



Exact Modeling of Binary Decision Diagram Complexity

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Abstract

An exact model of Binary Decision Diagram (BDD) complexity is derived in this paper. An estimate of BDD complexity using models has already been proposed. The estimated BDD complexity in previous papers is just an approximation of experimental data using a curve that fits the data. Mathematical analysis indicates that many curves can be fitted to the same data. This paper derives the exact model for the BDD complexity by analysis. In order to derive the exact model, relation between the SOP (Sum Of Product) terms and Min-terms was analyzed. Further the variation of BDD size for various Min-terms was also analyzed. Finally the plot between the experimental BDD size and the derived exact model proves the accuracy of the derived model.

Keywords: Binary Decision Diagrams, Boolean Functions Complexity, Modeling.

1. Introduction

BDD is a direct non-cyclic diagram portrayal of a Boolean capacity proposed by Akers and Bryant [1], [2]. The accomplishment of this strategy has pulled in numerous scientists in the zone of amalgamation and check of VLSI and CAD frameworks [3], [4] and they have turned out to be exceptionally well known information structures, since they permit effective portrayal of numerous down to earth capacities.

The productivity of BDDs depends fundamentally on the measure of their diagram portrayals. Throughout the years the multifaceted nature of BDD execution expanded, and the quantity of hubs constituting the BDD turned into a noteworthy worry since it is straightforwardly identified with the time and space necessities of the computerized circuit spoke to by this BDD. Specialists around there are effectively associated with discovering strategies to limit the span of BDDs to a worthy esteem.

It has been seen that the variable requesting in a given Boolean capacity additionally assumes a noteworthy part in limiting the span of the BDD diagram, which decreases the aggregate number of hubs required to assemble the BDD [4]. Customarily this errand was done by methods for recreation. In this approach one needs to make the total BDD which speaks to the Boolean capacity with the most ideal variable requesting. With a specific end goal to diminish the reproduction time, examine has been done to decrease the BDD measure calculation by displaying the BDD estimate variety [5,6,7]. Fig.1 represents the connection between the BDD measure and the quantity of terms in SOP.

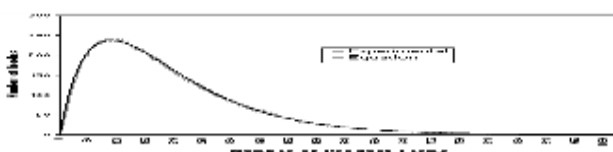


Figure1: Connection between BDD Size and number of terms in SOP for 11 factors.

The main motivation of the work is from the perspective the many curves can be fitted to the same data and the estimation in [5,6,7] is one of the many cures that can be fit

The fundamental target of this paper is to reclassify the model in [5,6,7] to an exact model that is derived from analysis. In [5,6,7] the connection between number of terms in SOP and BDD measure was assessed. In order to find the exact relation, analysis was divided in to two steps. The first step is the relation between SOP terms and Min-terms and second is the relation between the Min-terms and the BDD size. Both of these steps provide an exact relationship. The two relationships were combined to generate the target relation that redefined the model in [5,6,7]. The relation between Min-terms and the BDD size is an extension of the analysis in [8]. In [8] the relation between the BDD size and XOR/XNOR Min-terms was analyzed as depicted in Fig.2.

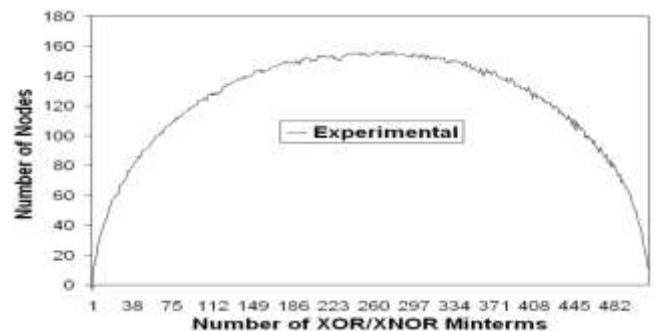


Figure2: BDD Size variation for XOR/XNOR Min-terms for 10 variables

In the second segment of this paper, the impediments of the current model are clarified. The third segment of the paper clarifies the connection between the quantity of terms in SOP and relating Min-terms secured. Area four examines the connection between the quantity of Min-terms and the BDD measure. Area five joins the connection in areas three and four to infer the correct connection between number of terms in SOP and the BDD estimate lastly the paper is finished up.

