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Research paper

Expected Time to Recruitment for a Three Grade Manpower System with Bivariate Policy of Recruitment Using Geometric Process for Inter Decision Times

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Abstract

In this paper the time to recruitment is studied using bivariate max policy of recruitment for a three grade manpower system with attrition generated by its policy decisions. It is assumed that the threshold distribution for all the three grades is exponential and the inter decision times forms geometric process. The analytical results are substantiated with numerical illustration.

Keywords: *Manpower Planning, Ordinary Renewal process, Bivariate max policy of recruitment, Geometric Process, Threshold, Expected Mean.*

1. Introduction

In organizations, when policy decisions regarding revision of wages, incentives, and revised sales targets are announced, exit of personnel happens. This in turn leads to depletion of manpower which can be computed in terms of man hours. It would be uneconomical to go in for frequent recruitments. Hence the organization goes for recruitments as and when the cumulative loss of manpower on successive occasions crosses a random threshold level. In [1] , [2] and [6] the authors have discussed about the manpower planning models. The authors have studied the mean and variance using univariate and bivariate recruitment policies in [3] and [5]. In [4] the authors have discussed about the mean and variance using geometric process. In [7] and [8] the expected time to recruitment in a single grade manpower system has been calculated with different epochs for decision and exits and the same using geometric process has been discussed in [9]. For a three grade manpower system with bivariate max policy the mean and variance are calculated in [10] using i.i.d exponential random variable. The objective of this paper is to obtain the expected time to recruitment using bivariate max policy of recruitment for a three grade manpower system using geometric process.

2. Model Description

Consider an organization having three grades A , B and C taking decisions at random epochs in $[0,\infty)$ and at every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower (depletion) is linear and cumulative. For i = 1,2,3… let $\{W_i\}$, be the continuous random variable denoting amount of depletion caused to the system due to the exit of persons on ith decisions in grade A ,B and C respectively .Let $Z_k{}^A = \sum_{i=1}^{\infty} W_i{}^A$, $Z_k{}^B = \sum_{i=1}^{\infty} W_i{}^B$ and $Z_k{}^C = \sum_{i=1}^{\infty} W_i{}^C$ be the cumulative amount of depletion caused in grade A, B and C in the first k decision respectively. The inter-decision times forms geometric process with parameter "a". Each grade has its own threshold level. Let T be a continuous random variable denoting the time to recruitment in the organization with density function l(·) and distribution function L(.). The recruitment policy employed in this paper is as follows: Recruitment in the organization is made whenever the maximum of the cumulative loss of man- hours crosses the threshold level, or the number of decision exceeds its threshold d whichever is earlier.

Let $V_k(t)$ be the probability that there are exactly k- decisions in (0, t]. From renewal theory, $V_k(t) = G_k(t)- G_{k+1}(t)$ with $G_0(t)=1$. Let E(T) be the mean of the time for recruitment.

3. Notations

W_i : Loss of man power to the organization due to i-th decision (i=1, 2, 3……)

Z_A, Z_B, Z_C : Exponential random variable denoting the threshold level for the grades A, B and C respectively.

Z : Continuous random variable denoting the threshold level for the organization.

G(.) : Density function of W_i.
G_k(.) : k-fold convolution of g(.) with itself.
H(.) : The distribution of Z_A, Z_B and Z_C respectively.
U_k : a continuous random variable denoting time between (k-1)th and kth decisions,
k =1,2, with pdf f(.) and cdf F(.).
d : the threshold for the number of decisions.
$V_k(t)$: Probability that there are exactly k- decision epochs in (0,t].
T: Continuous random variable denoting the time for recruitment in the organization.
L(.): Cumulative distribution functions of T.
E(T): Mean time for recruitment.

4. Result

The threshold level for the organization is defined as,
$Z = \max(\min(Z_A, Z_B), Z_C)$
It is known that
$P[\min(Z_A,Z_B) > z] = P[Z_A > z].P[Z_B > z]$ and
$P[\max(Z_A,Z_B) > z] = 1- P[Z_A \leq z]\ P[Z_B \leq z]$
Z_A,Z_B,Z_C follows exponential distribution with
parameters $\theta_1, \theta_2, \theta_3$ respectively, we have

$$\begin{aligned} P[\max(\min(Z_A, Z_B), Z_C) > z] &= 1 - P[\min(Z_A, Z_B) \leq z]\ P[Z_C \leq z] \\ &= 1 - (1 - P(Z_A > z)P(Z_B > z))P[Z_C \leq z] \\ &= 1 - \left(1 - e^{-\theta_1 z} e^{-\theta_2 z}\right)(1 - e^{-\theta_3 z}) \\ &= 1 - \left(1 - e^{-(\theta_1+\theta_2)z}\right)(1 - e^{-\theta_3 z}) \\ &= e^{-\theta_3 z} + e^{-(\theta_1+\theta_2)z} - e^{-(\theta_1+\theta_2+\theta_3)z} \end{aligned}$$

