



# Neutrosophic Fuzzy Soft Game

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## Abstract

The Soft set theory, originally proposed by Molodtsov, can be used as a general mathematical tool for dealing with uncertainty. This paper is devoted to the discussions of Neutrosophic fuzzy soft set. A new game model is proposed and called Neutrosophic fuzzy soft game since it is based on Neutrosophic fuzzy soft set theory. We concentrate on discussing a class of two person zero-sum games with Neutrosophic fuzzy soft payoffs. The proposed scheme is illustrated by an example regarding the pure strategy problem.

**Keywords:** Soft set, Fuzzy soft set, Neutrosophic fuzzy soft set, two person Neutrosophic fuzzy soft games, soft payoff functions.

## 1. Introduction

The concept of modern Game theory was introduced by John Von Neumann and Oskar Morgenstern in 1944, who described the word 'Game' for the first time by systematically specifying the rules of the game, the move of players, the information they possess during their moves and the outcome for each player at the end of the game.

In 1999, Molodtsov introduced soft set theory for modeling vagueness and uncertainty. Soft set theory is a newly emerging mathematical tool to deal with uncertain problems. The main advantage of soft set theory in data analysis is that it does not need any grade of membership as in the fuzzy set theory. Molodtsov et al applied the soft sets to field such as game theory, operations research, probability and so on.

Neutrosophic logic has been proposed by Florentine Smarandache (13) which is based on non-standard analysis that was given by Abraham Robinson in 1960. In neutrosophic set indeterminacy is quantified explicitly whereas the truth membership, indeterminacy membership and falsity membership are independent.

In recent years, many interesting applications of game theory have been expanded by embedding the ideas of fuzzy sets. Especially, fuzzy games depend on the fuzzy set that is described to set the membership function. The proposed new game is called a Neutrosophic fuzzy soft game since it is based on soft set theory.

## 2. Preliminaries

### 2.1 Soft Set (7)

Suppose that U is an initial universe set and E is a set of parameters, Let P(U) denotes the power set of U. A Pair (F,E) is called a *soft set* over U where F is a mapping given by  $F:E \rightarrow P(U)$ . A soft set (F,E)

on the universe U is defined by the set of order pairs  $(F,E) = \{(e, F(e)) : e \in E, F(e) \in P(U)\}$  where  $F:E \rightarrow P(U)$  s.t.  $F(e) = \emptyset$  if  $e \notin A$ .

### 2.2 Fuzzy Soft Set (5)

Let U be an initial Universe, E be the set of all parameters and  $A \subseteq E$ . A Pair (F,A) is called a fuzzy soft set over U where  $F:A \rightarrow \tilde{P}(U)$  is a mapping from A into  $\tilde{P}(U)$ . Where  $\tilde{P}(U)$  denotes the collection of all subsets of U.

## 3. Fuzzy Neutrosophic Set (12)

A fuzzy neutrosophic set A on the universe of A discourse X is defined as

$A = \{x, T_A(x), I_A(x), F_A(x) : x \in X\}$  where  $T, I, F: X \rightarrow [0,1]$  and  $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$  where  $T_A(x)$  is membership,  $I_A(x)$  is indeterministic function and  $F_A(x)$  is non-deterministic function.

### 3.1. Definition

Let  $U = \{C_1, C_2, \dots, C_m\}$  be the universal set and E be the set of parameters given by  $E = \{e_1, e_2, \dots, e_n\}$ . Let  $A \subseteq E$ . A pair (F,A) be a fuzzy neutrosophic soft set. Then fuzzy neutrosophic soft set (F,A) in a matrix form as  $A_{m \times n} = [a_{ij}]_{m \times n}$ ,  $i=1,2,\dots,m, j=1,2,\dots,n$  where

$$a_{ij} = \begin{cases} T_j(c_i), I_j(c_i), F_j(c_i) & , & \text{if } e_j \in A \\ 0, 0, 1 & , & \text{if } e_j \notin A \end{cases}$$

Where  $T_j(c_i)$  represents the membership of  $(c_i)$ ,  $I_j(c_i)$  represent the indeterminacy of  $(c_i)$  and  $F_j(c_i)$  represent the non-membership of  $(c_i)$  in the fuzzy neutrosophic set  $F(e_j)$ .



**3.2 Definition**

Let (F,A) be fuzzy neutrosophic soft set over U. Then the value function of (F,A) is defined as  $V(F,A)=T_A + (1 - I_A) - F_A$  where  $T_A, I_A, F_A$  denotes the truth value, indeterministic value and false value of (F,A) respectively.

**4. Two Person Fuzzy Soft Game (Tfsg) (4)**

In this, we construct two person fuzzy soft games with fuzzy soft payoffs.

In soft games, the strategy sets and the soft Payoffs are crisp. But in fuzzy soft game, while the strategy sets are crisp, the fuzzy soft payoffs are fuzzy subsets of U.