2 Impediments of the Current Bdd Measure Estimation Show

Figure 1 demonstrates the connection between BDD measure and the quantity of terms in SOP as clarified in [5,6,7]. In Fig 3, the Poisson circulation with $\lambda=4$, yields a similar bend. Hence, proved that for the experimental BDD size, many curves can be fit and the model in [5,6,7] is one of the many fits. In [5,6,7] no theoretical explanation is provided for the model. In this paper an exact model is derived and proved that the exact model fits the experimental data.

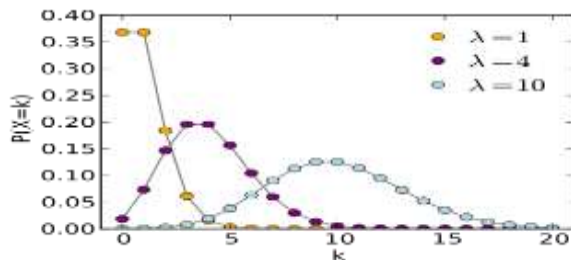


Figure 3: Poisson distribution

3. Connection between the Quantity of Terms in Sop and Number of Min-Terms Secured

A product term can cover more than one Min-term. For example, in the following equation, (assuming three variables a, b and c).

$$bc = abc + a'bc \tag{1}$$

the product term bc covers the Min-terms $abc + a'bc$. An SOP term can cover more Min-terms that are covered by one product term. In the following equation,

$$ab + bc = abc + abc' + a'bc \tag{2}$$

The SOP expression stomach muscle + bc covers three Min-terms. In condition (2) there are two terms in SOP term and three Min-terms. It ought to be noticed that the quantity of Min-terms conceivable is $2n$ and number of item terms conceivable is $3n$, where n is the quantity of factors. So as to discover the connection between the quantity of terms in a SOP articulation and the relating Min-terms secured, an analysis was led. The examination has the accompanying advances:

- Stage 1: For j differing from 1 to $3n$, rehash stages 2 to 4
- Stage 2: Generate 100 SOP with j item terms
- Stage 3: For each SOP, the quantity of Min-terms secured is found and recorded.
- Stage 4: The normal number of Min-terms secured for j SOP terms is ascertained and recorded.
- Stage 5: Plot a bend for the quantity of Min-terms secured and the quantity of terms in SOP.

Fig.4 it can be gathered that the Number of Min-terms secured raises converse exponentially contrasted with the quantity of Terms in SOP.

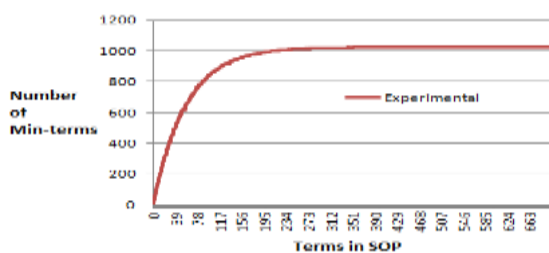


Figure 4: Relation between Number of Terms in SOP and Min-terms covered (Experimental)

The curve in Fig. 4 was modeled using the following equation.

$$MIN_T = 1024 (1 - e^{-0.17403 SOP}) \tag{3}$$

Where, MIN_T is the number of Min-terms, and SOP is the number of terms in SOP.

Fig. 5 shows the same relation as in Fig. 4 for experimental and equation (3).

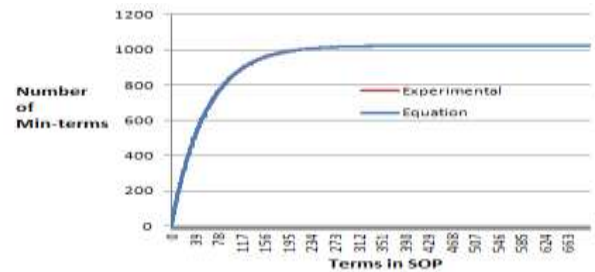


Figure 5: Relation between Number of Terms in SOP and Min-terms covered (Experimental and Equation)

4. Relation between Number of Min-Terms and Bdd Size

The analysis in [5,6,7] was repeated for Min-terms, and was an extension of analysis in [8] for complete list of Min-terms (Rather than only XOR/XNOR Min-terms). Fig. 6. shows the relation between the number of Min-terms (1024 for 10 variables) and corresponding BDD size.

