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Research paper

Application of Level Set Method for Shape and Topology Optimization in an Electromagnetic System

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Abstract:

Level set methods(LSM) are conceptual methods for using level sets as tools for numerical analysis of surfaces and geometries. The advantage of the level set model is that you can perform numerical calculations involving curves and surfaces in a fixed grid without parameterizing these objects. In recent years, it has been applied to structural design optimization of mechanical engineering, and it has been actively studied as a new design technique overcoming the problems of existing shape sensitivity analysis method and phase optimization method. You can also use the level setting method to easily follow how the topology changes. In this study, we show the application of the level set method to the optimal design using sensitivity analysis. This paper presents applications of dielectric electrode design, moving objects and particle dynamics in magnetic field systems.

Keywords: *Level set method, optimal design, shape optimization, sensitivity analysis, topology, zero level set.*

1. Introduction

THE LEVEL SET METHOD (LSM) was used to process x-ray or MRI images (P. Y. Muhammed Anshad and S. S. Kumar, 2015; S. U. Aswathy et al., 2015; G. Raghotham Reddy et al., 2016). It is now being actively introduced into the field of electromagnetic field system design. Shape modeling has been one of the most important and difficult problems in the numerical analysis of electromagnetic systems. For example, appropriate and easy representation of shape changes for the analysis problem, optimal shape design problem, and inverse problems of systems with a moving object might involve many technical difficulties. If one of the typical methods of electromagnetic numerical analysis such as the finite difference method, finite element method (FEM), or boundary element method, is used to analyze the abovementioned problems, to express the change of medium boundary according to the change in the shape of model and to appropriately discretize a space will be a complicated process. The present study proposes to use the level set method (LSM) in the electromagnetic system to overcome these difficulties. LSM was first proposed by Osher and Sethian, who were mathematicians (Osher S. and Fedkiw R, 2003; Sethian, J. A., 1999). LSM has been utilized extensively in recent times as a useful tool for representing shape changes in various fields such as fluid mechanics, material science image processing, and computer vision (Michael Yu Wang et al., 2003). Furthermore, this method has been applied to optimize structural shapes in the mechanical engineering field and to overcome problems that appear in the shape sensitivity analysis method and topology optimization method (Il-han Park et al., 1993; Young Sun Kim et al., 2008; Young Sun Kim et al., 2008). In this paper, basic concepts and theories of LSM are briefly explained and the LSM-based optimization method for electromagnetic systems combined with continuum sensitivity analysis is described. Finally, numerical analysis of the optimal design problem of dielectric distribution in electric field, modeling and analysis of a moving object in magnetic field, and the motion analysis problem of dielectric particles in electric field are carried out as the case studies of applying LSM.

2. Concept and Theory of Level Set Method

LSM expresses the regions and boundaries in the level set function, which is a continuous function. This function is an implicit and high order function, and a boundary transformation is represented by the properties of this function. The level set function, $\phi(x)$, which is an implicit function on the region Ω that has a certain boundary, is defined as (1) and Fig. 1.

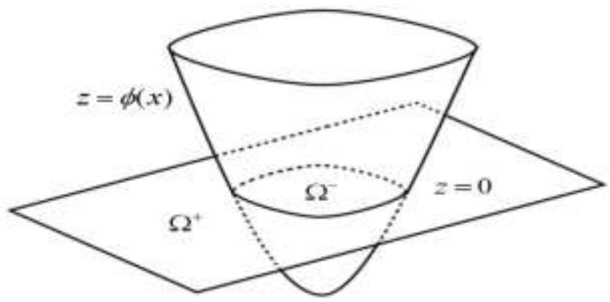


Fig. 1: Design domain Ω- and distribution of level set function

$$\begin{aligned} &\phi(x) > 0 \; x \in \Omega^+ \; : \text{air} \\ &\phi(x) = 0 \; x \in \partial\Omega \; : \text{boundary} \\ &\phi(x) < 0 \; x \in \Omega^- \; : \text{Material} \end{aligned} \tag{1}$$

