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Research paper

Use of MEMS based Vibration Data for REB Fault Diagnosis

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Abstract

Bearing failure could lead to unpredicted productivity loss to the industries or catastrophic failure of mechanical components. Hence, bearing fault detection and diagnosis is an essential part of the predictive maintenance procedure. Nowadays, the usage of micro-electro mechanical system (MEMS) based sensors for machine fault diagnosis have shown increasing trend due to their size, cost, portability, and flexibility. This paper presents the use of cost effective MEMS based accelerometer to acquire vibration signals for three conditions of REB namely, normal (N), defect on inner race (IR), defect on outer race (OR) at variable load and high shaft speed on a customised bearing test rig. The acquired vibration signals are denoised and wavelet packet transform is used to decompose the signal. Statistical features have been extracted from the wavelet packet coefficients. These coefficients are given as inputs to two widely used classifiers namely Artificial Neural Network (ANN) and k-Nearest Neighbor (kNN) to check the reliability of MEMS accelerometer data for REB fault diagnosis. From the results, it is shown that the proposed fault diagnosis scheme using MEMS accelerometers yield 83.3% and 77.8% classification accuracy using ANN and kNN respectively. Accordingly, MEMS based accelerometers are enough to replace conventional accelerometers that are used for REB fault diagnosis.

Keywords: *Fault diagnosis, REB, MEMS accelerometer, ANN, k-NN.*

1. Introduction

Rolling element bearing (REB) is termed as heart of any rotating machinery. It supports and reduces friction between two rotating parts. Its failure may lead to whole machine or series of machines to fail. A failure survey of rotary machines conducted by Institution of Electrical & Electronic Engineering (IEEE) and Electric Power Research Institution (EPRI) showed that 45% is mainly due to bearing failure [1]. Therefore incipient detection of bearing faults is vital to avoid the system failure. Vibration analysis is the most extensively used method in condition monitoring. The stiffness and geometric accuracy of REB contribute to the vibration characteristics of rotary machine; hence vibration monitoring furnishes insight into the mechanical condition of the bearings [2].

General principles behind vibration signal for monitoring consists of those components in mechanical system that vibrates during operation [3]. Due to fault development, the system dynamics vary which results in deviation in the vibration configurations. By using proper data analysis techniques, it is possible to identify change in vibration signal due to faulty component and a proper reason for that fault can be found [4]. Vibration based diagnostic mainly consists of data/signal acquisition, signal processing/feature extraction, and fault diagnosis using different computational techniques. Here the computational techniques include signal denoising, fast Fourier transform (FFT) analysis, wavelet transform technique, etc.

The acquired raw signal contains noise, which needs to be removed for efficient diagnosis. Hence, the signal is denoised to eliminate noise. It is impossible to classify machine condition based on the discrete sample of the vibration signal [5]. Therefore, feature extraction technique is used, which involves pre-processing of acquired time series vibration signal to get suitable parameters which can be used to evaluate the fault. Time and frequency domain are the primary analysis process for fault diagnosis using vibration signals. Using time domain analysis, it is possible to find the presence of defect but the location of the defect cannot be determined [6]. Frequency domain is not suitable for analysis of non stationery signals, as it doesn't provide sufficient information that is required for fault diagnosis.

Wavelet transform (WT) acquires information from time and frequency domains and therefore more reliable features can be extracted. WT methods are classified as Continuous wavelet transform (CWT), Discrete wavelet transform (DWT), and Wavelet packet transform (WPT). CWT is obtained by summing all the time of the signal and multiplying it by scaled and shifted versions of the wavelet function [7]. It requires a lot of time to calculate the wavelet coefficients. Moreover, there is a possibility of data loss if improper scaling is chosen. To overcome this, DWT is used. In DWT the signal is passed over set of filters and decomposed into its detailed version and approximated version. The wavelet packet transform (WPT) [8], on the other hand, uses a rich library of redundant bases with arbitrary time-frequency resolution. Therefore, it allows the extraction of features from signals that mix stationary and non-stationary characteristics [9].