**4.1 Definition**

Let E be a set of strategy and  $X, Y \subseteq E$ . A choice of behavior in a fuzzy soft game is called an action. The elements of  $X \times Y$  are called action pairs. i.e  $X \times Y$  is the set of available actions.

**4.2 Definition**

Let U be a set of alternatives,  $F(U)$  be all fuzzy sets over U, E be a set of strategies,  $X, Y \subseteq E$ . Then, a set valued function  $\gamma_{X \times Y} : X \times Y \rightarrow F(U)$  is called a fuzzy soft payoff function. For each  $(x, y) \in X \times Y$ , the value  $\gamma_{X \times Y}(x, y)$  is called a fuzzy soft payoff.

**4.3 Definition**

Let X and Y be a set of strategies of player 1 and 2, respectively, U be a set of alternatives and  $\gamma_{X \times Y}^k : X \times Y \rightarrow F(U)$  be a fuzzy soft payoff function for player K, (K=1,2). Then, for each player K, a two person fuzzy soft game is defined by a fuzzy soft set over U as

$$\Gamma_{X \times Y}^k = \{(x, y), \gamma_{X \times Y}^k(x, y) / (x, y) \in X \times Y\}$$

If  $X = \{x_1, x_2, \dots, x_n\}$  and  $Y = \{y_1, y_2, \dots, y_n\}$ , then the fuzzy payoffs of  $\Gamma_{X \times Y}^k$  can be arranged in the form of the  $m \times n$  matrix shown in table.

$\Gamma_{X \times Y}^k$	$y_1$	$y_2$	...	$y_n$
$x_1$	$\gamma_{X \times Y}(x_1, y_1)$	$\gamma_{X \times Y}(x_1, y_2)$	...	$\gamma_{X \times Y}(x_1, y_n)$
$x_2$	$\gamma_{X \times Y}(x_2, y_1)$	$\gamma_{X \times Y}(x_2, y_2)$	...	$\gamma_{X \times Y}(x_2, y_n)$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
$x_m$	$\gamma_{X \times Y}(x_m, y_1)$	$\gamma_{X \times Y}(x_m, y_2)$	.	$\gamma_{X \times Y}(x_m, y_n)$

**5. Two Person Neutrosophic Fuzzy Soft Game (Tfnfsg)**

Let X and Y be a set of strategies of player 1 and 2 respectively, U be a set of alternatives and  $T_{X \times Y}^k : X \times Y \rightarrow F(U)$ ,  $I_{X \times Y}^k : X \times Y \rightarrow F(U)$ ,  $F_{X \times Y}^k : X \times Y \rightarrow F(U)$  be a truth, indeterministic, falsity function of fuzzy neutrosophic soft payoff function for player K,

(K=1,2). Then, for each player K, a two person neutrosophic fuzzy soft game is defined by a neutrosophic fuzzy soft set over U as

$$\Gamma_{X \times Y}^k = \{(x, y), T_{X \times Y}^k(x, y), I_{X \times Y}^k(x, y), F_{X \times Y}^k(x, y) / (x, y) \in X \times Y\}$$

At a certain time player 1 chooses a strategy  $x_i \in X$ , simultaneously Player 2 chooses a strategy  $y_j \in Y$ .

If  $X = \{x_1, x_2, \dots, x_n\}$  and  $Y = \{y_1, y_2, \dots, y_n\}$ , then the fuzzy payoffs of  $\Gamma_{X \times Y}^k$  can be arranged in the form of the  $m \times n$  matrix shown in table.

$\Gamma_{X \times Y}^k$	$y_1$	$y_2$	...	$y_n$
$x_1$	$(T_{X \times Y}^k, I_{X \times Y}^k, F_{X \times Y}^k)_{(x_1, y_1)}$	$(T_{X \times Y}^k, I_{X \times Y}^k, F_{X \times Y}^k)_{(x_1, y_2)}$	...	$(T_{X \times Y}^k, I_{X \times Y}^k, F_{X \times Y}^k)_{(x_1, y_n)}$
$x_2$	$(T_{X \times Y}^k, I_{X \times Y}^k, F_{X \times Y}^k)_{(x_2, y_1)}$	$(T_{X \times Y}^k, I_{X \times Y}^k, F_{X \times Y}^k)_{(x_2, y_2)}$	...	$(T_{X \times Y}^k, I_{X \times Y}^k, F_{X \times Y}^k)_{(x_2, y_n)}$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
$x_m$	$(T_{X \times Y}^k, I_{X \times Y}^k, F_{X \times Y}^k)_{(x_m, y_1)}$	$(T_{X \times Y}^k, I_{X \times Y}^k, F_{X \times Y}^k)_{(x_m, y_2)}$	.	$(T_{X \times Y}^k, I_{X \times Y}^k, F_{X \times Y}^k)_{(x_m, y_n)}$

