



# Maximizing Energy Efficiency in Downlink Massive MIMO Systems by Full-complexity Reduced Zero-forcing Beamforming

Adeeb Salh, Lukman Audah\*, Nor. S. M. Shah, Shipun. A. Hamzah

Wireless and Radio Science Centre (WARAS)  
Faculty of Electrical and Electronic Engineering, Universiti Tun Hussein Onn Malaysia  
\* Corresponding author E-mail:hanif@uthm.edu.my

## Abstract

Energy efficiency (EE) is one of the key design goals for fifth-generation (5G) cellular networks due to the intermittent properties of renewable energy sources and limited battery capacity. In this paper, we analyze the EE of downlink (DL) massive multi-user multiple-input multiple-output (MIMO) system based on circuit power consumption for the transmit antenna configuration. We designed full complexity reduced zero-forcing (ZF) beamforming to cancel out interbeam interference when the number of antennas  $\rightarrow \infty$  and minimized the power consumption model, when formulating the power allocation optimization problem with the Lagrange duality method, in order to maximize EE. Simulation results revealed that the EE in the base station (BS) could be improved when the number of radio frequency (RF) chains was proportional to the optimal transmit power allocation and when the consumption circuit power was comparable to the transmit power.

**Keywords:** Massive MIMO; Fifth Generation (5G); Energy Efficiency; Zero Forcing; Downlink.

## 1. Introduction

The next generation of cellular networks faces an increasing demand for data traffic. Massive MIMO systems in 5G are very important in maximizing both data rates and EE for cellular networks using the same time-frequency resources. Among other improvements, this requires a focus on maximizing energy efficiency based on the transmit power when the number of antennas is large enough and the number of used RF chains is small. The massive MIMO system has great potential to improve data rates without increasing the bandwidth. Moreover, massive MIMO systems exploit a huge number of antenna arrays at the BS to provide a sufficient data rate to many users. Massive MIMO systems are able to provide higher EE than single antenna when only take the account transmit power of users [1], and [2]. Energy consumption that reduces transmit power is due to the direct impact of the carbon emission problem of BS; therefore, it is necessary to reduce the high level of energy consumption. Transmitting more power supports an achievable high data rate that is inversely proportional to an increasing number of antenna elements at the BS [3]. Moreover, we considered RF chains a key to practical consumption circuit power, where the transmit power increases proportionally with the number of antennas. In contrast, the large number of RF chains increases the cost of both the RF circuit and wireless cellular networks.

Signal detection increases exponentially with the number of used antenna and increases the hardware complexity. Optimized transmit power can be obtained under a predefined signal for a user and reduced interference for signal-to-interference noise ratio (SINR) [4], and [5].

Most previous studies have focused on RF chains, where the consumption of power is better at transmitting higher numbers of antennas in the transmitter. According to [6] the author to compute

the achievable higher EE, digital and used to reduce the cost and power consumption to achieve spectral efficiency were required for ideal phase shifters resolution based on perfect channel state information (CSI) and baseband processing for ZF precoding. In addition to the author in [6] derived the EE based on all transmitted antennas from BS to users (UEs), based on analyze spectral efficiency to the total transmit power. While, in our research we derived only the numbers of RF chains at proportional to the optimal transmit power allocation at transmitted signals from BS to UEs. The author [7] focused on reducing the power consumption of RF chains in mm-Wave by proposing the Butler-matrix-based hybrid analog/digital beamforming based on evaluated spectral efficiency to maximize EE. The author in [7], analyzed the result based on the increase of EE with spectral efficiency (SE) and concluded that when the spectral efficiency was low, the EE was proportional to the SE. After the EE had reached the saturation point the EE decreased with increased spectral efficiency. The other author in [8] maximized EE in mm-Wave based on the hybrid transmitter designs for a number of transceiver antennas and RF chains. Liu and Lau [9] proposed bi-convex approximation by using a large number of antennas with limited RF chains to maximize the data rate. Ni, Chiang and Dey [10] proposed hybrid beamformers to provide quality of service and reduce consumption power at the BS by using ZF beamforming to maximize high data rates. Salh et al. [11] derived the optimal number of RF chains from all available antennas at the BS by using ZF beamforming for an achievable data rate. Mensah et al. [12] proposed resource allocations for both the quality of service constraint and transmit power constraint to maximize energy efficiency. In another study, Salh et al. [13] maximized EE when all antennas were available while minimizing power consumption by obtaining the optimum number of antennas in a DL. In this research, we maximized EE based on the full complexity reduced ZF beamforming to cancel out interbeam interference when the number of antennas  $\rightarrow \infty$  and improved the power con-

sumption model, formulated under the power allocation optimization problem for Lagrange duality method. The effects of circuit power consumption in massive MIMO system were more serious not only in transmitting power, but also in analyzing the activating of the fundamental circuits power consumption at the transmitter.