Recently, there has been an increasing trend in identifying appropriate techniques for various stages of fault diagnosis, which have accurate classification and shorter computation time [10]. Artificial neural networks (ANNs) are profitably applied for fault diagnosis applications. They are developed to copy humans in decision-making and recognition. ANNs are effective for studying certain status or operation condition of the rotary machines. Tang et al [11] used Elman neural network to create a model for fault classification of the rotating machinery. They estimated the effectiveness of the model by testing it for deep groove bearing fault diagnosis. Castejon et al [12] proposed a new methodology for data collected from a quasi-real industrial machine. Multiresolution analysis (MRA) was used to extract interesting features in the signal. They evaluated the performance using ANN classifier. Moosavian et al [10] proposed new programme for fault identification of journal bearings of internal combustion (IC) engine using two classifiers, namely ANN and kNN. They varied the value of k in kNN and number of hidden neurons (N) in ANN in the scale of 1 to 20. They found performance of ANN to be better when compared with kNN. Moura et al [13] used ANN to diagnose the severity of bearing outer race fault.

One of the major drawback of conventional accelerometers is that they have slightly higher mass compared to MEMS. In addition to this, the MEMS accelerometers are much cheaper compared to conventional accelerometers. Today MEMS devices are available in the form of small chips with mechanical elements, actuators, sensors, and electronics on it. Even though MEMS-based sensors can be applied to fault diagnosis of machines their usage in real life fault applications is restricted because their suitability for the same is not validated yet. Many authors have used MEMS accelerometers in machine condition monitoring. Albarbar et al [13] gave insight into design, working principles and practical guidance to use of MEMS accelerometers in condition monitoring. They provided particulars of experimental setup, signal conditioning and DAQ system to construct simple experimental setup and evaluated its performance. Son et al [14] used MEMS-based accelerometers and current sensors for fault diagnosis of induction motor. They captured vibration and current signals and validated the capability of MEMS accelerometers in motor fault diagnosis. Albarbar et al [15] studied performance of MEMS accelerometers. They conducted experiments on CNC machines and compared the results with ICP type accelerometers. Chaudhury et al [16] integrated the basic sensor and intelligence of vibration analysis to develop a vibration sensor using an inexpensive MEMS accelerometer. They tested the sensitivity and performance of the proposed intelligence system. The results obtained were satisfactory and MEMS can be used to acquire vibration data for condition monitoring.

The present work carries out a study on use of MEMS accelerometer for fault diagnosis of REB. The vibration signals were acquired using ADXL-335 MEMS accelerometer and a conventional accelerometer for three bearing conditions namely- N, IR, and OR using a customised test rig at 1.7 kN load and at 355 rpm & 622 rpm shaft speed. Vibration signals can prove as an efficient diagnostic tool, if unwanted noise is removed. The denoised signals were decomposed using WPT up to three levels. Statistical features were extracted from the wavelet packet coefficients. These features are given as inputs to ANN and kNN classifiers. The best classifier in this work is identified based on the classification results of the classifier.

2. Experimental Setup

The experimental setup consists of a shaft having diameter of 32mm and is supported with the help of two bearings. The induction motor with variable speed drives the shaft. A belt and pulley arrangement is made which drives motor and bearing. The speed ratio for the time pulley arrangement is 2.25 and the motor speed can be varied from 0 to 1400rpm. In this work, a 6206 deep groove ball bearing is mounted to support the shaft on which the tests are carried out. A hydraulic loading arrangement is made through which radial load is applied on the bearing under test.

The ADXL335 MEMS accelerometer is placed on the housing of the test bearing. The accelerometer is connected to NI PCI 6221 Data Acquisition system (DAQ) hardware. The DAQ is then connected to a computer through cables. After bearing gains steady speed, the acceleration signals from the transducer fixed on the test bearing is acquired using Data Acquisition board. A customized LABVIEW (VI) program was developed to collect the signals at sampling rate of 10000 samples per second and stored in the system as an .xls file. Fig 2.1 shows photograph of experimental set up.

