International Journal of Engineering & Technology, 7 (3.34) (2018) 654-659

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

Vague Separation

1 R. Ramya Swetha, 2 T. Anitha, 3V.Amarendra Babu

1. Department of Mathematics , Gandhiji Institute of Science and Technology, Jaggayyapet, A.P, India .
2. Department of Mathematics, KL University, Vaddeswaram, Guntur, A.P, India.
3.Department of Mathematics, Acharya Nagarjuna University,Nagarjuna Nagar-522510, India.
**Corresponding author Email: ramya.swetha19@gmail.com , anitha.t537@gmail.com , amarendravelisela@ymail.com,*

Abstract

In this paper we are introducing VT_1 space, vague haussdorff space (VT_2) and then we derive every vague subspace of VT_1 space is VT_1 and also for VT_2. And also we derive the Cartesian product of two vague closed sets is also vague closed set in the vague product topological space X x Y .Finally we define Vague limit point, Vague isolated point, Vague adherent point, Vague perfect, Vague derived set, vague exterior and also derive some theorems on this .

Keywords:

1. Introduction

Many authors have introduced different types of generalisation of Zadeh's fuzzy set theory [8] and have been applied to many branches in mathematics. The theory of fuzzy topology was developed by C.L.Chang [2] in 1967. In a fuzzy set, we cannot express both the evidence of supporting and opposing the value of a variable. As a remedy to this the concept of vague set was introduced in 1993 by Gau and Burherer [4]. The basic concept of Vague Set theory and its extensions defined in [4]. The theory of Vague concept is applied to topology as vague Topology was introduced by Mariapresenti.L and Arockia Rani [5].

The aim of this paper to introduce the notion of vague T_1 space (VT_1),vague haussdorff space (VT_2) ,Cartesian product of vague sets. Vague limit point, Vague adherent point, Vague isolated point, Vague derived set, Vague perfect.

2. Preliminaries

Definition 2.1[4]:
A vague set P in the universe of discourse L is characterised by two membership functions given by, a true membership function $t_P : L \to [0,1]$ and a false membership function $f_P : L \to [0,1]$. The grade of membership of y in the vague set P is bounded by a sub interval $[t_P(y), 1-f_P(y)]$ of [0,1]. This indicates that ,if the actual grade of membership μ(x),then $t_P(y) \le \mu(y) \le 1-f_P(y)$. The vague set P is written as $P = \{y, [t_P(y), 1-f_P(y)] / y \in X\}$, where the interval $[t_P(y), 1-f_P(y)]$ is called the vague value of y in P and is denoted by $V_P(y)$.

The zero vague set of P and denoted by **0**,and defined as **0** = {<x,[0,0]> / x ϵ P }.
The unit vague set of P and denoted by **1**, and defined as **1**= {<x,[1,1]> / x ϵ P }

Definition 2.2[1]:
A vague topology on P is a family σ of vague sets on P satisfying the following conditions:

1. **0**, **1** $\in \sigma$
2. $A_1 \cap A_2 \in \sigma$, for any $A_1, A_2 \in \sigma$.
3. $\cup A_i \in \sigma$, for any arbitrary family $\{A_i / A_i \in \sigma, i \in I\}$.

The pair (P, σ) is called a vague topological space. The elements of σ are called vague open sets.

Definition 2.3[6]:
A map g :(P ,τ) →(Q ,σ) is called a vague closed mapping if g(F) is a vague closed set in Q for each vague closed set 'F' in 'P'.

Definition 2.4[7]:
A T_1- space is a topological space(P, γ) in which given any pair of distinct points, each has a neighbourhood which does not contain the other .

Definition 2.5[7]:
A Haussdorff space is a topological space in which each pair of distinct points can be separated by open sets.

Definition 2.6[7]:
Let (P,σ) be a vague topological space and A⊆ P .A point p in P is said to be a limit point of P if each of its neighbourhood contains a point of P different from p. The derived set of P is denoted by D(P),is the set of all limit points of P.

Definition 2.7[7]:
Let (P,σ) be a vague topological space and A⊆ P .A point p in A is said to be an isolated point of A if each of its neighbourhood contains no other point of A.