Variation of the domain and shape can be defined by converting the level set function into independent variables, x and t, which are space and time variables, respectively. This will then determine the domain change, Ω-.

$$\Omega^{-}(t)=\{\phi(x,t)<0\} \quad (2)$$

Here, the boundary of Ω- (t), Γ(t), can be determined by the zero level set.

$$\Gamma(t)=\{\phi(x,t)=0\} \quad (3)$$

The shape of the domain changes with time and this change can be expressed in terms of a velocity field, *V*, which is defined in the entire domain. Then, the following relationship can be obtained.

$$\frac{dx(t)}{dt}=V(x(t),t) \quad (4)$$

The zero level set, where the function $\phi(x,t)$ has a constant zero, represents a surface in a three-dimensional space. Then, this set can be expressed in the first-order Hamilton-Jacobi equation through the total derivative representation of the zero level set in terms of time.

$$\frac{\partial\phi}{\partial t}+V\cdot\nabla\phi=0 \quad (5)$$

In general, because only the normal component of velocity vector contributes to the change of domain, the velocity on zero level set surfaces can be expressed as follows.

$$V=V_n\,\mathbf{n} \quad (6)$$

Here, V_n is a scalar function and n represents an outward-pointing unit normal vector on the boundary, *Γ*. Since the unit vector can be expressed as $\mathbf{n}=\nabla\phi/|\nabla\phi|$, the level set equation can be expressed as follows.

$$\frac{\partial\phi}{\partial t}+V_n\nabla\phi=0 \quad (7)$$

Two functions are normally used to utilize the LSM. The first function is the Heaviside function and is expressed as follows.

$$H(\phi(x))=\begin{cases}0 & if \quad \phi\le 0\\ 1 & if \quad \phi\le 0\end{cases} \quad (8)$$

The other function is the Dirac delta function, which is expressed as follows.

$$\delta(\phi(x))=\frac{dH(\phi(x))}{d\phi} \quad (9)$$

where ϕ is a function of x and the medium constant of the system is determined by the level set function. Using these functions, the surface and the boundary integrals for an arbitrary function in the practical problems can be expressed as follows.

$$\int_{\Omega} f(x)d\Omega=\int_{\Omega} f(x)H(\phi(x))d\Omega \quad (10)$$

$$\int_{\Gamma} f(x)d\Gamma=\int_{\Omega} f(x)\delta(\phi(x))|\nabla\phi(x)|d\Omega \quad (11)$$

The following is an example of the electrostatic field. It shows the final level set equation and the governing equation of the electric field. In the electromagnetic field equation, the spatial and medium distributions are determined by the level set function. Moreover, in the level set equation, the velocity V_n is determined through the electric field analysis. Therefore, the following two differential equations can be used to form one combined system equation.

$$\frac{\partial\phi}{\partial t}+V_n|\nabla\phi|=0$$
$$\nabla^2 V=-\frac{\rho_v}{\varepsilon(\phi)} \quad (12)$$

where V is the electric scalar potential, ρv is the charge density, and ε is the permittivity.

3. Optimization in the Electromagnetic System

In order to perform optimization using LSM in the electromagnetic field system, an analytical sensitivity equation for continuum sensitivity is necessary to represent changes in the objective function as the shape on zero level set surface changes. Furthermore, a process to express constraint conditions for the shape in terms of the level set function is also required.

A. Objective Function and Constraints

For example, the basic formula for design optimization in an electrostatic field can be expressed by an objective function and a constraint equation.
Min :
$F(V(x,y))$: objective function
Subject to :
$\nabla^2 V=-\rho_v/\varepsilon(\phi)$: System equation

$$\int_{\Omega} H(\phi)d\Omega=S^{*} \quad \text{: Spatial constraint} \quad (13)$$

where Ω is a design domain. These equations are iterated in a direction that repeatedly minimizes the objective function to find the level set function while satisfying both the Poisson equation and the spatial constraints.
The design variable becomes a boundary between a dielectric material and air, which can be expressed by a zero level set function as follows.

$$\phi(x,t)=0 \quad (14)$$

Because the level set function is a function of time and position, its total derivative at the boundary can be expressed using Eulerian equation as follows.

$$\frac{d\phi}{dt}=\frac{\partial\phi}{\partial t}+V_n|\nabla\phi|=0 \quad (15)$$

Here, the shape of the design domain is changed by velocity, and it is determined by a derivative of the objective function, which is the continuum sensitivity obtained by the electromagnetic field analysis.