## 2. System Model

We considered a downlink massive MIMO system in a single cell equipped with  $M$  antennas to communicate with  $K$  users, where  $M \gg K$ . We assumed complex channel gain between  $M$  antennas and  $K$  users over the flat fading channel with perfect channel state information (CSI). The channel matrix between BS antennas and the  $k$ th UEs is  $H = [h_1^H, \dots, h_k^H]^H \in \mathbb{C}^{K \times M}$ , and the transmitted signal from BS to  $K$  UEs can be written as

$$\mathbf{y}_k = \sqrt{Q_d} \mathbf{h}_k \mathbf{u}_k \mathbf{v}_k + \sqrt{Q_d} \sum_{i=1, i \neq k}^K \mathbf{h}_k \mathbf{u}_i \mathbf{v}_i + \mathbf{n}_k \quad (1)$$

where  $\mathbf{u}_k \in \mathbb{C}^{M \times 1}$  represents the transmit beamformer to users  $\mathcal{U} = [\mathbf{u}_1, \dots, \mathbf{u}_k] \in \mathbb{C}^{M \times K}$ ,  $\mathbf{v}_k$  is the data symbol for UE,  $Q_d$  is the downlink transmit power and  $\mathbf{n}_k$  is the complex white Gaussian noise  $\mathbf{n}_k \sim \mathcal{CN}(0, \sigma^2 I_K)$ . According to the channel matrix  $\mathbf{h}_k^H \mathbf{u}_k$ , based on properties of ZF precoding, this can be represented as

$$\begin{cases} \mathbf{h}_k^H \mathbf{u}_k \neq 0 & i = k \\ \mathbf{h}_k^H \mathbf{u}_k = 0 & i \neq k \end{cases} \quad (2)$$

According to the uncorrelated Rayleigh fading channel, the transmit power from BS to different users can be written as

$$Q_S = \sqrt{Q_d} |\mathbf{h}_k \mathbf{u}_k|^2 \quad (3)$$

The interference can be reduced by using conjugate beamforming with inter-user interference forced to zero with a proposed full complexity of ZF precoding, which can be written as

$$Q_I = \sqrt{Q_d} \sum_{i=1, i \neq k}^K |\mathbf{h}_k \mathbf{u}_i|^2 + 1 \quad (4)$$

The full complexity ZF precoding beamforming played an important role to provide near-optimal performance and cancel out interbeam interference. Where the optimal beam selection was able to reduce the RF chains' complexity and achieve near-optimal performances for high-dimensional signal space, both the complexity and cost of BS increased. The SINR of users can be simplified as

$$\Gamma_k = \frac{\sqrt{Q_d} |\mathbf{h}_k \mathbf{u}_k|^2}{\sqrt{Q_d} \sum_{i=1, i \neq k}^K |\mathbf{h}_k \mathbf{u}_i|^2 + 1} \quad (5)$$

$$s. t. \mathbf{u}_{RF}^k \in \mathcal{U}, \quad (6)$$

$$\|\mathbf{u}_{RF}\|^2 = \mathcal{N}_{RF} \quad (7)$$

From the closed form for achievable data rate based on Jensen inequality [14], and [15], the baseband precoding matrix for the equivalent channel is

$$\Gamma_k^{ZF} = \frac{Q_d}{H(HH^H)^{-1} \mathcal{L}} \quad (8)$$

where  $\mathcal{L}$  is a  $K \times K$  diagonal matrix, from the minimum number of RF chains arranged and from the Rayleigh fading channel  $H = \mathbf{h}_k \mathbf{u}_{RF}^k \in \mathbb{C}^{K \times \mathcal{N}_{RF}}$ . The signal transmit power was equal to every UE in the DL at increasing  $M \rightarrow \infty$ , based on random matrix theory [16]. Some overlapping antennas were connected to more RF chains; in these cases, we needed a  $N = M_k \mathcal{N}_{RF}$  phase shifter

because a large number of antennas connected to one RF chain in higher array gain; this can be written as

