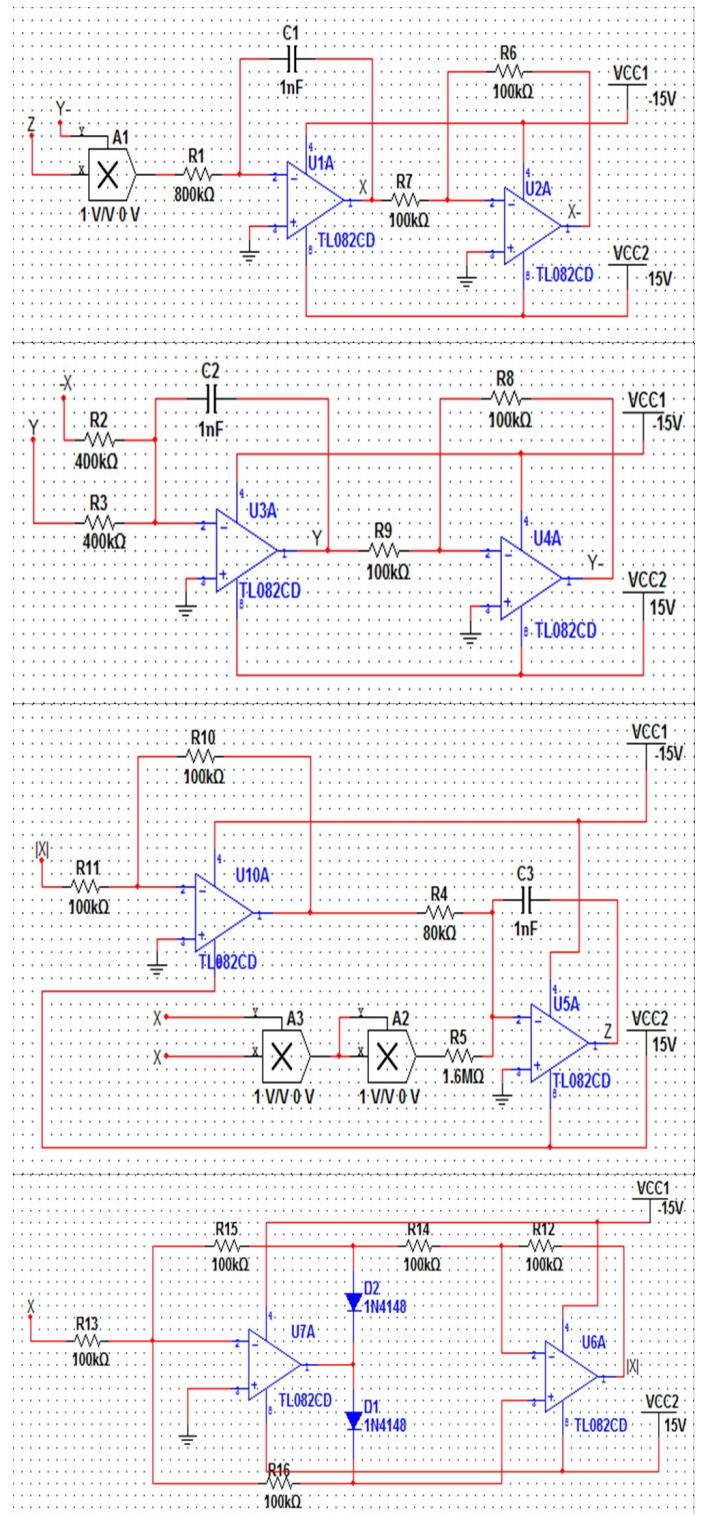
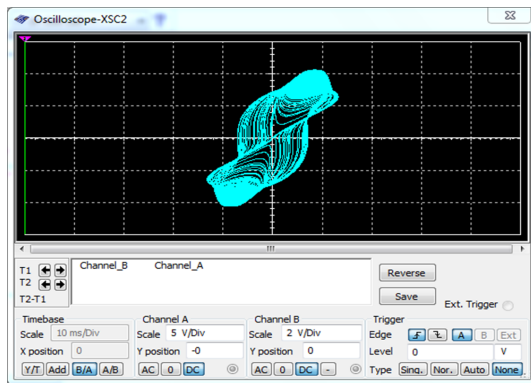


Figure 11: Circuit design of the new chaotic system (1)

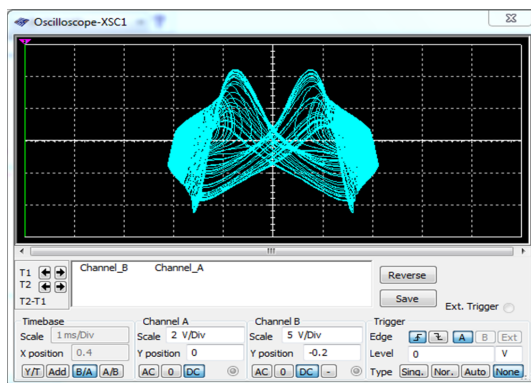
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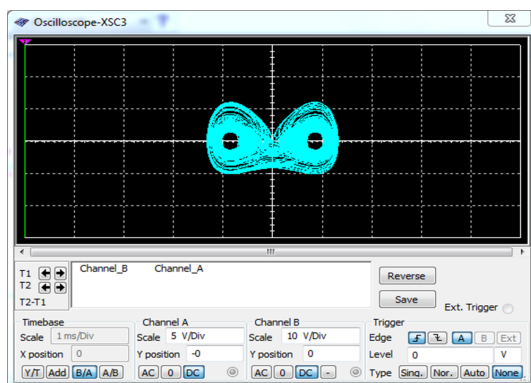
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(a)



(b)



(c)

Figure 12: Chaotic attractors of the new chaotic system (1) using MultiSIM circuit simulation: (a) $x - y$ plane, (b) $y - z$ plane and (c) $x - z$