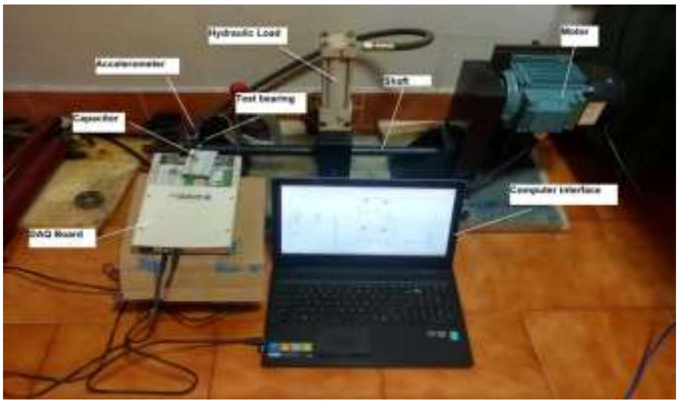


Fig 2.1: Photograph of experimental set up.

2.1 Accelerometers

2.1.1. ADXL335 MEMS Accelerometer

ADXL335 is a commercial MEMS accelerometer from the maker Analog Devices. Since, it is a low cost, low noise, and low power accelerometer, it has been selected as vibration sensor. It is a monolithic integrated circuit (IC) accelerometer with signal conditioning circuit and is shown in Fig 2.2.

The ADXL335 is a single structure accelerometer which senses X, Y, and Z axis acceleration. It has high orthogonality in all three directions and has small cross-axis sensitivity. ADXL335 has a built in temperature compensation circuitry, hence it is not necessary to use any additional circuitry to reduce temperature effects.

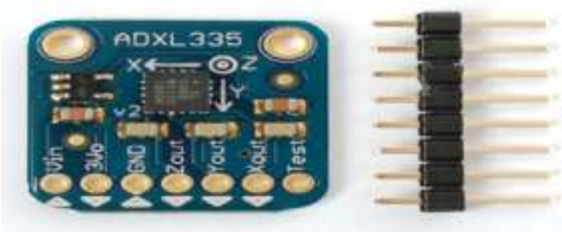


Fig 2.2: ADXL335 accelerometer**.**

2.1.2. Isotron 65-100 Accelerometer

The high sensitivity and high performance of model 65 distinguishes this triaxial accelerometer from comparable products. Model 65 is packaged in a 10-mm cube of welded titanium construction. Interface to the model 65 is via a Microtech 4-pin connector.

The specification of ADXL335 and Isotron 65-100 accelerometer are given in the Table 2.1.

Table 2.1: Accelerometer Specifications.

Sl No.	Specification	ADXL 335	Isotron 65-100
1	Measurement range	$\pm 3g$	± 50
2	Sensitivity	300mV/g	100mV/g
3	Bandwidth:		
	X, Y	60Hz	3 - 1500Hz
	Z	50Hz	
4	Power supply	3V	23 – 30V
5	Noise performance	$150\frac{\mu g}{\sqrt{Hz}}rms$	$800\frac{\mu g}{\sqrt{Hz}}rms$

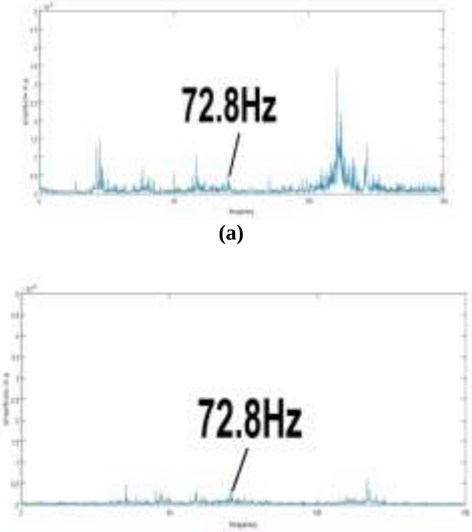
2.1.3. Bandwidth Selection

For a sensor, the maximum input frequency for which the sensor respond explicitly is the band width. In frequency analysis of vibration signals, the machine condition is identified by observing the eminent periodic frequencies. The maximum bandwidth of the ADXL335 accelerometer is 90Hz. But the bandwidth necessary for condition monitoring of REB is 128Hz. So to achieve this range the existing capacitor in the ADXL335 was removed and a capacitor of value 0.03 μF is added in series to get the appropriate frequency characteristic or range [17].