Distribution function of Z is,

$$H(z) = 1 - e^{-\theta_3 z} + e^{-(\theta_1+\theta_2)z} - e^{-(\theta_1+\theta_2+\theta_3)z} \tag{1}$$

$$P(T > t) = \sum_{k=0}^{\infty} P\left[\begin{array}{l} \textit{exactly } k \textit{ decision epochs in } (0,t], k < d \textit{ and the threshold level is not} \\ \textit{crossed by } Z \end{array}\right]$$

$$P(T > t) = \sum_{k=0}^{\infty} V_k(t) P\left[\sum_{i=1}^{k} W_i < Z, k < d\right]$$

$$= \sum_{k=0}^{d-1} V_k(t)\, P\left[\sum_{i=1}^{k} W_i < Z\right]$$

By the law of total probability,

$$P\left[\sum_{i=1}^{k} W_i < Z\right] = \int_0^{\infty} g_k(z)(1 - H(z))dz$$

$$= \left(\bar{g}(\theta_3)\right)^k + \left(\bar{g}(\theta_1+\theta_2)\right)^k - \left(\bar{g}(\theta_1+\theta_2+\theta_3)\right)^k$$

From renewal theory $V_k(t) = F_k(t) – F_{k+1}(t)$ with $F_0(t) = 1$

$$\begin{aligned} P(T > t) &= \sum_{k=0}^{d-1} [F_k(t) - F_{k+1}(t)](\bar{g}(\theta_3))^k + \sum_{k=0}^{d-1} [F_k(t) - F_{k+1}(t)](\bar{g}(\theta_1+\theta_2))^k \\ &\quad - \sum_{k=0}^{d-1} [F_k(t) - F_{k+1}(t)](\bar{g}(\theta_1+\theta_2+\theta_3))^k \\ &= 1 - [1-\bar{g}(\theta_3)]\sum_{k=1}^{d-1} F_k(t)[\bar{g}(\theta_3)]^{k-1} - [1-\bar{g}(\theta_1+\theta_2)]\sum_{k=1}^{d-1} F_k(t)[\bar{g}(\theta_1+\theta_2)]^{k-1} \\ &\quad + [1-\bar{g}(\theta_1+\theta_2+\theta_3)]\sum_{k=1}^{d-1} F_k(t)[\bar{g}(\theta_1+\theta_2+\theta_3)]^{k-1} \end{aligned}$$

$L(t) = 1-P(T > t)$

$$\begin{aligned} &= [1-\bar{g}(\theta_3)]\sum_{k=1}^{d-1} F_k(t)[\bar{g}(\theta_3)]^{k-1} + [1-\bar{g}(\theta_1+\theta_2)]\sum_{k=1}^{d-1} F_k(t)[\bar{g}(\theta_1+\theta_2)]^{k-1} \\ &\quad - [1-\bar{g}(\theta_1+\theta_2+\theta_3)]\sum_{k=1}^{d-1} F_k(t)[\bar{g}(\theta_1+\theta_2+\theta_3)]^{k-1} \end{aligned}$$

$$\begin{aligned} l(t) &= [1-\bar{g}(\theta_3)]\sum_{k=1}^{d-1} f_k(t)[\bar{g}(\theta_3)]^{k-1} + [1-\bar{g}(\theta_1+\theta_2)]\sum_{k=1}^{d-1} f_k(t)[\bar{g}(\theta_1+\theta_2)]^{k-1} \\ &\quad - [1-\bar{g}(\theta_1+\theta_2+\theta_3)]\sum_{k=1}^{d-1} f_k(t)[\bar{g}(\theta_1+\theta_2+\theta_3)]^{k-1} \end{aligned}$$

Since, {Fk} is a geometric process with rate “a” it is known that

$$\bar{f}_k(s) = \prod_{n=1}^{k} \bar{f}\left(\frac{s}{a^{n-1}}\right)$$

$$\begin{aligned} \bar{l}(s) &= [1-\bar{g}(\theta_3)]\sum_{k=1}^{d-1}\prod_{n=1}^{k} \bar{f}\left(\frac{s}{a^{n-1}}\right)(\bar{g}(\theta_3))^{k-1} + [1-\bar{g}(\theta_1+\theta_2)]\sum_{k=1}^{d-1}\prod_{n=1}^{k} \bar{f}\left(\frac{s}{a^{n-1}}\right)(\bar{g}(\theta_1+\theta_2))^{k-1} \\ &\quad - [1-\bar{g}(\theta_1+\theta_2+\theta_3)]\sum_{k=1}^{d-1}\prod_{n=1}^{k} \bar{f}\left(\frac{s}{a^{n-1}}\right)(\bar{g}(\theta_1+\theta_2+\theta_3))^{k-1} \end{aligned}$$

Let g(t) follows exponential distribution with parameter μ > 0,

$$\bar{g}(s) = \frac{\mu}{\mu + s}$$

and $E(U_1)$ is the mean of U_1 with parameter $\frac{1}{\lambda}$.