B. Continuum Sensitivity Analysis

In the electromagnetic field system, a total derivative of the objective function can be derived using the changes of both the medium boundary and supplementary variable technique in the continuum sensitivity analysis method. For instance, the continuum sensitivity in electrostatic systems can be expressed as follows.

$$\frac{dF}{dt}=\int_{\Gamma} G(V,\lambda)V_n d\Gamma \tag{16}$$

Where

$$G(V,\lambda)=(\frac{\varepsilon_1}{\varepsilon_2}-1)[\varepsilon_1 E_n(V^*)E_n(\lambda^*)+\varepsilon_2 E_t(V^*)E_t(\lambda^*)]$$

where V_n is a normal component of the velocity term, λ is a supplementary variable, and Γ is a design boundary. The sensitivity equation represents a relationship between an objective function and a velocity term. If a velocity is chosen as follows, the objective function would decrease with time.

$$V_n=-G(V,\lambda) \text{ on } \Gamma \tag{17}$$

Then, the obtained velocity value can be substituted into the Hamilton-Jacobi equation to obtain the level set equation, where the objective function decreases as follows.

$$\frac{\partial\phi}{\partial t}-G(V,\lambda)|\nabla\phi|=0 \tag{18}$$

The above equation is a partial differential equation in the time domain and its analysis is to perform the optimization in the time domain. Therefore, the variable, *t*, is called "pseudo time" in the optimization process.

4. Application Examples of LSM in the Electromagnetic Field System

In this section, several application examples of LSM in electromagnetic field systems are presented. Those examples are dielectric redistribution in the electric field, moving object analysis in the magnetic field, and the particle motion problem in the electric field.

C. Dielectric Distribution in Electric Field

In this example, the goal is to optimize the topology and the shape of a dielectric material so that the system energy of the electric system with the dielectric between the electrodes can be maximized. Here, the objective function is the energy in the electric field system, which is as follows.

$$W \int_{\Omega}\frac{1}{2}E(\phi)\cdot D(\phi)d\Omega \tag{19}$$

A velocity term can be obtained through continuum sensitivity analysis in the electrostatic field as follows, and it is applied to the level set equation.

$$V_n=(\frac{\varepsilon_1}{\varepsilon_2}-1)[\varepsilon_1 E_n^2(V)+\varepsilon_2 E_t^2(V)] \tag{20}$$

The Fig. 2 shows an initial shape of 12 dielectric rods placed between curved electrodes and the zero level set function of the optimized final shape.

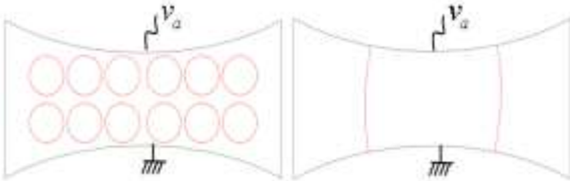


Fig. 2: Zero level set of initial and final shape.

Fig. 3 shows distributions of level set functions for both the initial shape and the final shape. Fig. 4 shows a zero level set where the topology and shape changes occur at each computation step.