In this work, the acceleration signals were collected at 10000 samples per second for a duration of 10 seconds. Signals were collected for three conditions: N, IR, and OR under a radial load of 1.7 kN and maximum shaft speed of 355 rpm and 622 rpm. To check the repeatability of the accelerometer, total three set of signals have been acquired. Each time the amplitude was found varying, but the trend in FFT was same in all set of signals.

3. Fft and Denoising

The frequency domain analysis (FFT) has been used on bearing vibration signals to determine the exact location of the fault in the bearing. FFT for the recorded vibration signals has been plotted using MATLAB. The FFT of good and fault bearing vibration signals signifying different peaks at fault frequencies are shown Fig. 3.1 to Fig. 3.3. The DAQ systems used in this work to acquire the vibration signals from conventional and MEMS accelerometers are different. So, the FFT plots of MEMS and conventional accelerometers are not in good match. The shaft rotation speed is 23.33Hz, and the bearing inner and outer fault characteristic frequencies are 128.412Hz and 87.588Hz respectively. These frequencies are found in spectra of IR and OR. In these figure (a) corresponds to signal from conventional accelerometer and figure (b) corresponds to signal from MEMS accelerometer. The profile and energy of the signal are higher at significance in conventional accelerometer than in MEMS accelerometer.

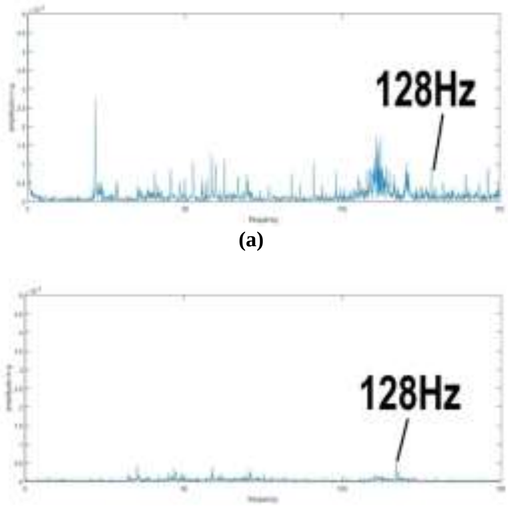


(a)

(b)

Fig. 3.1: FFT of normal bearing.

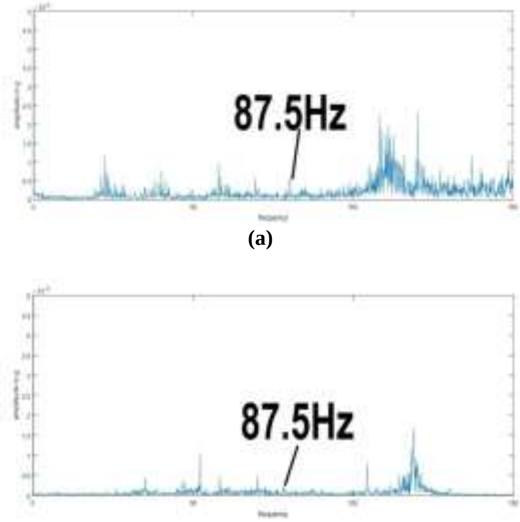
An important use of the wavelets in signal processing is wavelet based denoising. It is most widely used in many studies, as it is effective in extracting hidden information and enhancing the impulsive components of complex, non-stationery signals with background noise. The main theory of wavelet thresholding is that the signal energy will concentrate in a fewer wavelet coefficients, while energy of noise spreads all over in the resulting wavelet coefficients.



(a)

(b)

Fig. 3.2: FFT of bearing with fault in IR.



(a)

(b)

Fig. 3.3: FFT of bearing with fault in OR

The selection of mother wavelets plays an important role here, which makes energy of signal to concentrate in a fewer coefficients and effective denoising. The wavelet based denoising has been able to overcome the limitations of most of the conventional denoising methods. The raw vibration signal has been denoised using customized codes available in MATLAB. The signal energy and kurtosis values for the raw and denoised vibration signals acquired by using conventional and MEMS accelerometers for three conditions of bearing are shown in the Table 3.1.

