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Research paper

Analysis of transformation methods for mathematical modeling of wind resource

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Abstract

In the current global renewable source summit environmental policy, wind power industry has been growing six times fast in recent years. This paper describes and compares the techniques of modeling the wind speed while assessing the wind energy potential of the geographic location of the region. The probability density functions are discussed to designate the wind speed density functions. Transformation method proposed to obtain a wind power density model and its statistical properties are discussed particularly from three pdfs. The wind power density and cumulative density functions are derived using the transformation method. The parameters of those distributions are estimated using the maximum likelihood method. The quality of the goodness of fit is analyzed and compared using the Kolmogorov-Smirnov test. An application of the mathematical model is demonstrated by a case study that involves wind speed data from several stations in India. Also, the descriptive statistics such as mean, standard deviation, skewness and kurtosis of the wind speeds of the different stations are deliberated which provides better intuition about the characteristics and properties of power density. Among the discussed distribution functions, the Burr probability density function appears to be the most reliable statistical distribution for the stations taken for the analysis.

Keywords: *Burr Probability Density Function; Transformation; Mathematical Modeling; Statistical Properties*

1. Introduction

The awareness of the wind features at a certain site leads to the operative consumption of wind power. The dispersal of wind speediness is one of the best essential factor in wind energy valuation, and the functional performance of the wind speed distribution used for the selected site depends on the thorough understanding of the wind features. The outlook of provincial wind energy progress is assessed in terms of the proficiency of mounted wind turbines, social, financial and conservational issues. The energy consumption from the source of wind has been used throughout the world during the last few decades, but its boom has been utmost noteworthy in recent years. The distribution of wind speed is one of the most vital feature in wind resource valuation. It is significant to exactly predict the resources of wind energy in the designated zone when quantifying the potentials of application of those resources. Several research works have been carried out using different probability distributions to describe the wind speed of the system. Industrializations and economic development necessitate further energy sources specifically as clean alternatives that are friendly with the environment and especially help to reduce the global climate change effectively.

Nowadays the increasing energy demand is fulfilled by the consumption of wind energy [6]. Likewise, the wind resource avoids harmful effects associated to the use of conservative fuels for instance greenhouse gas discharges. It has considered to be essential to improve the energy source after renewables since the request for eco-friendly energy foundations is always cumulative. So as to increase the use of renewable energy sources in the energy division we should have dependable climatological measurements that will regulate the finest probable positions for placing the wind turbines. The wind speed is unpredictable and disparity of wind speed over time period is exemplified by numerous probability density functions [11]. This necessitates the existence of time series wind speed data.

2. Methods

The power equation formula to calculate the obtainability of the wind power in a particular region, is given as:

$$P(X) = (0.5)\mu\rho X^3 \tag{1}$$

where ρ is a constant value of air density and μ is a blade sweep area and. Using this wind power, one can calculate the mean wind power of that area. Then from the Betz's law of power factor Bp, the power equation has become

$$P_{B_p}(X) = (0.5)\mu\rho X^3 B_p(u,v) \tag{2}$$

Where $B_p(u,v)$ is the power factor value. Thus the several calculations are needed to get the wind power using the above equation (2). Henceforth to avoid these calculations, we have used the transformation method to get the wind power from the perceived wind speed data. This is generally used procedure in numerical exploration to originate the probability density function for the function of arbitrary variable, h(X). Then let P = h(X), where X is a arbitrary variable for wind speed data and $f_X(x)$ is probability density function of X. Thus, h(X) is a monotonic function as X is always greater than zero. Then the power probability density function can be resultant by

$$f_p(p) = f_X\left(h^{-1}(p)\right)\left|\frac{d\left[h^{-1}(p)\right]}{dp}\right|, \quad p \in P = 0 \tag{3}$$

Otherwise

Let $u = (0.5)\mu\rho$.

Then from the equation (1), we get $P = uX^3$

Hence from P = h(X),

We get h-1(p) = X

$$= \left(\frac{p}{u}\right)^{1/3}$$

And

$$\frac{d}{dp}\left(h^{-1}(p)\right) = \frac{1}{3u^{1/3}p^{2/3}}$$

Thus by substituting this in equation (3), we can derive the wind power pdf $f_p(p)$.

