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Research paper

Groups, Algorithms and Programming (GAP) and the Nonabelian Tensor Square of Groups of Order 8*q*

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Abstract

In this paper, the software package Groups, Algorithms and Programming (GAP) is used to verify the hand calculation of the nonabelian tensor square for groups of order 8*q*, where *q* is an odd prime.

Keywords: *Groups of order 8q, commutator subgroup , the Schur Multiplicator, nonabelian tensor square, GAP*

1. Introduction

For a group G, the nonabelian tensor square $G\otimes G$ is the group generated by the symbols $g\otimes h$ and defined by the relations $gg'\otimes h=({}^{g}g'\otimes {}^{g}h)(g\otimes h)$, $g\otimes hh'=(g\otimes h)({}^{h}g\otimes {}^{h}h')$ for all $g, g', h, h'\in G$, where ${}^{g}g'=gg'g^{-1}$. The nonabelian tensor square is a special case of the nonabelian tensor product which has its origin in homotopy theory and was introduced by Brown and Loday [1, 2]. The exterior square $G\wedge G$ is obtained by imposing the additional relations $g\otimes g =1_{\otimes}$ for all $g\in G$ on $G\otimes G$. The commutator map induces homomorphisms $\kappa : g\otimes h \in G\otimes G \rightarrow \kappa(g\otimes h)= [g, h] \in G'$ and $\kappa' : g\wedge h \in G\wedge G \rightarrow \kappa'(g\wedge h)= [g, h] \in G'$ and $J_2(G) =\ker(\kappa)$.
The results of Brown and Loday [1, 2] give the following commutative diagram as shown in Figure 1 with exact rows and central extensions as columns:

$$\begin{array}{ccccc}
 & & 0 & & 0 \\
 & & \downarrow & & \downarrow \\
\Gamma(G^{ab}) & \rightarrow & J_2(G) & \rightarrow & M(G) \rightarrow 0 \\
 & & \downarrow & & \downarrow \\
\Gamma(G^{ab}) & \rightarrow & G\otimes G & \rightarrow & G\wedge G \rightarrow 1 \\
 & & \kappa\downarrow & & \downarrow\kappa' \\
 & & G' & = & G' \\
 & & \downarrow & & \downarrow \\
 & & 1 & & 1
\end{array}$$

Fig. 1.: The Commutative Diagram

where G' is the commutator subgroup of G, $M(G)$ the Schur multiplier of G and Γ the Whitehead's quadratic function (Whitehead 1950).
The determination of $G\otimes$G for $G = GL(2, q)$ and other linear groups was mentioned as an open problem by Brown *et al*. [2]. In the latter paper, there is a list of open problems on the computation of the nonabelian tensor square of finite groups. Among these, there is the problem to find an explicit value of the nonabelian tensor square of linear groups. Hannebauer [3] determined the nonabelian tensor square of $SL(2, q)$, $PSL(2, q)$, $GL(2, q)$ and $PGL(2, q)$ for all $q \geq 5$ and $q = 9$.
The Schur multiplier and nonabelian tensor square of the nonabelian groups of order p^2q, groups of order p^3q *and* special orthogonal groups have been computed by Rashid *et al*. in [4,5,6, 7, 8, 9], where p and q are distinct primes.

In this paper, we used Groups, Algorithms and Programming (GAP) [10] to verify the hand calculation of the nonabelian tensor square of finite nonabelian groups of order 8*q*, where *q* is an odd prime.

In1975, Miah [11] classified groups of order 8*q* that in the following Theorem we state this classification.

Theorem 1: Let G be a nonabelian group of order 8*q*, where *q* is an odd prime. Then G is isomorphic to exactly one group of the following types.

$D_4\times C_q$, (1.1.1)

$Q_2\times C_q$, (1.1.2)

$D_{2q}\times C_2$, (1.1.3)

$Q_q\times C_2$, (1.1.4)

$D_q\times C_4$, (1.1.5)

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$<a, b| a^8=b^q=1, a^{-1}ba=b^{-1}>$, (1.1.6)

D_{4q}, (1.1.7)

$<a,b,c|a^4=b^2=c^q=1,a^{-1}ba=a^{-1},a^{-1}ca=c^{-1},bc=cb>$, (1.1.8)

Q_{2q}, (1.1.9)