3. T_1- Spaces and Haussdorff Spaces

Definition 3.1:
A vague topological space (P, σ) is called

a) VT_1 space if all pair of distinct vague points $p_a^{(\alpha,\beta)}$, $p_b^{(\gamma,\delta)}$ of X there exist U,V ∈ τ suchthat

$p_a^{(\alpha,\beta)}$ ∈U, $p_a^{(\alpha,\beta)}$ ∉ V and $p_b^{(\gamma,\delta)}$ ∈V, $p_b^{(\gamma,\delta)}$ ∉U.

b) VT_2 space or Vague Haussdorff space if for all pair of distinct vague points $p_a^{(\alpha,\beta)}$, $p_b^{(\gamma,\delta)}$ of X there exist U,V ∈ τ suchthat U∩ $V = \emptyset$ and $p_a^{(\alpha,\beta)}$ ∈U, $p_b^{(\gamma,\delta)}$ ∈ V.

Example3.2:-
Let P={a,b}, σ ={0,A,B,C}where 0={x<[0,0],[0,0]>} A={x,<[0.4,0.7],[0,0]>},
B={ x,<[0,0],[0.3,0.4]>},C={x<[0.4,0.7],[0.3,0.4]>} then (P, σ) is a vague topological space .
Let $p_a^{(0.38,0.7)}$, $p_b^{(0.29,0.39)}$ be two distinct vague points of P there exist two open sets A and B such that $p_a^{(0.38,0.7)}$ ∈A ,$p_a^{(0.38,0.7)}$ ∉ B and , $p_b^{(0.29,0.39)}$ ∈B, $p_b^{(0.29,0.39)}$ ∉ A and also A∩ $B = 0$.
So (P,σ) is VT_1 and VT_2 space.

Proposition 3.3:
Every vague subspace of VT_1 space is VT_1.
Proof :-Let (P,σ) be a VT_1 vague topological space and Z be a subspace of P. So
T_Z={G $_z$=<x, [t $_{G/Z,}$ 1-f$_{G/z}$]>} where G= <x, [t $_{G,}$ 1-f$_G$]>}. Let a ,b ∈ Z such that a ≠ b. Then as Z ⊆ P, we have a ,b ∈ P such that a ≠ b. Since P is T_1,therefore
∃ U=[t_u,1-f_u],V=[t_v,1-f_v]∈ σ such that $p_a^{(\alpha,\beta)}$ ∈U, $p_a^{(\alpha,\beta)}$ ∉ V and $p_b^{(\gamma,\delta)}$ ∈V, $p_b^{(\gamma,\delta)}$ ∉U.
.Thus ∃ U_Z= [t $_{u/z,}$1-f $_{u/z}$], V_Z = [t $_{v/z,}$1-f $_{v/z}$] ∈ T_Z such that $p_a^{(\alpha,\beta)}$ ∈U_Z, $p_a^{(\alpha,\beta)}$ ∉ V_Z and $p_b^{(\gamma,\delta)}$ ∈V_Z, $p_b^{(\gamma,\delta)}$ ∉U_Z .This proves that the subspace Z is alsoVT_1.

Proposition 3.4:-
Every subspace of VT_2 space is VT_2.
Proof :- Let X be a vague haussdorff topological space and A be a subspace of X .
Let $p_x^{(\alpha,\beta)}$, $p_y^{(\gamma,\delta)}$ be two vague points in A such that $p_x^{(\alpha,\beta)} \neq p_y^{(\gamma,\delta)}$.
Since X is a haussdorff space there exist U,V ∈ τ such that $p_x^{(\alpha,\beta)} \in U$ *and* $p_y^{(\gamma,\delta)} \in V$ and U∩ $V = 0$.
Since U and V are vague open subsets of X and t_u (z) ∧ t_v(z) =0 for every z∈X.There fore U_A=$< x,(t_{U/A}, 1-f_{U/A}) >$ and V_A =$< y,(t_{V/A}, 1-f_{V/A}) >$ are two vague open subsets of A such that $p_x^{(\alpha,\beta)} \in U_A$ *and* $p_y^{(\gamma,\delta)} \in V_A$ and U_A∩ V_A=0 .
Hence (A ,τ_A) is a vague haussdorff topological space .