From the Table 3.1, it is clear that kurtosis is lower and energy values for raw vibration signals are high when compared to denoised vibration signals. Also the values of Energy and Kurtosis are higher for bearing with defect in IR and OR when compared to normal bearing for both raw and denoised vibration signals and can be used as an indicator to detect the defective bearings. Also the energy and kurtosis value of the vibration data acquired from MEMS accelerometer is less when compared to conventional accelerometer. Energy value is also very small.

4. Wavelet Analysis

Since, the WPT has greater advantage over other WT methods, in this work WPT is used for signal decomposition. The denoised signal is decomposed up to three level using WPT selecting ‘haar’ as mother wavelet in MATLAB using customised codes.

Table.4.1: Energy Of Odd Wavelet Packet Coefficients Acquired For Vibration Signal.

Co-efficient	Normal	Inner Race fault	Outer race fault
(3,1)	0.2891	4.09	2.87
(3,3)	0.0386	0.789	0.543
(3,5)	0.0106	0.16	0.012
(3,7)	**0.08**	**0.09**	**0.0987**

The energy of each odd coefficient of the signal at the third level is calculated. The energy obtained is tabulated in Table.4.1. The energy value for different conditions of the bearing shows a clear increasing trend (3, 7) coefficient. Whereas for other coefficients, the energy value increases for normal to IR and then decreases to OR. Hence the statistical features are extracted from this wavelet coefficient.

5. Feature Extraction

The denoised vibration signal (100000×1) was split into 10 non-overlapping bins each with 10000 samples. A three level WPT has been applied to the denoised signal using Haar mother wavelet. From each bin 11 statistical features have been extracted selecting the odd nodes of decomposed signal. The features extracted and these formulae used in this work are shown in Table 5.1.

Table 5.1: Features and Equation

Bearing condition	Conventional accelerometer				MEMS accelerometer			
	K_{raw}	K_{den}	E_{raw}	E_{den}	K_{raw}	K_{den}	E_{raw}	E_{den}
Normal	5.8	10.9	82.16	74.39	3.17	3.31	0.02	0.018
IR	8.03	17.20	450.21	211.52	5.15	7.80	0.061	0.055
OR	8.78	46.73	910.45	347.33	6.37	46.73	0.08	0.064

6. Classifiers

6.1 Artificial Neural Network (ANN)

ANN is used to simplify the complex systems, so as to analyze it. It is based on a collection of connected units or nodes called artificial network. Each artificial network is interconnected such that they can transfer signal from within. An artificial neuron is a computational model inspired by natural neurons.

Table6.1:-

Sl. No.	Features	Formulae	Sl. No.	Features	Formulae
T1	Mean	$\frac{\sum_{i=1}^{n} x_i}{n}$	T7	Variance	$\left(\frac{1}{N}\right)\sum_{n=1}^{N}(x(n) - \bar{x})^2$
T2	RMS	$\sqrt{(\frac{1}{N})\sum_{n=1}^{N} x(n)^2}$	T8	Crest Factor	T4/T2
T3	Standard deviation	$\sqrt{\left(\frac{1}{N}\right)\sum_{n=1}^{N}(x(n) - \bar{x})^2}$	T9	Latitude Factor	$T4/\sqrt{T1^2}$
T4	Peak-Peak value	$\max x(n) - \min x(n)$	T10	Impulse Factor	T4/T1
T5	Skewness	$\frac{\left(\left(\frac{1}{N}\right)\sum_{n=1}^{N}(x(n) - \bar{x})\right.}{\left(\sqrt{\left(\frac{1}{N}\right)\sum_{n=1}^{N}(x(n) - \bar{x}\right.}\right.}$	T11	Log-Log Ratio	$(\frac{1}{\log T3})\sum_{i=1}^{n}(\log\vert + 1)$
T6	Kurtosis	$\frac{\left(\frac{1}{N}\right)\sum_{n=1}^{N}(x(n) - \bar{x})^4}{(\left(\frac{1}{N}\right)\sum_{n=1}^{N}(x(n) - \bar{x})^2}$			

In ANN the signal sent to artificial neurons is processed and it is then sent to others neurons connected to it. Each connection in the network associated with the numeric number called weight [12].