$<a, b| a^8=b^q=1, a^{-1}ba=b^{\alpha}>$ (1.1.10)

where α is a primitive root of $\alpha^4 \equiv 1(\text{mod } q)$, 4 divides $q-1$,
$<a, b, c| a^4=b^2=c^q= 1, ab=ba, a^{-1}ca=c^{\alpha}, bc=cb>$ (1.1.11)

where α is a primitive root of $\alpha^4 \equiv 1(\text{mod } q)$, 4 divides $q-1$,
$<a, b| a^8=b^q=1, a^{-1}ba=b^{\alpha}>$ (1.1.12)

where α is a primitive root of $\alpha^8 \equiv 1(\text{mod } q)$, 8 divides $q-1$,
$C_4 \times A_4$ (1.1.13)

$SL(2,3)$, (1.1.14)

S_4 (1.1.15)

$<a, b, c, d | a^2=b^2=c^2= d^q= 1, ab=ba, ac=ca, bc=cb, d^{-1}ad=b, d^{-1}bd=c, d^{-1}cd=bb>$ (1.1.16)

In 2013, Rashid *et al.* [4] determined the Schur multiplicator of groups of order 8*q*.

2. Basic Definition and Theorems

This section includes some definitions and results on the nonabelian tensor squareof finite groups.

Theorem 2: If G is a group in which G' has a cyclic complement C, then $G' \cong (G \wedge G) \times G^{ab}$ and $|G \otimes G| = |G||M(G)|$.

Theorem 3: If G is a cyclic group of order m,then $G \otimes G = G$.

Theorem 4: If G be a group and G' be a cyclic group then G⊗G is abelian.

Theorem 5 (Caushy): If a prime p divides the order of a finite group,then group contains an element of order p.

Theorem 6: Let G be a nonabelian group of order 8q, where q is an odd prime. Then G is isomorphic to exactly one group of the following types:

$$G \otimes G \cong \begin{cases} \phi_{4q} \times (\phi_2)^3; G \text{ is of type (1.1.1),} \\ (1.1.4),(1.1.5),(1.1.7),(1.1.11), \\ \phi_{4q} \times \phi_4 \times (\phi_2)^2; G \text{ is of type (1.1.2),(1.1.9),} \\ \phi_{8q}; G \text{ is of type (1.1.6),(1.1.10),(1.1.12),} \\ \phi_{2q} \times (\phi_2)^8; G \text{ is of type (1.1.3),} \\ \phi_{4q} \times M; G \text{ is of type (1.1.8),} \\ \phi_6 \times Q_2; G \text{ is of type (1.1.13),} \\ \phi_3 \times Q_2; G \text{ is of type (1.1.14),} \\ \phi_2 \times A_4; G \text{ is of type (1.1.15),} \\ \phi_{14} \times (\phi_2)^2; G \text{ is of type (1.1.16).} \end{cases}$$

3. GAP Computations

In this section, we compete the nonabelian tensor square, $G \otimes G$ by the use of Groups, Algorithms and Programming (GAP) and Theorem 1.1 for finite nonabelian groups of order 8q, where q is an odd prime.

Groups of Type (1.1.1)

```
gap> f:=FreeGroup(3);
<free group on the generators [ f1, f2, f3 ]>
q=3
gap> G:=f/[f.1^4,f.2^2,f.3^3,f.2*f.1*f.2*f.1,
f.1*f.3*f.1^3*f.3^2,f.2*f.3*f.2*f.3^2];
<fp group on the generators [ f1, f2, f3 ]>
gap> Size(G);
24
gap> StructureDescription(G);
"C3 x D8"
gap> G:=PcGroupToPcpGroup(G);
Pcp-group with orders [ 2, 2, 3, 2 ]
gap> N:=NonAbelianTensorSquare(G);
Pcp-group with orders [ 2, 2, 2, 6, 2 ]
gap> StructureDescription(N);
"C12 x C2 x C2 x C2"

q=5
gap> G:=f/[f.1^4,f.2^2,f.3^5,f.2*f.1*f.2*f.1,
f.1*f.3*f.1^3*f.3^4,f.2*f.3*f.2*f.3^4];
<fp group on the generators [ f1, f2, f3 ]>
gap> Size(G);
40
gap> G:=PcGroupToPcpGroup(G);
Pcp-group with orders [ 2, 2, 5, 2 ]
gap> N:=NonAbelianTensorSquare(G);
Pcp-group with orders [ 2, 2, 2, 10, 2 ]
gap> StructureDescription(N);
"C20 x C2 x C2 x C2"
```