Theorem3.5:
The homomorphism image of a VT_1 space is a VT_1 space.

proof :- Let (P, τ) be a vague VT_1 space and f be a one-one mapping from (P, τ) to another vague topological space (Q,σ).
Now we have to show that (Q,σ) is a VT_1 space.
Let $p_{y1}^{(\alpha,\beta)}$ and $p_{y2}^{(\gamma,\delta)}$ be two distinct vague points of Q.
Since f is onto ∃ distinct vague points $p_{x1}^{(a,b)}$, $p_{x2}^{(c,d)}$ of P such that $p_{x1}^{(a,b)} \in G$ and $p_{x2}^{(c,d)} \notin G$
and $p_{x2}^{(c,d)} \in H$ and $p_{x1}^{(a,b)} \notin H$,where G and H are vague open sets under the vague T_1 space τ.
Since f is an open mapping ,f(G) is open in Q ,f(H) is open in Q such that
y_1=f(x_1)∈ $f(G)$ but y_2 =f(x_2)∉f(G)
y_2=f(x_2)∈f(H) , X-{y} =H is an open set which contains x but not y.
Hence (P ,τ) is VT_1-space.

Remark3.6:-
Every finite VT_1-space is discrete.
Proof:- Let (P,τ) be a VT_1-space where P is finite. From the above proof every singleton subset{x} of P is closed .Since the finite union of closed sets is closed. So every subset of P is closed .So the VT_1-space (X,τ) is discrete.

Theorem3.7:-
Let σ and σ* be two vague topologies on a set P such that $\boldsymbol{\sigma}$* is finer than σ .If (P, σ) is a haussdorff space then (P, σ*) is also haussdorff space.
Proof:- Given (P, σ) is a Haussdorff space .
Let $p_x^{(\alpha,\beta)}$, $p_y^{(\gamma,\delta)}$ be any two distinct vague points of P then ∃ two vague open sets G and H of σ such that G∩ H=0, $p_x^{(\alpha,\beta)} \in G$and $p_y^{(\gamma,\delta)} \in H$.
since σ^* is finer than σ ,G and H are open sets under σ^*
and having the conditions $p_x^{(\alpha,\beta)} \in G$and $p_y^{(\gamma,\delta)} \in H$ and G∩ H=0.
Hence (X,σ^*) is also haussdorff space.

Theorem 3.8:-
A vague point $p_x^{(\alpha,\beta)}$ and a vague compact set K such that $p_x^{(\alpha,\beta)}$ ∩ K =[0,0] in a Haussdorff space can be separated by disjoint open sets.
Proof :- Let (X, τ) be a vague topological space. Let K be a vague compact set in (X, τ). Since $p_x^{(\alpha,\beta)}$ ∩ K =[0,0] .Therefore $p_x^{(\alpha,\beta)}$ ∉ K and t_k(x)=0,1-f_k(x)=0. Let $p_y^{(\gamma,\delta)}$ ∈ K then clearly $p_x^{(\alpha,\beta)} \neq p_y^{(\gamma,\delta)}$. Thus $p_x^{(\alpha,\beta)}$ and $p_y^{(\gamma,\delta)}$ be two distinct vague points . Since X is a Haussdorff space therefore ∃ two vague open sets U and V such that U∩V =**0** .Thus ,corresponding to each vague point in K ∃ two disjoint open sets with separating that point with $p_x^{(\alpha,\beta)}$ .Clearly K ⊆ $\bigcup_{y(\gamma,\delta)} V$,Since K is compact ∃ finitely many open sets such that K is contained into their union. Suppose that ,the union of these finitely many open sets be represented by H and the intersection of corresponding vague open sets containing $p_x^{(\alpha,\beta)}$ be given by G.
Now we want to show that G∩H =0.On the contrary way, suppose that G∩H ≠ 0 . Then ∃
a vague point say z which will belong to the intersection of G and H, But this will contradict the existence of vague open sets of the type U∩V =0,Hence G∩H =0.