$$\begin{aligned} |\mathbf{h}_k \mathbf{u}_{RF}^k|^2 &= \frac{1}{M_k \mathcal{N}_{RF}} \{ |\sum_{m=1}^M \cos(\vartheta_m)|^2 + |\sum_{m=1}^M \sin(\vartheta_m)|^2 \} \\ &= \frac{1}{M_k \mathcal{N}_{RF}} (\mathbb{E}[\cos(\vartheta_m)]^2 + \mathbb{E}[\sin(\vartheta_m)]^2) = \frac{4}{\pi^2 \mathcal{N}_{RF}} \end{aligned} \quad (9)$$

where  $\vartheta_m$  represents the elevation angle,  $\mathcal{N}_{RF}$  antennas for RF and  $N$  is the available number of antennas. The achievable data rate can be expressed as

$$\mathcal{R}_k = B \log_2(1 + \Gamma_k) \quad (10)$$

$$\mathcal{R}_k = \log_2 \left( 1 + \frac{Q_k}{K[(HH^H)^{-1}]_{k,k}} \right) \quad (11)$$

$$\mathcal{R}_k = K B \log_2 \left( 1 + \mathbb{E} \left( Q_k |H|_{k,k}^2 \right) \right) = K B \log_2 \left( 1 + \frac{Q_k \left( \frac{\pi \mathcal{N}_{RF} - 2}{4\pi} \right)}{K} \right) \quad (12)$$

where  $B$  represents the bandwidth of the baseband signal, based on full complexity ZF converging to conjugate beamforming to satisfy the performance for maximizing power.

### 2.1. EE Formulation for Massive MIMO

This section presents the analysis of the total power consumption for maximizing EE based on the transmit power and fundamental power for the circuit at the transmitter, where  $Q_K$  represents the allocated power consumed for the  $K$ th users,  $Q_C$  is the power consumption for all circuit power,  $Q_{RF}$  is the power consumed at each RF chain and  $Q_{BB}$  is the power consumed by the baseband processing. The maximum transmit power in DL can be written as

$$Q_{max} = Q_K + Q_C + K(Q_{BB} + Q_{RF}) \quad (13)$$

Maximal EE could be increased when the number of antennas increased, which increased the transmitted power consumption and yielded a concave shape for EE. In contrast, the maximal EE obtained when the transmit power was less than the maximum transmit power or when the consumption circuit power was comparable to the transmit power can be written as

$$EE = \frac{K \log_2 \left( 1 + \frac{Q_k \left( \frac{\pi \mathcal{N}_{RF} - 2}{4\pi} \right)}{K} \right)}{Q_k + Q_C + K(Q_{BB} + Q_{RF})} \quad (14)$$

The optimization problem of transmission power can be formulated as follows

$$s. t \quad \mathcal{C}_1: \mathcal{R} \geq \mathcal{R}_{min} \quad (15)$$

$$Q_k + Q_C + K(Q_{BB} + Q_{RF}) \leq Q_{max} \quad (16)$$

The objective function was equivalent to maximal EE. Following Liang, Xu and Dong [15], we used the constrained data rate and transmit power  $r_t(Q) - \varepsilon(Q_k + Q_C + K(Q_{BB} + Q_{RF}))$ . The updated value of EE was  $\tilde{\varepsilon} = \max_Q \frac{r_{tot}(Q)}{Q_k + Q_C + K(Q_{BB} + Q_{RF})}$ . We used the Lagrange dual decomposition method to solve the optimization problem under constrained data rate and transmit power, as in equations (15) and (16) [16], as follows:

$$\begin{aligned} L(Q_k, \mathcal{F}_1, \mathcal{F}_2) &= \mathcal{R}_k - \tilde{\varepsilon}(Q_{AP} + Q_C + K(Q_{BB} + Q_{RF})) + \\ &\mathcal{F}_1(\mathcal{R}_k - \mathcal{R}_{min}) + \mathcal{F}_2(Q_{max} - f(R_k)) \end{aligned} \quad (17)$$

$$\mathcal{L}(Q_k, \mathcal{F}_1, \mathcal{F}_2) = \frac{K_B \log_2 \left( 1 + \frac{Q_k}{K} \left( \frac{\pi N_{RF} - 2}{4\pi} \right) \right)}{\partial Q_k} - \tilde{\varepsilon} (Q_{AP} + Q_C + K(Q_{BB} + Q_{RF})) + \mathcal{F}_1 (\mathcal{R} - \mathcal{R}_{min}) - \mathcal{F}_2 (Q_{AP} + K(Q_{BB} + Q_{RF}) - Q_{max}) \quad (18)$$