Groups of Type (1.1.2)

```
gap> f:=FreeGroup(3);
<free group on the generators [ f1, f2, f3 ]>
q=3;
gap> G:=f/[f.1^4,f.2^4,f.3^3,f.2^2*f.1^2,f.2^3*f.1*f.2*f.1,
f.1*f.3*f.1^3*f.3^2,f.2*f.3*f.2^3*f.3^2];
<fp group on the generators [ f1, f2, f3 ]>
gap> Size(G);
24
gap> G:=PcGroupToPcpGroup(G);
Pcp-group with orders [ 2, 3, 2, 2 ]
gap> N:=NonAbelianTensorSquare(G);
Pcp-group with orders [ 2, 3, 2, 4, 4 ]
gap> StructureDescription(N);
"C12 x C4 x C2 x C2"

q=5;
gap> G:=f/[f.1^4,f.2^4,f.3^5,f.2^2*f.1^2,f.2^3*f.1*f.2*f.1,
f.1*f.3*f.1^3*f.3^4,f.2*f.3*f.2^3*f.3^4];
<fp group on the generators [ f1, f2, f3 ]>
gap> Size(G);
40
gap> G:=PcGroupToPcpGroup(G);
Pcp-group with orders [ 2, 2, 5, 2 ]
gap> N:=NonAbelianTensorSquare(G);
Pcp-group with orders [ 2, 2, 4, 20 ]
gap> StructureDescription(N);
"C20 x C4 x C2 x C2"
```

Groups of Type (1.1.3)

```
gap> f:=FreeGroup(4);
<free group on the generators [ f1, f2, f3, f4 ]>
q=3;
gap> G:=f/[f.1^2,f.2^2,f.3^3,f.4^2,f.1*f.2*f.1*f.2,f.1*f.4*f.1*f.4,
f.2*f.4*f.2*f.4,f.1*f.3*f.1*f.3^2,f.2*f.3*f.2*f.3^2,f.4*f.3*f.4*f.3];
<fp group on the generators [ f1, f2, f3, f4 ]>
gap> Size(G);
24
gap> G:=PcGroupToPcpGroup(G);
Pcp-group with orders [ 2, 2, 2, 3 ]
gap> N:=NonAbelianTensorSquare(G);
Pcp-group with orders [ 3, 2, 2, 2, 2, 2, 2, 2, 2, 2 ]
gap> StructureDescription(N);
"C6 x C2 x C2 x C2 x C2 x C2 x C2 x C2 x C2"

q=5;
gap> G:=f/[f.1^2,f.2^2,f.3^5,f.4^2,f.1*f.2*f.1*f.2,f.1*f.4*f.1*f.4,
f.2*f.4*f.2*f.4,f.1*f.3*f.1*f.3^4,f.2*f.3*f.2*f.3^4,f.4*f.3*f.4*f.3];
<fp group on the generators [ f1, f2, f3, f4 ]>
gap> Size(G);
40
gap> G:=PcGroupToPcpGroup(G);
Pcp-group with orders [ 2, 2, 2, 5 ]
gap> N:=NonAbelianTensorSquare(G);
Pcp-group with orders [ 5, 2, 2, 2, 2, 2, 2, 2, 2, 2 ]
gap> StructureDescription(N);
"C10 x C2 x C2 x C2 x C2 x C2 x C2 x C2 x C2 x C2"
```

Groups of Type (1.1.4)

```
gap> f:=FreeGroup(3);
<free group on the generators [ f1, f2, f3 ]>
q=3;
gap> G:=f/[f.1^4,f.2^2,f.3^3,f.1*f.2*f.1^3*f.2,
f.1^3*f.3*f.1*f.3,f.2*f.3*f.2*f.3^2];
<fp group on the generators [ f1, f2, f3 ]>
gap> Size(G);
24
gap> G:=PcGroupToPcpGroup(G);
Pcp-group with orders [ 2, 2, 2, 3 ]
gap> N:=NonAbelianTensorSquare(G);
Pcp-group with orders [ 3, 2, 2, 2, 4 ]
gap> StructureDescription(N);
"C12 x C2 x C2 x C2"

q=5;
gap> G:=f/[f.1^4,f.2^2,f.3^5,f.1*f.2*f.1^3*f.2,
f.1^3*f.3*f.1*f.3,f.2*f.3*f.2*f.3^4];
<fp group on the generators [ f1, f2, f3 ]>
gap> Size(G);
40
gap> G:=PcGroupToPcpGroup(G);
Pcp-group with orders [ 2, 2, 2, 5 ]
gap> N:=NonAbelianTensorSquare(G);
Pcp-group with orders [ 5, 2, 2, 2, 4 ]
gap> StructureDescription(N);
"C20 x C2 x C2 x C2"
```