Definition 3.9:-
Let P and Q be two non-empty sets and A ={ <x, [t $_A$(x), 1-f_A(x)]>} , B ={ <y, [t $_B$(y), 1-f_B(y)]>} be two vague sets of P and Q respectively. Then A x B is a vague set of P x Q defined by

A x B (x ,y) ={ < (x ,y) ,[min ($t_A(x)$, $t_B(y)$),min($1-f_A(x)$, $1-f_B(y)$)]>}

Lemma 3.10:-
If A is a vague set of P, B is a vague set of Q then i)(Ax1)∩(1xB) =AxB
ii) (Ax1)∪(1xB) =1-(AXB).
iii) 1- (AXB) =($AX1$)∪(1XB).
Proof :- Let A ={ <x, [$t_A(x)$, $1-f_A(x)$]>} ,B ={ <y, [$t_B(y)$, $1-f_B(y)$]>}
i) Since Ax1 ={ < x, ,[min ($t_A(x)$, 1),min($1-f_A(x)$, 1)]>}= { < x, , [$t_A(x)$,$1-f_A(x)$]>}=A.
and 1xB = ={ < x, ,[min (1,$t_B(x)$),min(1,$1-f_B(x)$]>} = { < x, , [$t_B(x)$,$1-f_B(x)$]>}=B.
(Ax1)∩(1xB) = A(x)∩B(y)
=< (x ,y)($t_A(x) \wedge t_B(y)$, $1-f_A(x) \wedge 1-f_B(y)$) > = AxB
ii) ($AX1$)∪ (1XB) = A(x)∪B(y) =1- (A(x) ∩ B(y))
=1- < (x,y)($t_A(x) \wedge t_B(y)$, $1-f_A(x) \wedge 1-f_B(y)$) >
= 1- AxB

Lemma 3.11:-
Let A and B be two vague closed sets in vague topological spaces respectively then A x B is a vague closed set in the vague product topological spaces of X x Y.
Proof :- Let A = { < x, , [$t_A(x)$,$1-f_A(x)$]>} ,B = { < x, , [$t_B(x)$,$1-f_B(x)$]>}
(1-AxB) (x ,y) =$(AX1) \cup (1XB)\ (x,y)$.
since $(AX1)\ and\ (1XB)$ are vague open sets of XxY respectively. Hence $(AX1)\ and\ (1XB)$ is also a vague open set of X x Y and consequently A x B is a vague closed set of X x Y.

Theorem 3.12:-
Let {(X_i, τ_i) / i∈I} be a family of VT_1 vague topological space and (X, τ) be their product vague topological space. Then (X, τ) is VT_1 iff (X_i, τ_i) are VT_1 for all i ∈ I.
Proof :- Let (X_i,τ_i) be VT_1 for all i∈I.
To show that (X, τ) is VT_1, let x ,y ∈ X and x ≠ y.
Let x= $\prod x_i$, y= $\prod y_i$ then ∃ j∈J such that $x_j \neq y_j$.
Now since (X_j,τ_j) is VT_1 ∃ U_j =[t_{uj},$1-f_{uj}$] and V_j=[t_{vj},$1-f_{vj}$] ∈ τ_j such that $t_{Uj}(x_j)$ =1,
$1-f_{Uj}(x_j)$ =1, $t_{Vj}(x_j)$ =0, $1-f_{Vj}(x_j)$ =0 and $t_{Vj}(y_j)$ =1, $1-f_{Vj}(y_j)$ =1, $t_{Uj}(y_j)$ =0, $1-f_{Uj}(y_j)$ =0 .Now consider the basic vague open sets $\prod U_k$, $\prod V_k$ where U_k = V_k =[1,1] for k∈J, k≠ j and U_k =U_j and V_k =V_j when k=j. Then $\prod U_k$, (x) =[inf $t_{Uk}(x_k)$,inf $1-f_{Uk}(x_k)$]=[1,1], $\prod U_k$, (y) =[inf $t_{Uk}(y_k)$,inf $1-f_{Uk}(y_k)$]=[0,0]. Similarly $\prod V_k$ (y) =[inf $t_{Vk}(y_k)$,inf $1-fv_k(y_k)$]=[1,1], $\prod V_k$ (x) = [0,0].Therefore (X, τ) is VT_1.
Conversely ,let (X, τ) be VT_1
To show that (X_j, τ_j) is VT_1.
Choose x_j, y_j ∈ X_j such that $x_j \neq y_j$
Now, consider x=π x_i ,y= π y_i where $x_i = y_i$, i ≠ j and the j th co - ordinate of x ,y are x_j, y_j respectively.
Then x ≠ y therefore since (X, τ) is VT_1 ∃ U ∈ [t_u ,$1-f_u$] and V= [t_v ,$1-f_v$] ∈ τ such that U(x)=[1,1],U(y)=[0,0], V(x)=[0,0],V(y)=[1,1].
Now, consider the vague points $p_x^{(\alpha,\beta)}$ ∈ U , $p_y^{(\gamma,\delta)}$ ∈V then ∃ basic vague open sets ΠU_i ,ΠV_i in X such that $p_x^{(\alpha,\beta)} \in \Pi U_i \subseteq U$ and $p_y^{(\gamma,\delta)} \in \Pi V_i \subseteq V$