The optimization power allocation for the activated number of  $M$  in (18) can be solved using Karush–Kuhn–Tucker conditions to obtain the optimal transmission power  $Q_k$ , which is as follows:

$$\frac{\partial \mathcal{L}(Q_k, \mathcal{F}_1, \mathcal{F}_2)}{\partial Q_k} = \frac{\partial r(Q_k)}{\partial Q_k} + \mathcal{F}_1 \frac{\partial r}{\partial Q_k} - \tilde{\varepsilon} (Q_{AP} + K(Q_{BB} + Q_{RF})) - K\mathcal{F}_2 \geq 0 \quad (19)$$

where  $\mathcal{F}_1$  is the Lagrange multiplier vector corresponding to the data rate constraints and  $\mathcal{F}_2$  is the Lagrange multiplier corresponding to the transmit power constraints in equations (15) and (16). From the Lagrange multiplier, the dual optimization problem can be expressed as

$$\min_{\mathcal{F}_1, \mathcal{F}_2} \max_{Q_k} \mathcal{L}(\mathcal{F}_1, \mathcal{F}_2) \text{ s.t. } \mathcal{F}_1 \geq 0, \mathcal{F}_2 \geq 0 \quad (20)$$

Transmit power should be allocated to every user by using high channel gain. From (20), the optimal transmit power can be obtained based on fixed and iterative Lagrange multipliers, where  $\mathcal{F}_1, \mathcal{F}_2 \geq 0$ :

$$Q_k = \left[ \left( \frac{(B + \mathcal{F}_1)}{(\tilde{\varepsilon} + \mathcal{F}_2) \ln 2} - \frac{4\pi}{(\pi N_{RF} - 2)} \right) K \right]^{\square} \quad (21)$$

Based on the applied Lagrange dual method [16], the Lagrange multiplier was updated using the gradient method

$$\mathcal{F}_1(\varphi + 1) = \mathcal{F}_1(\varphi) + \varkappa(\varphi)(\mathcal{R}_k - \mathcal{R}_{min}) \quad (22)$$

$$\mathcal{F}_2(\varphi + 1) = \mathcal{F}_2(\varphi) + \gamma(\varphi)(Q_{max} - f(\mathcal{R}_k)) \quad (23)$$

where  $\varkappa$  and  $\gamma$  are step sizes and  $\varphi$  is the iteration index.

### 3. Numerical Results

The numerical results were tested via MATLAB simulation; and this section presents the numerical results from Monte Carlo simulations. Figure 1 illustrates EE versus the number of transmit antennas. This figure shows that the EE was affected by a large number of antenna arrays and users at transmitting power  $Q_k = 30\text{dBm}$ , wherein a single cell the inter-cell interference was not considered. In addition, the EE increased with a large number of antennas when  $N = 400$ , whereas for unwanted power consumption in the RF, the circuit was still needed for transmission. After this value, the EE started to decrease, due to more distributed users  $K$  and the consumption circuit power became higher than the transmitted power, which made the EE concave shape. Figure 1 shows that the EE started to increase and become saturated until the number of antennas  $N = [370, 390, 420]$ , after which the EE started to decrease, according to the sufficient number of antennas for serving the number of users in the cell,  $K = [16, 8, 4]$ , respectively. While, from Figure 4, in [6] conclude that the total EE started to increase at the increase number of antenna elements in the BS until the EE became saturated then the EE decreased with increasing numbers of antennas array based on proposed bit resolution and ideal phase shifters.

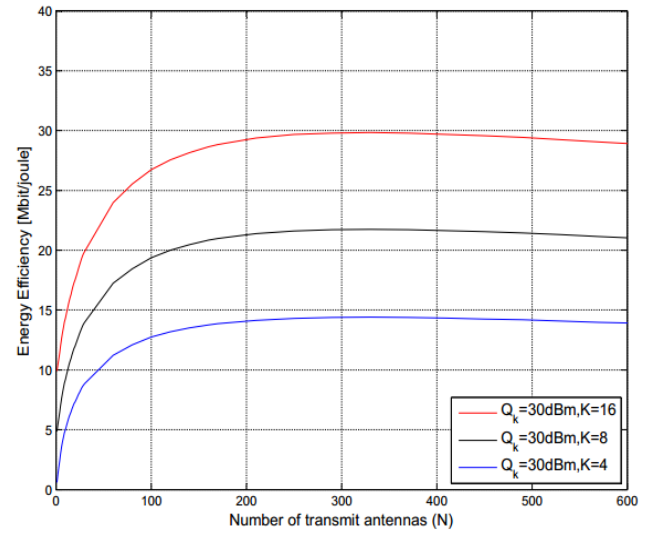


Fig. 1: EE with number of transmit antennas  $N$ .