Groups of Type (1.1.5)

```
gap> f:=FreeGroup(3);
<free group on the generators [ f1, f2, f3 ]>
q=3;
gap> G:=f/[f.1^4,f.2^2,f.3^3,f.1*f.2*f.1^3*f.2,
f.1*f.3*f.1^3*f.3^2,f.2*f.3*f.2*f.3];
<fp group on the generators [ f1, f2, f3 ]>
gap> Size(G);
24
gap> G:=PcGroupToPcpGroup(G);
Pcp-group with orders [ 2, 2, 2, 3 ]
gap> N:=NonAbelianTensorSquare(G);
Pcp-group with orders [ 3, 2, 2, 2, 4 ]
gap> StructureDescription(N);
"C12 x C2 x C2 x C2"

q=5;
gap> G:=f/[f.1^4,f.2^2,f.3^5,f.1*f.2*f.1^3*f.2,
92
f.1*f.3*f.1^3*f.3^4,f.2*f.3*f.2*f.3];
<fp group on the generators [ f1, f2, f3 ]>
gap> Size(G);
40
gap> G:=PcGroupToPcpGroup(G);
Pcp-group with orders [ 2, 2, 2, 5 ]
gap> N:=NonAbelianTensorSquare(G);
Pcp-group with orders [ 5, 2, 2, 2, 4 ]
gap> StructureDescription(N);
"C20 x C2 x C2 x C2"
```

Groups of Type (1.1.6)

```
gap> f:=FreeGroup(2);
<free group on the generators [ f1, f2 ]>
q=3;
gap> G:=f/[f.1^8,f.2^3,f.1^7*f.2*f.1*f.2];
<fp group on the generators [ f1, f2 ]>
gap> Size(G);
24
gap> G:=PcGroupToPcpGroup(G);
Pcp-group with orders [ 2, 2, 2, 3 ]
gap> N:=NonAbelianTensorSquare(G);
Pcp-group with orders [ 3, 8 ]
gap> StructureDescription(N);
"C24"

q=5;
gap> G:=f/[f.1^8,f.2^5,f.1^7*f.2*f.1*f.2];
<fp group on the generators [ f1, f2 ]>
gap> Size(G);
40
gap> G:=PcGroupToPcpGroup(G);
Pcp-group with orders [ 2, 2, 2, 5 ]
gap> N:=NonAbelianTensorSquare(G);
88
Pcp-group with orders [ 5, 8 ]
gap> StructureDescription(N);
"C40"
```

Groups of Type (1.1.7)

```
gap> f:=FreeGroup(3);
<free group on the generators [ f1, f2, f3 ]>
q=3;
gap> G:=f/[f.1^4,f.2^2,f.3^3,f.2*f.1*f.2*f.1,
f.1*f.3*f.1^3*f.3^2,f.2*f.3*f.2*f.3];
<fp group on the generators [ f1, f2, f3 ]>
gap> Size(G);
24
gap> G:=PcGroupToPcpGroup(G);
Pcp-group with orders [ 2, 2, 2, 3 ]
gap> N:=NonAbelianTensorSquare(G);
Pcp-group with orders [ 2, 3, 2, 2, 2, 2 ]
gap> StructureDescription(N);
"C12 x C2 x C2 x C2"

q=5;
gap> G:=f/[f.1^4,f.2^2,f.3^5,f.2*f.1*f.2*f.1,
f.1*f.3*f.1^3*f.3^4,f.2*f.3*f.2*f.3];
<fp group on the generators [ f1, f2, f3 ]>
gap> Size(G);
40
```

```
gap> G:=PcGroupToPcpGroup(G);
Pcp-group with orders [ 2, 2, 2, 5 ]
gap> N:=NonAbelianTensorSquare(G);
Pcp-group with orders [ 2, 5, 2, 2, 2, 2 ]
gap> StructureDescription(N);
"C20 x C2 x C2 x C2"
```