$p_x^{(\alpha,\beta)} \in \Pi U_i \subseteq U \Rightarrow$ α < $\inf_i t_{Ui}\ \alpha$ and β < $\inf_i 1-f_{Ui}\ \alpha\ (x_i)$
$\Rightarrow$ α < $t_{Ui}\ \alpha\ (x_i)$, β < $1-f_{Ui}\ \alpha\ (x_i)$ ∀ i ∈ J
Similarly , we can show that $p_y^{(\gamma,\delta)} \in \Pi V_i \subseteq V \Rightarrow \gamma$ < $t_{vi}\ \gamma\ (y_i)$ and
δ < $1-f_{vi}\ \gamma\ (y_i)$ ∀ i ∈ J
Now , $V_j = \bigcup \{ v_j \}$ is such that $t_{vj}(yj)$ = 1 , [$1-f_{vj}$] (y_j)=1
Further , since $x_i = y_i$ for i ≠ j
α < $t_{Ui}(y_i)$, β < $1-f_{Ui}(y_i)$ ∀ i∈I , i ≠ j and
γ < $t_{vi}(x_i)$, δ < $1-f_{vi}(x_i)$ ∀ i∈I.
Therefore U(y) =[0,0] $\Rightarrow \pi U_i\ (y)$=[0,0] $\Rightarrow$ $\inf_i t_{Ui}(y_i)$ =0 , $\inf_i [1-f_{Ui}]\ (y_i)$ =0
$\Rightarrow$ $t_{Ui}(y_i)$ =0 , [$1-f_{Ui}$]α (y_i) =0 . Similarly $t_{vj}(x_j)$ =0 , [$1-f_{vj}$] (x_j)=0.
Hence (X_j , τ_j) is T_1.

4. Limit Point and Derived Set

Definition 4.1**:-**
Let (X, τ) be a vague topological space and let A be a subset of X. A Vague point $p_x^{(\alpha,\beta)}$ ∈ X is called a vague limit point of A iff every neighbourhood of that vague point contains a vague point of A other than that point.

Example 4.2:-
Let X={a,b,c}, τ={0,G_1,G_2,1} where
G_1={⟨a, [0.4,0.7]⟩, ⟨b, [0.5,0.6]⟩, ⟨c, [0.3,0.7]⟩},
G_2 ={⟨a, [0.5,0.7]⟩, ⟨b, [0.5,0.8]⟩, ⟨c, [0.4,0.8]⟩}.
Let A={a,c} is a subset of X.
$p_a^{(0.4,0.6)}$ is not a vague limit point of A, because the neighbourhood of $p_a^{(0.4,0.6)}$ is G_1 does not contain any point of A other than $p_a^{(0.4,0.6)}$.
$p_b^{(0.3,0.8)}$, $p_c^{(0.4,0.5)}$ are vague limit points of A.

Definition 4.3:-
Let (X, τ) be a vague topological space and let A be a subset of X. A point $p_x^{(\alpha,\beta)}$ is said to be a vague isolated point of a subset A of a topological space X if ∃ some neighbourhood N of $p_x^{(\alpha,\beta)}$ such that N contains no point of A other than $p_x^{(\alpha,\beta)}$.