3. Literature review

Morgan et al., & Islam MR et al., have recommended that the pdf of Weibull distribution is cast-off to evaluate the wind power for innumerable regions [7]. On the other hand, it is not possible to model the Weibull pdf for all the wind regimes. In modeling the wind speed, the blend in the Weibull pdf would be superior than using a bimodal distribution [4]. Brano et al., have analysed best modal for wind system in the metropolitan area of Italy and they proposed that the best modal for the tested region as Burr distribution [1]. Chang, have done the comparative study between the unimodal and bimodal wind speed data with six different pdf's. He found that there won't be a significant difference between the fittings of each pdf for the unimodal data. Also he extended that if it is a bimodal wind speed data then the mixture pdf's will provide the better fit than the Weibull pdf [3]. Nurullkamal Masseran, have introduced the transformation method of random variables to derive the wind power directly from the experimental wind speed data. Also he proposed the wind power density model using the Weibull, Gamma and Inverse Gamma distributions and discussed the characteristics of power density using the statistical properties [8]. Ju-Young Shin et al., have introduced Heterogeneous mixture distributions for the wind speed modelling in the region of Arabian Peninsula. They have compared the performance of the non-mixer distributions and the mixer distributions and revealed that the mixer distribution has given the better fit based on all the goodness tests [5]. Talha Arslan et al., have introduced two new distributions, Generalized Lindley (GL) and Power Lindley (PL) in the field of wind speed modelling. Also they have compared these two distributions with Weibull distribution and proved that the GL distribution offers the finest fitting to the wind speed data in the region of Turkey [9]. Yeliz Mert Kantar et al., have presented the Extended Generalized Lindley Distribution (ELD), and proved that it can be substituted to the other distributions for the calculation of wind power potential in the different areas of Turkey [10].

4. Wind speed distribution model

Approximation of the wind turbine power output is a very essential phase in the wind speed precise modeling. It is noteworthy to describe the appropriate distribution that offers the finest fit to the wind speed data at a specific place. We have considered three distributions namely: Weibull (W), Gamma (G) and Burr (B) to define the dispersal of wind speed at three different stations. A thorough explanation about the selected distribution functions and their parameter approximation method is specified. The Maximum Likelihood method gives the parameters of the above distributions and the performance of those distributions are tested with the Kolmogorov-Smirnov's statistic test (KS-test). The descriptive statistics of the empirical wind speed data are calculated and the characteristics of the wind speed are discussed. The wind power of the considered stations are calculated by applying the proposed transformation method and the corresponding wind power density and the cumulative power density curves are also deliberated.

a) The Burr
b) Distribution

The Burr Type XII distribution or simply the Burr distribution is an unremitting probability distribution of a positive arbitrary variable. As well it is recognized as the Singh-Maddala distribution and at times called the "generalized log-logistic distribution. The Burr pdf for the wind speed variable is given as:

$$f_X(x) = \frac{\alpha k\left(\frac{x}{\beta}\right)^{\alpha-1}}{\beta\left(1+\left(\frac{x}{\beta}\right)^{\alpha}\right)^{k+1}} \tag{4}$$

Where α and k are continuous shape constraints (k>0 , α>0) and β is a continuous scale parameter (β>0). Then by applying the transformation method in equation (4), the wind power density function from the Burr distribution is derived as:

$$f_p(p) = \frac{\alpha k\left(\frac{(p/u)^{\frac{1}{3}}}{\beta}\right)^{\alpha-1}}{\beta\left(1+\left(\frac{(p/u)^{\frac{1}{3}}}{\beta}\right)^{\alpha}\right)^{k+1}} \cdot \frac{1}{3u^{1/3}p^{2/3}}$$

$$f_p(p) = f_X(h^{-1}(p))\left|\frac{d\left[h^{-1}(p)\right]}{dp}\right| \tag{5}$$

The cumulative distribution function of Burr distribution is

$$F_X(x) = 1 - \left(1+\left(\frac{x}{\beta}\right)^{\alpha}\right)^{-k}$$

Also the corresponding cumulative power density function can be derived as:

$$F_p(p) = 1 - \left(1+\left(\frac{(p/u)^{1/3}}{\beta}\right)^{\alpha}\right)^{-k} \tag{6}$$

a) The Gamma (G) Distribution

The Gamma distribution is a family of 2 constraint unremitting probability distribution. The erlang, exponential and chi-squared distributions are special cases of the Gamma distribution. The Gamma distribution has two parameters α>0, continuous shape parameter and β>0, continuous scale parameter. Its probability density function is given as:

$$f_X(x) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1}\exp\left(-\frac{x}{\beta}\right) \tag{7}$$