Groups of Type (1.1.8)

```
gap> f:=FreeGroup(3);
<free group on the generators [ f1, f2, f3 ]>
q=3;
gap> G:=f/[f.1^4,f.2^4,f.3^3,f.2^2*f.1^2,f.2^3*f.1*f.2*f.1,
f.1*f.3*f.1^3*f.3^2,f.2^3*f.3*f.2*f.3];
<fp group on the generators [ f1, f2, f3 ]>
gap> Size(G);
24
gap> G:=PcGroupToPcpGroup(G);
Pcp-group with orders [ 2, 2, 2, 3 ]
gap> N:=NonAbelianTensorSquare(G);
Pcp-group with orders [ 2, 3, 2, 4, 4 ]
gap> StructureDescription(N);
"C12 x C4 x C2 x C2"

q=5;
gap> G:=f/[f.1^4,f.2^4,f.3^5,f.2^2*f.1^2,f.2^3*f.1*f.2*f.1,
f.1*f.3*f.1^3*f.3^4,f.2^3*f.3*f.2*f.3];
<fp group on the generators [ f1, f2, f3 ]>
gap> Size(G);
40
gap> G:=PcGroupToPcpGroup(G);
Pcp-group with orders [ 2, 2, 2, 5 ]
gap> N:=NonAbelianTensorSquare(G);
Pcp-group with orders [ 2, 5, 2, 4, 4 ]
gap> StructureDescription(N);
"C20 x C4 x C2 x C2"
```

Groups of Type (1.1.9)

```
gap> f:=FreeGroup(3);
<free group on the generators [ f1, f2, f3 ]>
q=3;
gap> G:=f/[f.1^4,f.2^2,f.3^3,f.2*f.1*f.2*f.1,
f.1^3*f.3*f.1*f.3,f.2*f.3*f.2*f.3^2];
<fp group on the generators [ f1, f2, f3 ]>
gap> Size(G);
24
gap> G:=PcGroupToPcpGroup(G);
Pcp-group with orders [ 2, 2, 3, 2 ]
gap> N:=NonAbelianTensorSquare(G);
110
Pcp-group with orders [ 3, 2, 2, 2, 2, 2 ]
gap> StructureDescription(N);
"C12 x C2 x C2 x C2"

q=5;
gap> G:=f/[f.1^4,f.2^2,f.3^5,f.2*f.1*f.2*f.1,
f.1^3*f.3*f.1*f.3,f.2*f.3*f.2*f.3^4];
<fp group on the generators [ f1, f2, f3 ]>
gap> Size(G);
40
gap> G:=PcGroupToPcpGroup(G);
Pcp-group with orders [ 2, 2, 2, 5 ]
gap> N:=NonAbelianTensorSquare(G);
Pcp-group with orders [ 2, 5, 2, 2, 2, 2 ]
gap> StructureDescription(N);
"C20 x C2 x C2 x C2"
```

Groups of Type (1.1.10)

```
gap> f:=FreeGroup(2);
<free group on the generators [ f1, f2 ]>
q=5, a=2;
<free group on the generators [ f1, f2 ]>
gap> G:=f/[f.1^8,f.2^5,f.1^7*f.2*f.1*f.2^3];
<fp group on the generators [ f1, f2 ]>
gap> G:=PcGroupToPcpGroup(G);
Pcp-group with orders [ 2, 2, 2, 5 ]
gap> N:=NonAbelianTensorSquare(G);
Pcp-group with orders [ 5, 8 ]
gap> StructureDescription(N);
"C40"
```

Groups of Type (1.1.11)

```
gap> GPrimitiveRoots(5,4);
[ 2, 3 ]
gap> f:=FreeGroup(3);
<free group on the generators [ f1, f2, f3 ]>
q=5, a=2;
<free group on the generators [ f1, f2, f3 ]>
gap> G:=f/[f.1^4,f.2^2,f.3^5,f.1*f.2*f.1^3*f.2,
f.1^3*f.3*f.1*f.3^3,f.2*f.3*f.2*f.3^4];
<fp group on the generators [ f1, f2, f3 ]>
gap> Size(G);
40
gap> G:=PcGroupToPcpGroup(G);
Pcp-group with orders [ 2, 2, 2, 5 ]
gap> N:=NonAbelianTensorSquare(G);
Pcp-group with orders [ 5, 2, 2, 2, 4 ]
gap> StructureDescription(N);
"C20 x C2 x C2 x C2"
```