Example 4.4:-
Let X= {a,b,c} ,τ={0,G_1,G_2,1}where
G_1={⟨a, [0.4,0.7]⟩, ⟨b, [0.5,0.6]⟩, ⟨c, [0.3,0.7]⟩},
G_2 ={⟨a, [0.5,0.7]⟩, ⟨b, [0.5,0.8]⟩, ⟨c, [0.4,0.8]⟩}.
Let A={a,c} is a subset of X.

$p_a^{(0.4,0.6)}$ is a vague isolated point of A. since the neighbourhood of $p_a^{(0.4,0.6)}$ contains no point of A other than this point.

Definition 4.5:-

Let A be a subset of a vague topological space (X, τ) .A point $p_x^{(\alpha,\beta)} \in$ X is said to be a vague adherent point iff every neighbourhood of $p_x^{(\alpha,\beta)}$ contains point of A.

Example4.6:-

Let X= $\{a,b,c\}$,τ={0,G_1,G_2,1}where
$G_1=\{\langle a,[0.4,0.7]\rangle,\langle b,[0.5,0.6]\rangle,\langle c,[0.3,0.7]\rangle\}$,
$G_2=\{\langle a,[0.5,0.7]\rangle,\langle b,[0.5,0.8]\rangle,\langle c,[0.4,0.8]\rangle\}$.
$p_a^{(0.4,0.6)}$, $p_b^{(0.3,0.45)}$, $p_c^{(0.4,0.5)}$ are vague adherent points of A={a,b} .

Definition 4.7:-

Let (X, τ) be a vague topological space and let A be a subset of X is said to be a vague perfect if it has no vague isolated points is called vague perfect.

Example 4.8:-

LetX= $\{a,b,c\}$,τ={0,G_1,G_2,1}where
$G_1=\{\langle a,[0.4,0.6]\rangle,\langle b,[0.5,0.8]\rangle,\langle c,[0.3,0.7]\rangle\}$,
$G_2=\{\langle a,[0.5,0.6]\rangle,\langle b,[0.5,0.9]\rangle,\langle c,[0.4,0.8]\rangle\}$ and let A={a,b}.
$p_a^{(0.4,0.6)}$, $p_b^{(0.3,0.8)}$, $p_c^{(0.4,0.5)}$ are vague limit points of A. So A is vague perfect.

Definition 4.9:-

Let (X, τ) be a vague topological space and let A be a subset of X .The set of all vague limit points of A is called the vague derived set of A.

Example4.10:-

LetX= $\{a,b,c\}$,τ={0,G_1,G_2,1}where
$G_1=\{\langle a,[0.4,0.6]\rangle,\langle b,[0.5,0.8]\rangle,\langle c,[0.3,0.7]\rangle\}$,
$G_2=\{\langle a,[0.5,0.6]\rangle,\langle b,[0.5,0.9]\rangle,\langle c,[0.4,0.8]\rangle\}$ and let A={a,b}.
$p_a^{(0.4,0.6)}$, $p_b^{(0.3,0.8)}$, $p_c^{(0.4,0.5)}$ are vague limit points of A. So V_D(A) ={a,b,c} .

Theorem 4.11:-

Let A,B be subsets of a topological space. Then

1) $V_D(\phi)=\phi$
2) A ⊂ B ⟹ V_D(A) ⊂ V_D(B).
3) $V_D(A\cap B)\subset V_D(A)\cap V_D(B)$
4) $V_D(A\cup B)=V_D(A)\cup V_D(B)$

Proof:-

1) $V_D(\phi)\subset\phi$ and always $\phi\subset V_D(\phi)$,so $V_D(\phi)=\phi$.

2) Let $A\subset B$,now show that $V_D(A)\subset V_D(B)$

let $p_x^{(\alpha,\beta)}\in V_D(A)$.So that $p_x^{(\alpha,\beta)}$ is a vague limit point of A ∃ a nbh N of $p_x^{(\alpha,\beta)}$ contains a point of A other than that point. Since $A\subset B$ ∃ a nbh N of $p_x^{(\alpha,\beta)}$ contains a point of B other than $p_x^{(\alpha,\beta)}$ .so $p_x^{(\alpha,\beta)}\in V_D(B)$ .So $V_D(A)\subset V_D(B)$.