**6. Fuzzy Soft Saddle Point**

Let  $(\mu_{X \times Y}^k, \nu_{X \times Y}^k)_{(x_m, y_n)}$  be a fuzzy soft payoff function of a two

person intuitionistic fuzzy soft game  $\Gamma_{X \times Y}^k$ . If the following properties hold.

$$(i) \bigcup_{i=1}^m (\mu_{X \times Y}^k, \nu_{X \times Y}^k)(x_i, y_j) = \gamma_{X \times Y}^k(x, y)$$

$$(ii) \bigcap_{j=1}^n (\mu_{X \times Y}^k, \nu_{X \times Y}^k)(x_i, y_j) = \gamma_{X \times Y}^k(x, y)$$

Then  $\gamma_{X \times Y}^k(x, y)$  is called a fuzzy soft saddle point value and  $(x, y)$  is called a fuzzy soft saddle point of player k s in the two person intuitionistic fuzzy soft game.

If  $\gamma_{X \times Y}^k(x, y)$  is a fuzzy soft saddle point of a two person intuitionistic fuzzy soft game  $\Gamma_{X \times Y}^k$ , then player 1 can win atleast by choosing the strategy  $x \in X$ , and player 2 can keep her/his loss to atmost  $\gamma_{X \times Y}^k(x, y)$  by choosing the strategy  $y \in Y$ . Hence the fuzzy soft saddle point is a value of the two person intuitionistic fuzzy soft game.

**7. Working Rule**

Step (1): First to convert the value matrix of Neutrosophic fuzzy soft game.

Step (2): To find the intersection of row value and union of column value.

Step (3): Apply fuzzy soft saddle point rule, we get the Neutrosophic fuzzy saddle point and value of the game.

**8. Numerical Example**

Let  $U = \{u_1, u_2, u_3, u_4, u_5\}$  be a set of alternatives,  $X = \{x_1, x_2\}$ ,  $Y = \{y_1, y_2\}$  be strategies player I and II respectively. Then, two person Neutrosophic fuzzy soft game of player I is given as follows:

**Player II**  
**Player I**

$\Gamma_{X \times Y}^{-1}$	$Y_1$	$Y_2$
$X_1$	$\{(0.7, 0.3, 0.2) / u_1, (0.4, 0.5, 0.7) / u_2, (0.7, 0.4, 0.1) / u_3\}$	$\{(0.6, 0.5, 0.7) / u_1, (0.5, 0.6, 0.7) / u_2, (0.2, 0.3, 0.7) / u_4\}$
$X_2$	$\{(0.3, 0.5, 0.7) / u_2, (0.5, 0.4, 0.2) / u_3\}$	$\{(0.7, 0.4, 0.1) / u_1, (0.5, 0.6, 0.7) / u_4\}$

**Solution:**

The value matrix of Neutrosophic fuzzy soft game is  $V(F,A)=T_A + (1 - I_A) - F_A$

**Player II**  
**Player I**

$\Gamma_{X \times Y}^{-1}$	$Y_1$	$Y_2$
$X_1$	$\{(1.2) / u_1, (0.2) / u_2, (1.2) / u_3\}$	$\{(0.4) / u_1, (0.2) / u_2, (0.2) / u_4\}$
$X_2$	$\{(0.1) / u_1, (0.1) / u_3\}$	$\{(1.2) / u_2, (0.2) / u_4\}$

$$\bigcup_{i=1}^2 \gamma_{X \times Y}^{-1}(x_i, y_1) = \{(1.2) / u_1, (0.2) / u_2, (1.2) / u_3\}$$

$$\bigcup_{i=1}^2 \gamma_{X \times Y}^{-1}(x_i, y_2) = \{(0.4) / u_1, (0.2) / u_2, (1.2) / u_3, (0.2) / u_4\}$$

$$\bigcap_{j=1}^2 \gamma_{X \times Y}^{-1}(x_1, y_j) = \{(0.4) / u_1, (0.2) / u_2, (1.2) / u_3, (0.2) / u_4\}$$

$$\bigcap_{j=1}^2 \gamma_{X \times Y}^{-1}(x_2, y_j) = \{(0.1) / u_2, (0.1) / u_3, (0.2) / u_4\}$$

Since the intersection of first row and union of second column is equal.

The Fuzzy soft saddle point is  $(x_1, y_2)$  and the value of the game is  $\{(0.4) / u_1, (0.2) / u_2, (0.2) / u_4\}$   
Therefore, Intuitionistic Fuzzy soft saddle point is  $(0.6, 0.5, 0.7) / u_1, (0.5, 0.6, 0.7) / u_2, (0.2, 0.3, 0.7) / u_4$

### 9. Conclusion

Soft set theory is a general method for solving problems of uncertainty. In this paper, two-person Neutrosophic fuzzy soft game is introduced and solved in view of Player I and in view of Player II separately using the function defining the degree of favour of element and degree of against of element belonging to the set. We considered aNFS game problem in which  $\Gamma'_{X \times Y}^{-1} = \Gamma''_{X \times Y}^{-2}$  and solved in view of both the players.

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