Groups of Type (1.1.12)

```
gap> f:=FreeGroup(2);
<free group on the generators [ f1, f2 ]>
q=17, a=2;
<free group on the generators [ f1, f2 ]>
gap> G:=f/[f.1^8,f.2^17,f.1^7*f.2*f.1*f.2^15];
<fp group on the generators [ f1, f2 ]>
gap> Size(G);
gap> G:=PcGroupToPcpGroup(G);
Pcp-group with orders [ 2, 2, 2, 17 ]
gap> N:=NonAbelianTensorSquare(G);
Pcp-group with orders [ 17, 8 ]
gap> StructureDescription(N);
"C136"
```

Groups of Type (1.1.13)

```
gap> f:=FreeGroup(4);
<free group on the generators [ f1, f2, f3, f4 ]>
gap> G:=f/[f.1^2,f.2^2,f.3^2,f.4^3,f.1*f.2*f.1*f.2,f.1*f.3*f.1*f.3,
f.2*f.3*f.2*f.3,f.1*f.4*f.1*f.4^2,f.4^2*f.2*f.4*f.3,f.4^2*f.3*f.4*f.
3*f.2];
<fp group on the generators [ f1, f2, f3, f4 ]>
gap> Size(G);
24
gap> G:=PcGroupToPcpGroup(G);
Pcp-group with orders [ 2, 3, 2, 2 ]
gap> N:=NonAbelianTensorSquare(G);
Pcp-group with orders [ 2, 2, 2, 2, 3 ]
gap> StructureDescription(N);
"C6 x Q8"
gap> StructureDescription(N);
"C6 x Q8"
```

Groups of Type (1.1.14)

```
gap> f:=FreeGroup(3);
<free group on the generators [ f1, f2, f3 ]>
gap> G:=f/[f.1^4,f.2^4,f.3^3,f.2^2*f.1^2,f.2^3*f.1*f.2*f.1,
f.3^2*f.1*f.3*f.2^3,f.3^2*f.2*f.3*f.2^3*f.1^3];
<fp group on the generators [ f1, f2, f3 ]>
gap> StructureDescription(G);
"SL(2,3)"
gap> Size(G);
24
gap> G:=PcGroupToPcpGroup(G);
Pcp-group with orders [ 3, 2, 2, 2 ]
gap> N:=NonAbelianTensorSquare(G);
Pcp-group with orders [ 2, 2, 2, 3 ]
gap> StructureDescription(N);
"C3 x Q8"
```

Groups of Type (1.1.15)

```
gap> f:=FreeGroup(3);
<free group on the generators [ f1, f2, f3 ]>
gap> G:=f/[f.1^4,f.2^2,f.3^3,f.2*f.1*f.2*f.1,f.3^2*f.1^2*f.3*f.2,
f.3^2*f.2*f.3*f.2*f.1^2,f.1^3*f.3*f.1*f.2*f.1^2*f.3];
<fp group on the generators [ f1, f2, f3 ]>
gap> Size(G);
24
gap> G:=PcGroupToPcpGroup(G);
Pcp-group with orders [ 2, 3, 2, 2 ]
gap> N:=NonAbelianTensorSquare(G);
Pcp-group with orders [ 3, 2, 2, 2, 2 ]
gap> StructureDescription(N);
"C2 x SL(2,3)"
```

Groups of Type (1.1.16)

```
gap> f:=FreeGroup(4);
<free group on the generators [ f1, f2, f3, f4 ]>
gap> G:=f/[f.1^2,f.2^2,f.3^2,f.4^7,f.1*f.2*f.1*f.2,f.1*f.3*f.1*f.3,
f.2*f.3*f.2*f.3,f.4^6*f.1*f.4*f.2,f.4^6*f.2*f.4*f.3,f.4^6*f.3*f.4*f.
2*f.1];
<fp group on the generators [ f1, f2, f3, f4 ]>
gap> Size(G);
56
gap> G:=PcGroupToPcpGroup(G);
Pcp-group with orders [ 7, 2, 2, 2 ]
gap> N:=NonAbelianTensorSquare(G);
Pcp-group with orders [ 2, 2, 2, 7 ]
gap> StructureDescription(N);
"C14 x C2 x C2"
```

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