3) $A\cap B\subset A, A\cap B\subset B$

$V_D(A\cap B)\subset V_D(A)$, $V_D(A\cap B)\subset V_D(B)$.

SO $V_D(A\cap B)\subset V_D(A)\cap V_D(B)$.

4) $A\subset A\cup B, B\subset A\cup B$

$V_D(A)\subset V_D(A\cup B), V_D(B)\subset V_D(A\cup B)$

$V_D(A)\cup V_D(B)\subset V_D(A\cup B)$

Now ,we have to show that $V_D(A\cup B)\subset V_D(A)\cup V_D(B)$

In a contrary way ,if $p_x^{(\alpha,\beta)}\notin V_D(A)\cup V_D(B)\Rightarrow$ $p_x^{(\alpha,\beta)}\notin V_D(A\cup B)$

$p_x^{(\alpha,\beta)}\notin V_D(A)\cup V_D(B) \Rightarrow p_x^{(\alpha,\beta)}\notin V_D(A)$ and $p_x^{(\alpha,\beta)}\notin V_D(B)$.

So $p_x^{(\alpha,\beta)}$ is neither a limit point of A nor a limit point of B .Hence

∃ neighbour hoods B_1 and B_2 of $p_x^{(\alpha,\beta)}$ such that $B_1-\{P_x^{(\alpha,\beta)}\}\cap A=\phi$, $B_2-\{P_x^{(\alpha,\beta)}\}\cap A=\phi$

So $p_x^{(\alpha,\beta)}\notin V_D(A\cup B)$.

So $V_D(A\cup B)\subset V_D(A)\cup V_D(B)$.

Definition 4.12:-

Let (X,τ) be a vague topological space and $A\subset X$ .The intersection of all vague closed supersets of A is called vague closure of A. It is denoted by A or Vcl(A).

Let X={k,l,m} and τ ={0,G_1,G_2,1} vague topology on X where

$$G_1=\{\langle k,[0.4,0.6]\rangle,\langle l,[0.5,0.7]\rangle,\langle m,[0.6,0.8]\rangle\}$$

$$G_2=\{\langle k,[0.5,0.7]\rangle,\langle l,[0.6,0.8]\rangle,\langle m,[0.6,0.9]\rangle\}$$

So the vague closed sets are $\tau^c=\{0,1,G_1^c,G_2^c\}$ where

$$G_1^c=\{\langle k,[0.4,0.6]\rangle,\langle l,[0.3,0.5]\rangle,\langle m,[0.2,0.4]\rangle\}$$

$G_2^c = \{\langle k,[0.3,0.5]\rangle, \langle l,[0.2,0.4]\rangle, \langle m,[0.1,0.4]\rangle\}$

.

Let A= $\{\langle k,[0.2,0.6]\rangle, \langle l,[0.3,0.5]\rangle, \langle m,[0.1,0.3]\rangle\}$

.So Vcl(A) = G_1^C .

Theorem 4.13:-

Let A be a subset of a vague topological space (X,τ) then

i) $\overline{A}$ is the smallest vague closed set containing A.
ii) A is vague closed iff Vcl(A)=A.

Proof:-
i) This is obvious.
ii) If A is a vague closed then A is the smallest vague closed set containing A .Hence Vcl(A)=A.

Conversly ,Let $\overline{A}$ = A, Since $\overline{A}$ is the smallest vague closed set containing A.
So A is vague closed.

Theorem 4.14:-

$\overline{A} = A \bigcup D(A)$

Proof:-

First we prove that $A \bigcup D(A)$ is closed.

instead of that $\left[A \bigcup D(A)\right]^c = A^c \cap D^c(A)$ is open.

Let $p_x^{(\alpha,\beta)} \in A^c \cap D^c(A)$.

Then $p_x^{(\alpha,\beta)} \in A^c$ and $p_x^{(\alpha,\beta)} \in D^c(A)$ .So $p_x^{(\alpha,\beta)} \notin A$ and $p_x^{(\alpha,\beta)} \notin$ D (A).