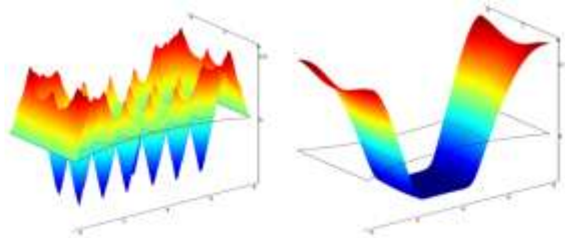


Fig. 3: Distributions of level set function for initial and final shapes

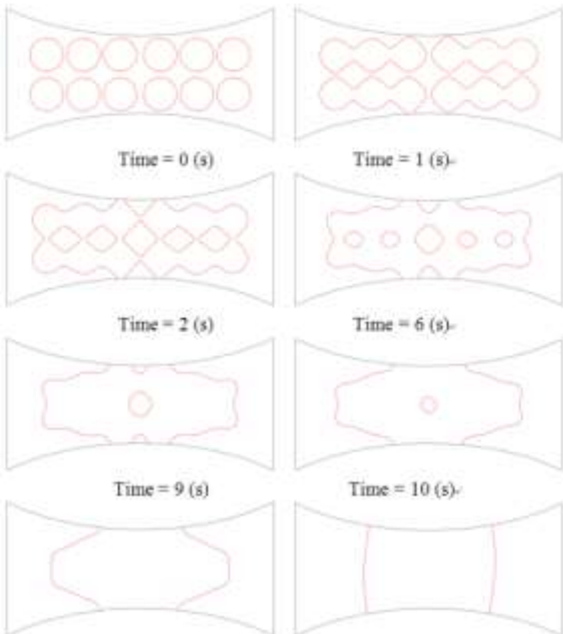


Fig. 4: Optimization process for rounded electrode model.

Fig. 5 shows a change of electrostatic system energy and an error variation of constraint during the optimization process.

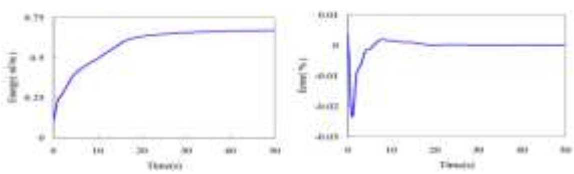


Fig. 5: System energy and error variation of constraint.

Fig. 6 shows an initial shape of 15 dielectric rods placed between funnel-shaped electrodes and the final shape in which the dielectric is concentrated towards the bottom after the optimization.

Fig. 6: Distribution of dielectric rods in the initial and final shapes.

Fig. 7 shows the optimization process at a different time. It was observed that the shape and the topology changed freely in the medium domain.

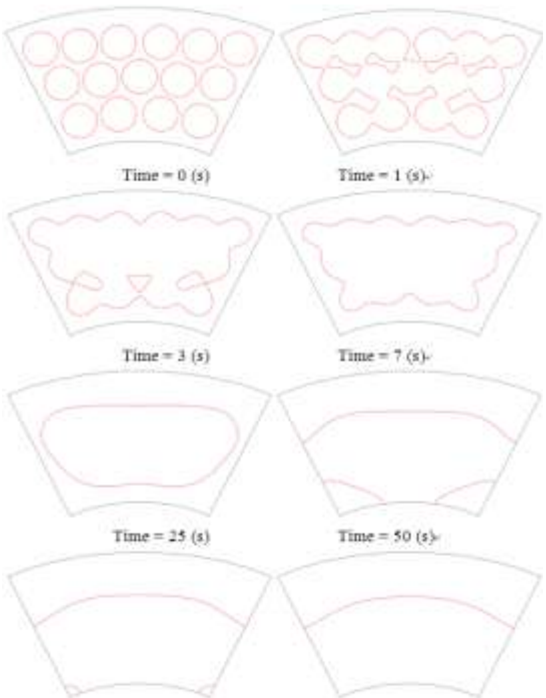
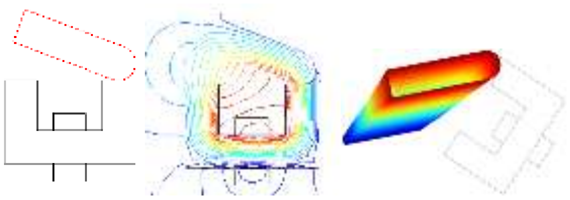


Fig. 7: Optimization process of the funnel-type electrode model.