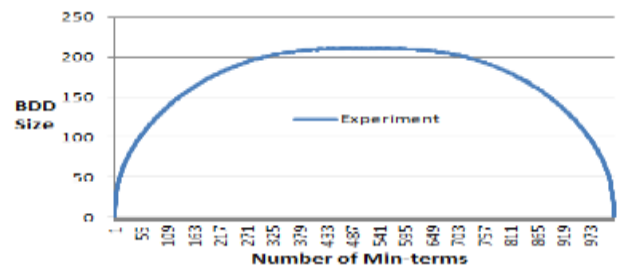


Figure6: Relation between Number of Min-terms and BDD Size (Experimental)

The relation in Fig.6 can be seen to be a semi-circle and can be modeled using the following equation.

$$BDD_S = 0.421 * SQRT(512^2 - (512 - MIN_T)^2) \tag{4}$$

Where, BSS_S is the size of BDD, and MIN_T is the Number of Min-terms

Fig. 7 shows the same relation as in Fig. 6 for experimental and equation (4).

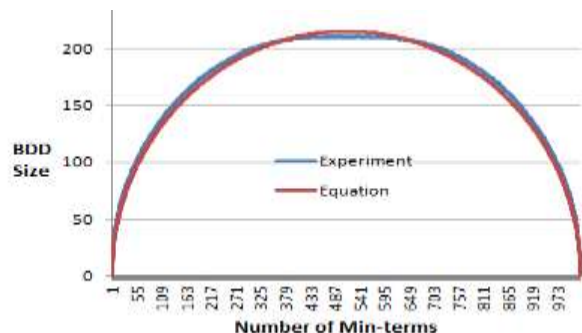


Figure7: Relation between Number of Min-terms and BDD Size (Experimental and Equation)

5. Redefined Exact Model for Bdd Size

The curves in Fig.4 and Fig.6 are exact exponential and semi-circle. Hence when the two are combined the relation should also be exact. Combining (3) and (4) we get,

$$BDD_S = 0.421 * \sqrt{512^2 - (512 - 1024(1 - e^{-0.17403 SOP}))^2} \quad (5)$$

Hence from analysis equation (5) should be the relation between BDD Size and Number of terms in SOP.

Fig.8 shows the variation of the size of BDD using experiment and from equation (5). From Fig8, the rise and fall of the experimental BDD size is obtained in the equation. The difference in height and the shift of the curve is due to the different variable reordering method used for BDD the different experiments. Thus the exact equation (5), that produces the same shape of the curve as in [5,6,7] is derived.

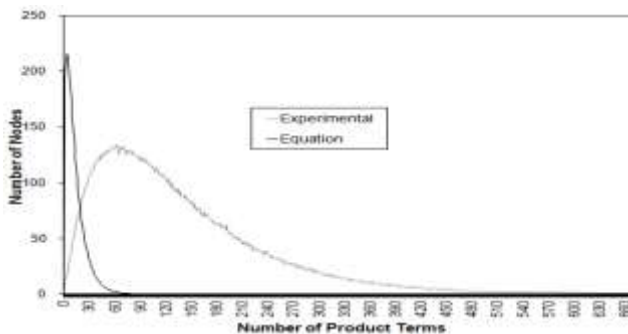


Figure 8: Variation of BDD Size (Experimental and Equation)

Future work of the paper will address the difference in height and shift of the curve. This future work will also include the re-run of the experiment in [5,6,7]

6. Conclusion

In this work, we discover the issue of precisely finding the condition for BDD many-sided quality. Different inquires about have been done beforehand to show the variety of BDD estimate. Those inquires about gave just a model however did not determine the bend. This paper inferred a condition for the BDD measure variety and was demonstrated that the state of the bend is obviously contrasted with the trial comes about. Future work will focus on blunder amendment of the extent of the BDD measure variety display.

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