In this work, the feed forward back propagation neural network model is used. The numeric symbols were used to represent the bearing conditions at the output of the classifier,

namely N (1), IR (2), OR (3). One hidden layer with different number of neurons (1 to 10) is used. The sigmoid activation function is used in the hidden layer and at the output layer. Neural networks were trained and then simulated with test data. By comparing actual class with predicted class, performance of each network is calculated.

6.2 k- Nearest Neighbor (kNN)

KNN classifier is a simple non-parametric classification technique. Even though it has simple algorithm, its performance is very good, and is an important benchmark method. It requires two parameters namely distance d and a positive integer k [10].

k-nearest neighbor rule holds position of training samples and their class by calculating the distance between training samples and query data, the decision about the new data can be calculated. k samples with minimum distance are selected based on defined threshold, out of those samples, the class with highest samples is the result. To get the result, a circle centred on input location is drawn; the radius is increased until k samples are embedded in that circle. Then the class with more samples inside the circle is the result.

7. Results and Discussion

Three bearings namely, N, IR, and OR are used in this work. The load was kept maximum (1.7 kN) and two shaft speed (355rpm & 622rpm) conditions. To capture vibration signals, a conventional and a MEMS accelerometer are used. The vibration signals are sampled at 10000Hz and are denoised. Statistical features are extracted from the denoised vibration signals. Hence, dimension of feature vectors containing these statistical values is 60×11. These feature vectors are divided into 80% as training data and 20% as test data and are given as inputs to the classifiers.

To check the performance of MEMS based accelerometer in bearing fault diagnosis, this work estimates diagnostic performance using ANN and kNN. To achieve high diagnostic performance, some parameters need to be set optimally for these classifiers. The performance of ANN is based on the parameters like, number of neurons and accuracy of the test data. The number of neurons (n) is set to 10. Similarly, the performance of kNN is evaluated based on the value of k. The value of k is set as 1 for kNN. The training has been carried out by setting 10 fold cross validation and the parameters are set to get optimum results.

Table 7.1: Classification Accuracies.

Accelerometer	ANN		kNN	
	Train (%)	Test (%)	Train (%)	Test (%)
MEMS	93.3	83.3	85.4	77.8
Conventional	95	90.81	86.09	71.36

Using the optimal parameters for classifiers, the fault diagnosis is carried out. Table 7.1 presents the results of ANN and kNN classifiers for both conventional and MEMS and accelerometers. The test accuracy of MEMS accelerometer is found to be 83.3% and 77.8% using ANN and kNN respectively, which is very close to that of conventional accelerometer, which is 90.81% and 71.36% for ANN and KNN respectively. The confusion matrix of ANN & kNN for MEMS accelerometer is shown in Table 7.2. The number of correctly classified instances is shown in diagonal elements of the confusion matrix. For example, in the first row, 9 samples are correctly classified as ‘N’ where as 1 sample is misclassified as ‘OR’. Out of 18 samples, 2 samples were wrongly classified by the ANN with a classification accuracy of 83.3%. It is evident that, the MEMS accelerometers show comparable performance to that of conventional accelerometers. The confusion matrix of kNN classifier with value of k as 1 is shown in the same table. Out of 18 samples, 4 samples were misclassified by kNN with a classification accuracy of 77.8%. ANN performance is better as it is having better generalisation capability compared to kNN under varying operating conditions and it is demonstrated by [10]

Table7.1: Confusion Matrices Of ANN & KNN For MEMS Accelerometer

ANN			KNN			
N	IR	OR	N	IR	OR	
6	0	1	7	0	0	N
0	4	0	1	5	1	IR
1	1	5	1	1	2	OR

7. Conclusions

A procedure was presented for fault diagnosis of REB using MEMS based accelerometer and comparing the results with that of conventional accelerometer. The proposed method was evaluated using vibration signals obtained from a customized test rig with ADXL335 MEMS accelerometer for three bearing conditions namely, N, IR, and OR. From the result of frequency analysis it is clear that, the MEMS based accelerometer show very similar frequency responses obtained from time domain vibration waveform acquired by conventional accelerometer. Addition to this, the bearing fault diagnosis capability of MEMS accelerometers has been tested using ANN and kNN classifiers. The classification results of both ANN and kNN are enough to conclude that conventional accelerometers can be replaced by MEMS accelerometers for low cost condition monitoring.

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