D. Moving Object Modeling in Magnetic Field

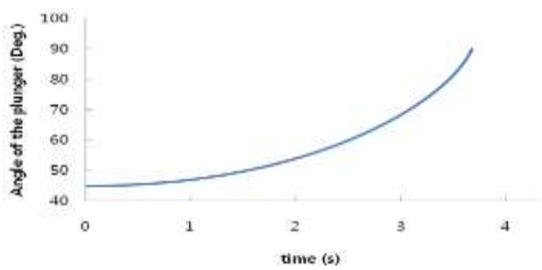
For the dynamic characteristics analysis of an electric system including a moving object, it is necessary to perform the iterative electromagnetic field analysis for the shape depending on the movement, and the element remeshing must be performed in every step. Using the LSM, it is possible to express change of the moving object without element remeshing for the topology change of the moving object. Fig. 8 shows an example of the dynamic characteristics analysis of an electromagnet system including a moving object.



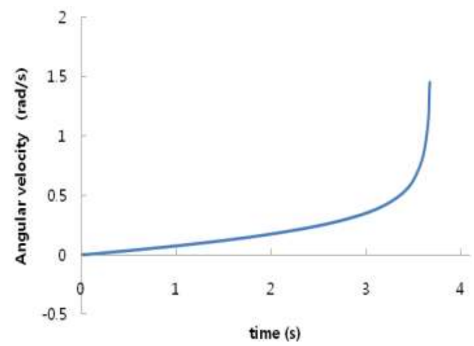
(a) (b) (c)

Fig. 8: (a) Zero level set, (b) field distribution and (c) level set function of the design area for the electromagnetic system.

The results of the dynamic characteristics analysis are shown in Fig. 9, in which the plunger shape of the electromagnet using the LSM was represented. The electromagnetic force was calculated from the electromagnetic field analysis, and finally, the motion equation of the plunger was combined with them.



(a)



(b)

Fig. 9: (a) Position and (b) velocity of plunger with respect to time.

E. Motion Of Dielectric Particles in Electric Field

As shown in the previous examples, a combination of the FEM and LSM facilitates the boundary representation and the electromagnetic field analysis of an object that varies with time using fixed elements. The Fig. 10 shows a numerical analysis example of modeling for the motion of dielectric particles in the electric field using the numerical analysis method. The initial and final zero level sets of the dielectrics are shown in Fig. 11. Fig. 12 shows the distribution of magnetic flux by external magnetic field and the movement path of dielectric particles.

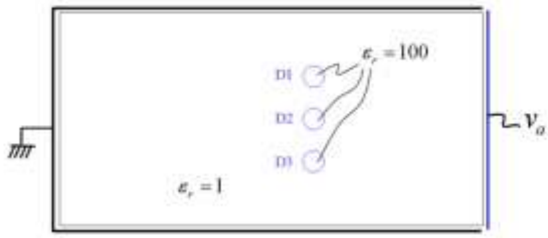
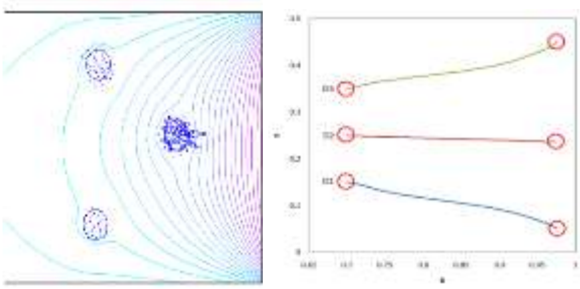


Fig. 10: Dielectric particles in the non-uniform electric field.

(a) (b)

Fig. 11: Initial and late zero level set of moving dielectric with respect to time.



(a) (b)

Fig. 12: (a) Magnetic field distribution and (b) trajectory of the particles.

5. Conclusion

The LSM is free to express various boundaries and it is easy to change the material properties for analyses with different mediums. Therefore, it has a much lower possibility to fall into the local minimum of the objective function, which is one of the largest problems in design optimization and in the inverse problem of the electromagnetic system. It can also provide convenient numerical shape modeling such as defining design variables.

Furthermore, we believe that this method will contribute to the topology design optimization, shape design optimization, and inverse problem solving in electromagnetic systems. Finally, we believe that this method would provide a convenient and useful insight for the analyses of the electromagnetic-dynamics coupling problem and the electromagnetic-fluid dynamics coupling problem.

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