Figure 2 illustrates EE versus the number of RF chains. When the number  $N$  was large, the EE deteriorated due to a large number of RF chains, which required an improved EE for the antenna array at the BS to reduce the consumption of circuit power. EE in the BS could be improved when the number of RF chains was proportional to the optimal transmit power allocation. Maximal EE could be obtained under the use of RF chains when  $N_{RF} = 80$ , as the EE became saturated and started to decrease because the full complexity of ZF used the same number of RF chains, which reduced the EE. Therefore, the EE for all curves is convex and nonlinear at transmit power, which should be allocated to every user to the transmitting power to select the optimal transmit power, according to (30). While, from Figure 5, in [8] the number of RF chains was independent with the number of employed phase shifters which maximized the EE. In addition, the EE start increases and then decreased dependent on each of the RF chains is connected to all of the available antennas.

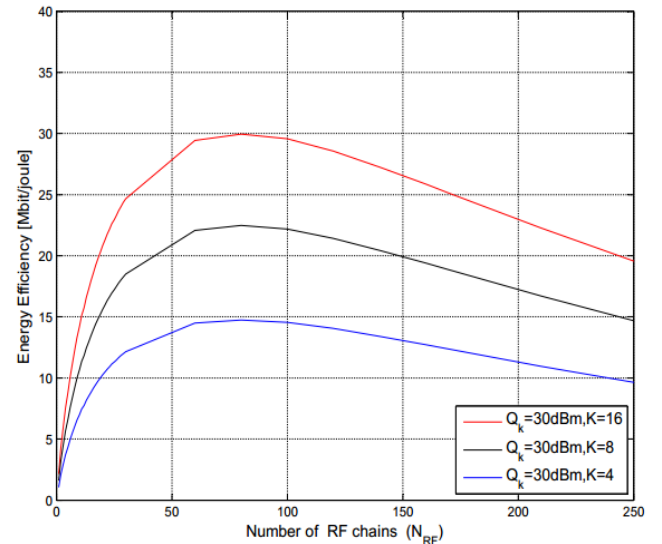


Fig. 2: Energy efficiency with number of RF chains.

Figure 3 illustrates EE versus the total transmission power. The effect of transmission power with an increased number of antennas made EE move toward zero due to circuit power consumption. Figure 3 shows how EE moved toward zero when the transmit power increased. Figure 3 also shows that when the transmitted power was very low, EE became constant, while when the transmitted power had large values, EE started to decrease, which required less circuit power consumption than transmit power. Therefore, when the consumption circuit power was comparable to the transmit power, EE could be maximized. Therefore, when the consumption circuit power was a counterpart to the transmit power,

this means that more antennas could be used, which decreased the EE.

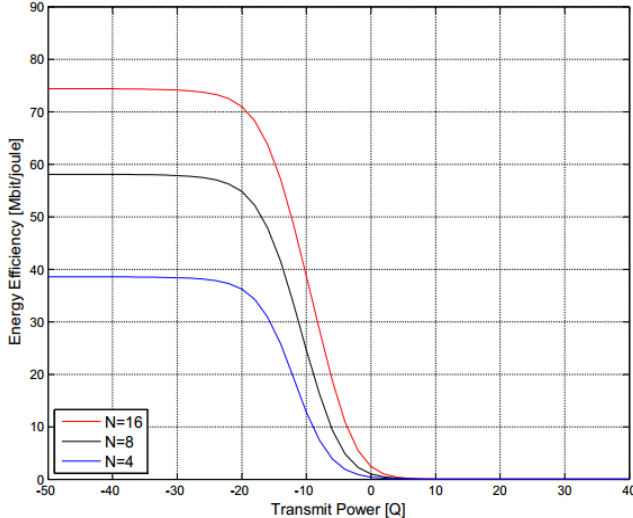


Fig. 3: Energy efficiency with transmit antennas power.

## 4. Conclusion

In this paper, we studied the maximal EE in DL with a single cell equipped with  $M$  antennas based on circuit power consumption. Moreover, the full complexity of ZF beamforming could cancel out interbeam interference when the number of antennas approached infinity. However, EE became saturated and started to decrease because the full complexity of ZF used the same number of RF chains, which reduced the EE. Therefore, EE in the BS could be improved when the number of RF chains was proportional to optimal transmit power allocation and when the consumption circuit power was comparable to the transmit power.

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