So $P_x^{(\alpha,\beta)}$ is not a limit point of A. There exist no neighbourhood N of $P_x^{(\alpha,\beta)}$ contains points of A.

So N$\subset D^c(A)$ and $N \subset A^c$.

$\Rightarrow N \subset A^c \cap D^c(A)$

$A^c \cap D^c(A)$ is open .So $A \bigcup D(A)$ is closed and $\overline{A}$ is the smallest closed set containing A.

So $\overline{A} \subset A \bigcup D(A)$ and in general $A \subset \overline{A}$ and $D(A) \subset \overline{A}$ .So $A \bigcup D(A) \subset \overline{A}$.

So $\overline{A} = A \bigcup D(A)$.

Definition 4.15:-

Let (X,τ) be a vague topological space. Let A $\subset X$. A point $p_x^{(\alpha,\beta)}$ is said to be an interior point of A iff A is a neighbourhood of $p_x^{(\alpha,\beta)}$ and $\exists$ an open set G such that $p_x^{(\alpha,\beta)} \in$ G$\subseteq A$. The set of all interior points of A is called interior of A. It is denoted by A^0.
A^0=U{G/G is open and G$\subset A$}.

Theorem 4.16:-

Let (X,τ) be a vague topological space. Let A $\subset X$ .Then
i)A^0 is the largest open set contained in A .
II) A is open iff A^0=A.
Proof:-
i) By the def of interior point of A , it is obvious.
ii) if A^0=A then by the def of A^0, A is open.

if A is open ,now show that A^0=A.

since A^0 contains every open subset of A ,so $A \subset A^0$ and always $A^0 \subset A$.

Theorem 4.17:-

Let (X,τ) be a vague topological space. Let A,B $\subset X$. Then

1) X^0 =X, $\phi^0 = \phi$.

2). $A^0 \subset A$.

3) $A \subset B \Rightarrow A^0 \subset B^0$

Proof:-

1) Since ϕ and X are vague open sets , $\phi^0 = \phi, X^0 = X$.

2)Let $p_x^{(\alpha,\beta)} \in A^0$, $p_x^{(\alpha,\beta)}$ is an interior point of A.

A is a neighbourhood of $p_x^{(\alpha,\beta)}$. $p_x^{(\alpha,\beta)} \in A$. Hence $A^0 \subset A$

3) $A \subset B$, $p_x^{(\alpha,\beta)}$ is an interior point of A.

Let $p_x^{(\alpha,\beta)} \in A^0$, $p_x^{(\alpha,\beta)}$ is an interior point of A .

$\Rightarrow A$ is a neighbourhood of $p_x^{(\alpha,\beta)}$ and $A \subset B$

So B is a neighbourhood of $p_x^{(\alpha,\beta)}$

$p_x^{(\alpha,\beta)} \in B^0$.

Definition 4.18:-

Let (X,τ) be a vague topological space. Let A $\subset X$ .A point $p_x^{(\alpha,\beta)} \in A$

is said to be an exterior point of A iff it is an interior point of A^c.

Definition 4.19:-

A point $p_x^{(\alpha,\beta)} \in A$ is said to be frontier of a subset A of X iff it is neither an interior nor an exterior.

Theorem 4.20:-

Let X be a topological space and let $A \subset X$ .Then a point x in X is a frontier point of A iff every neighbourhood of x intersects both A and

Proof:-

$p_x^{(\alpha,\beta)} \in$ Fr(A) Iff $p_x^{(\alpha,\beta)} \notin A^0$ and $p_x^{(\alpha,\beta)} \notin$ Ext (A)

$\Leftrightarrow$ neither A nor A^c is a neighbourhood of $p_x^{(\alpha,\beta)}$

$\Leftrightarrow$ no neighbourhood of $p_x^{(\alpha,\beta)}$ can be contained in A or in A^c.

$\Leftrightarrow$ Every neighbourhood of $p_x^{(\alpha,\beta)}$ intersects both A and A^C

5. Conclusion

In this paper the concepts Vague $T_{0,}T_{1,}T_2$ spaces, vague limit points ,vague adherent points and vague derived set are introduced and derive some theorems .So we hoped that these concepts will raise the concepts vague regular space,normal space and connectedness.

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