The wind power density function using the transformation method is derived as:

$$f_p(p) = \frac{1}{3\Gamma(\alpha)\beta^{\alpha}u^{\frac{\alpha}{3}}}(p)^{\frac{\alpha}{3}-1}\exp\left[-\frac{p^{1/3}}{u^{1/3}\beta}\right] \tag{8}$$

The cumulative distribution function and the corresponding cumulative power density function of Gamma distribution are given as: $F_X(x)=\frac{1}{\Gamma(\alpha)}\gamma\left(\alpha,\frac{x}{\beta}\right)$ where $\gamma\left(\alpha,\frac{x}{\beta}\right)$ is the lower incomplete gamma function.

$$F_p(p)=1-\exp\left[\left(-\frac{p}{u\alpha^3}\right)\right]^{\frac{\beta}{3}} \tag{9}$$

b) The Weibull (W) Distribution

The Weibull distribution is also a continuous probability distribution. It is named after Swedish mathematician Waloddi Weibull, who described it in detail in 1951. The Weibull distribution is a lot good estimate for the wind speed distribution. The Weibull pdf is a generalization of the Rayleigh pdf. When comparing the wind speed models between the Weibull and Rayleigh pdf's, the better result is provided from the Weibull pdf. The Weibull pdf for the wind speed variable is given by:

$$f_X(x)=\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}\exp\left[-\left(\frac{x}{\beta}\right)^{\alpha}\right] \tag{10}$$

Through the transformation method, we get the wind power density function as:

$$f_p(p)=\frac{\alpha}{3u\beta^3}\left(\frac{p}{u\beta^3}\right)^{\frac{\alpha}{3}-1}\exp\left[-\left(\frac{p}{u\beta^3}\right)^{\frac{\alpha}{3}}\right] \tag{11}$$

The cumulative density function and the cumulative power density function are given as:

$$F_X(x)=1-\exp\left[-\left(\frac{x}{\beta}\right)^{\alpha}\right]$$

And

$$F_p(p)=1-\exp\left[-\left(\frac{p}{u\beta^3}\right)^{\frac{\beta}{3}}\right] \tag{12}$$

5. Results and discussion

a) Description of Data

The real-time datasets were obtained from the National Renewable Energy Laboratory, which is operated by the Alliance for Sustainable Energy U.S. Department of Energy. The hourly data of October–December 2015 of three different wind farms have been used.

Table 1: Descriptive Statistics of Wind Speed in Various Stations

Parameters	Station 1	Station 2	Station 3
Sample size	5000	5000	5000
Max.Speed(m/s)	17.191	16.574	17.768
Mean Speed	8.9384	8.9792	9.1766
Variance	13.046	11.605	14.133
Std. Deviation	3.612	3.4066	3.7594
Std.Error	0.05108	0.04818	0.05317
Skewness	-0.01864	-0.15073	0.05971
Kurtosis	-0.7456	-0.59627	-0.64216

To provide a better understanding about procedures that have been deliberated above, an instance study involving a real time data set are offered in this paper. For the data from all these stations, three different distributions are estimated and paralleled. The expressive measurements of the wind speed data for the three different stations are presented in table I. The mean speed of the selected stations is in the range of 8 9 m/s, also the station 3 is having a highest mean speed among the three stations. Furthermore, the standard deviation for all of the stations ranges between 3 4 m/s in which the station 3 has got the greater standard deviation. As the skewness measures between 0.5 and 0.5 for all the stations, it clearly indicates that the wind speed data are fairly symmetrical. Kurtosis measures the tails of the wind speed data. It can be concluded that all our wind speed data are platykurtic distributions as the kurtosis value for all the stations are less than three. Thus from the Table I, many details regarding the potential of wind speed are observed.

b) Performance Metrics

The Kolmogorov-Smirnov statistic (KS) test is used to identify the goodness of fit among the three distributions. The KS test statistic will give the difference between the experimental and the theoretical wind speed data of the particular stations. The Kolmogorov-Smirnov test statistic for a given theorized cumulative distribution function S(x) and empirical distribution function $S_n(x)$ is given as:

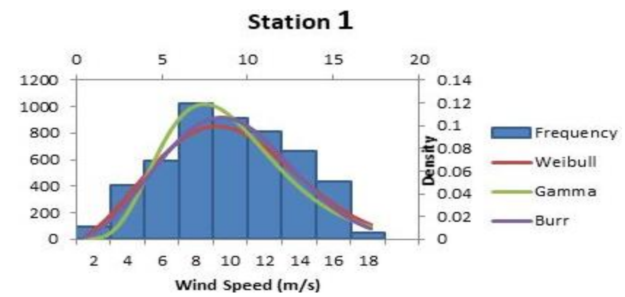
$$D_n=\underset{x}{Sup}\left|S_n(x)-S(x)\right|$$

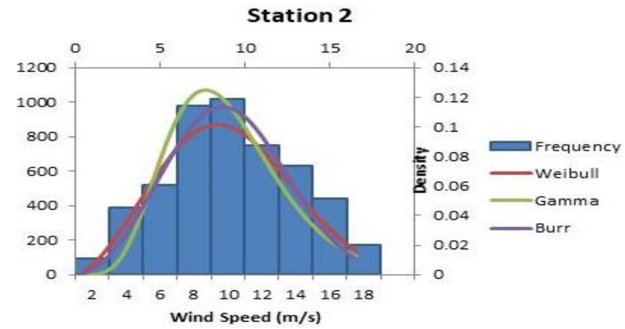
Where $\underset{x}{Sup}$ is the supremum of the set of distances. The KS test statistic values for all the distributions are provided in the table II.

Table 2: KS-Test Statistic Value for Various Distributions

Station	Kolmogorov Smirnov's Test Statistic		
	Burr (B)	Gamma (G)	Weibull (W)
Station 1	0.03535	0.05669	0.03511
Station 2	0.03073	0.05686	0.04607
Station 3	0.02700	0.05155	0.03803

The constraints of the three distributions are calculated by the Maximum Likelihood Estimation (MLE) which is a technique of obtaining the parameter value for a given distribution. MLE efforts to find the values of the parameter values by maximizing the probability function, assumed the annotations. The parameter values of the tested distributions are listed in the table III.





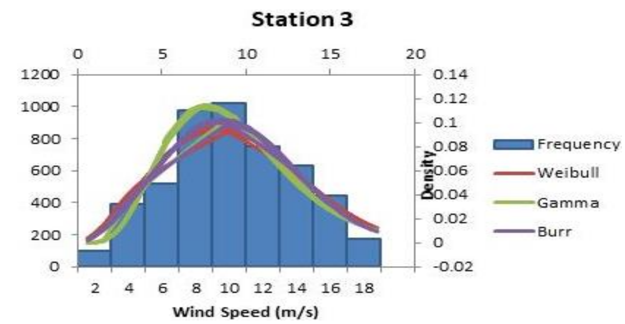


Fig. 1: Wind Speed Fitting of Various Distributions.

The fitting of the experimental wind speed to the assessed distributions for the various discussed stations are shown in the figure 1. The comparison of table 2 and the figure 1 indicates that the Weibull distribution is the finest fit for the station 1 and for stations 2 & 3, the Burr distribution provides the better fit.

Table 3: Parameters of the Distributions by MLE

Station	Burr (B)	Gamma (G)	Weibull (W)
Station 1	K=645.26 A=2.7024 B=110.08	A=6.1239 B=1.4596	A=2.4848 B=10.119
Station 2	K=1129.5 A=2.9030 B=113.39	A=6.9478 B=1.2924	A=2.5738 B=10.183
Station 3	K=1032.1 A=2.6389 B=143.09	A=5.9584 B=1.5401	A=2.4472 B=10.4

The wind power of the experimental wind speed data are derived by applying the transformation method and their corresponding density curves are shown in the figure2.