$$E(T) = E(U_1)\left\{\begin{array}{l} \frac{a}{a-\bar{g}(\theta_3)} + \left(\frac{\bar{g}(\theta_3)}{a}\right)^{d-1}\left(\frac{a(1-\bar{g}(\theta_3))}{(a-1)(a-\bar{g}(\theta_3))}\right) - \frac{a}{a-1}\bar{g}(\theta_3)^{d-1} + \frac{a}{a-\bar{g}(\theta_1+\theta_2)} + \\ \left(\frac{\bar{g}(\theta_1+\theta_2)}{a}\right)^{d-1}\left(\frac{a(1-\bar{g}(\theta_1+\theta_2))}{(a-1)(a-\bar{g}(\theta_1+\theta_2))}\right) - \frac{a}{a-1}\bar{g}(\theta_1+\theta_2)^{d-1} - \frac{a}{a-\bar{g}(\theta_1+\theta_2+\theta_3)} - \\ \left(\frac{\bar{g}(\theta_1+\theta_2+\theta_3)}{a}\right)^{d-1}\left(\frac{a(1-\bar{g}(\theta_1+\theta_2+\theta_3))}{(a-1)(a-\bar{g}(\theta_1+\theta_2+\theta_3))}\right) + \frac{a}{a-1}\bar{g}(\theta_1+\theta_2+\theta_3)^{d-1} \end{array}\right\}$$

$$E(T) = E(U_1)\left\{\begin{array}{l} \frac{a(\mu+\theta_3)}{a(\mu+\theta_3)-\mu} + \left(\frac{\mu}{a(\mu+\theta_3)}\right)^{d-1}\left(\frac{a\left(1-\frac{\mu}{\mu+\theta_3}\right)}{(a-1)\left(a-\frac{\mu}{\mu+\theta_3}\right)}\right) - \frac{a}{a-1}\left(\frac{\mu}{\mu+\theta_3}\right)^{d-1} + \\ \frac{a(\mu+\theta_1+\theta_2)}{a(\mu+\theta_1+\theta_2)-\mu} + \left(\frac{\mu}{a(\mu+\theta_1+\theta_2)}\right)^{d-1}\left(\frac{a\left(1-\frac{\mu}{\mu+\theta_1+\theta_2}\right)}{(a-1)\left(a-\frac{\mu}{\mu+\theta_1+\theta_2}\right)}\right) - \\ \frac{a}{a-1}\left(\frac{\mu}{\mu+\theta_1+\theta_2}\right)^{d-1} - \frac{a(\mu+\theta_1+\theta_2+\theta_3)}{a(\mu+\theta_1+\theta_2+\theta_3)-\mu} + \frac{a}{a-1}\left(\frac{\mu}{\mu+\theta_1+\theta_2+\theta_3}\right)^{d-1} - \\ \left(\frac{\mu}{a(\mu+\theta_1+\theta_2+\theta_3)}\right)^{d-1}\left(\frac{a\left(1-\frac{\mu}{\mu+\theta_1+\theta_2+\theta_3}\right)}{(a-1)\left(a-\frac{\mu}{\mu+\theta_1+\theta_2+\theta_3}\right)}\right) \end{array}\right\}$$

T
his gives the mean time to recruitment

4.1. Numerical Illustrations:

Table 1.1: The mean time to recruitment is numerically illustrated by varying one parameter and keeping the other parameters fixed.

μ	**λ**	**E(T)**
0.5	1.2	1.07346
1.0	1.2	1.05753
1.5	1.2	1.05625
2.0	1.2	1.02307
1.3	0.2	6.36743
1.3	0.4	3.18370
1.3	1.6	0.79593
1.3	2.0	0.63674

θ1 = 0.3, θ2 = 0.1, θ3 = 0.4, a = 4 and d =20

Table1.2: Effect of “a” on the Expected Time to Recruitment (μ = 1.0; λ = 1.0;

a	E(T)
2	1.71994
3	1.39274

4	1.26583
5	1.20416
6	1.16420

$\theta1 = 0.3$; $\theta2 = 0.1$; $\theta3 = 0.4$, $d = 20$)

5. Conclusion:

From the above tables we observe the following results:

(i) If μ increases and keeping other parameters are fixed the mean time for recruitment decreases. That is if μ increases the average loss of manpower increases which in turn decreases the time to recruitment.

(ii) If λ increases and keeping other parameters are fixed the mean time for recruitment decreases In this if λ increases the decisions are taken frequently on the average which in turn advances the time to recruitment.

(iii) If $a > 1$ and keeping other parameters are fixed the mean time for recruitment decreases. When $a > 1$ the geometric process of inter decision times is stochastically decreasing hence mean time to recruitment decreases.

The mean time to recruitment is obtained by assuming that the inter decision times form a geometric process. The influence of nodal parameters on the mean time to recruitment is also studied numerically. The model discussed in this paper is found to be more realistic and numerical analysis shows that the result obtained agrees with reality.

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