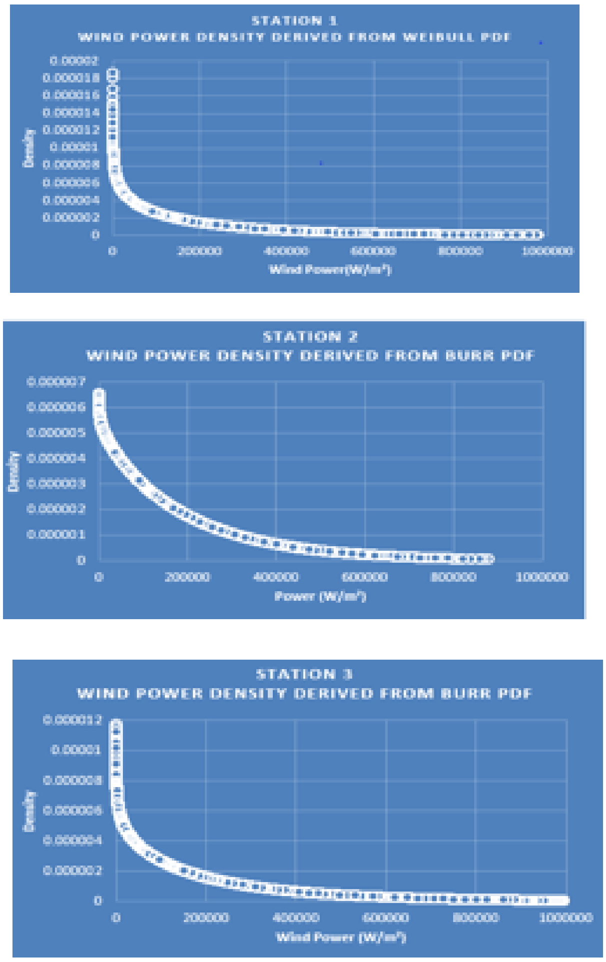


Fig. 2: Wind Power Density Curves.

The figure 3 gives the cumulative density curves for the best fitting distributions of the observed stations.

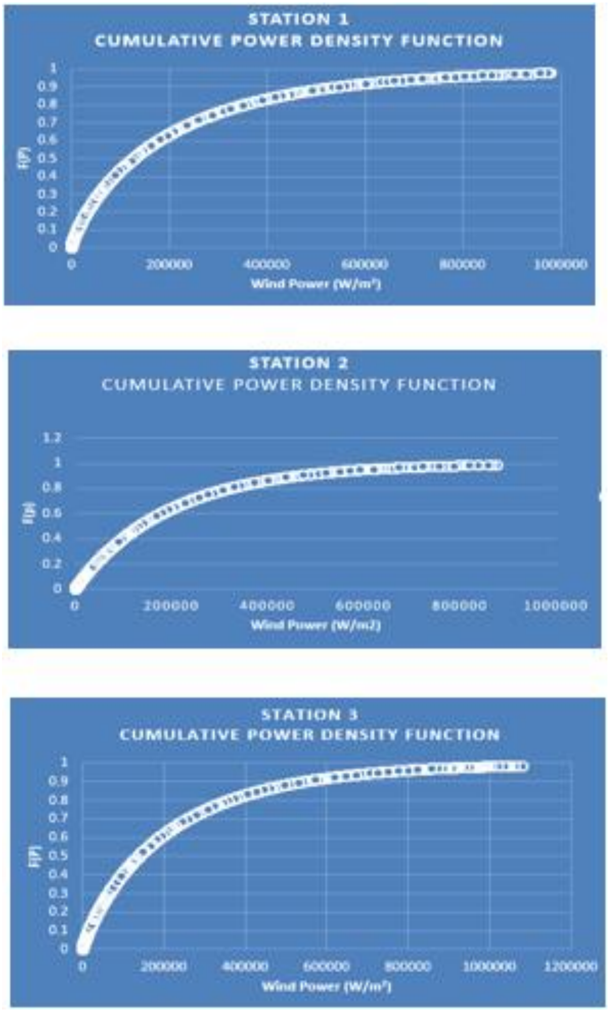


Fig. 3: The Cumulative Power Density Curves.

6. Conclusion

In this article, a comparison case study have been conducted to fit the finest mathematical model for wind speed data from various stations. To derive the theoretical wind power density function from the experimental wind speed data, the transformation method has been recommended. The hypothetical wind speed density fits well with the histogram of experimental wind speed data. Evaluations and comparison of three different distributions for various wind speed data have done in this study. A well-known method called MLE was used to estimate the constraints of those distributions. The goodness of fit is tested using the KS statistics test. The results illustrate that a specific distribution function cannot be suggested for all the measured stations. Largely, this analysis exhibited that the Burr distribution is more pertinent to model the wind speed data since it gives the best fit in two stations and ranked in second position in the other station. The scope of the mathematical model using the transformation method with other distribution functions can be carried out and the performance metrics can be assessed in real time applications.

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