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Research paper

Various space-time block codes over different modulation techniques using a new sphere decoder

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Abstract

In this paper, a detailed analysis based on the combination of spatial multiplexing and space time coding techniques under Rayleigh fading channel constraint in MIMO wireless communication systems is presented. The decoding algorithms of V-BLAST and Sphere Decoder are analyzed and their performance is evaluated using different Orthogonal Space-Time Block Codes techniques, with quasi and rotated quasi-orthogonal space-time block codes. The BER vs SNR curves of all the proposed algorithms have been verified for all modulation schemes including 64 QAM, 16QAM, QPSK and BPSK. A modified K and K1 Sphere decoder are proposed for the significant reduction of BER at higher modulation schemes. BER for 64 QAM modulation is calculated at optimum SNR of 20 dB and it has been shown, it gets significantly reduces when our proposed decoding algorithms are applied.

Keywords: *Multiple Input Multiple Output (MIMO); Orthogonal Space Time Block Codes (OSTBC); Rotated QOSTBC; Sphere Decoder (SD); Maximal Likelihood (ML); Vertical-Bell Laboratories Layered Space-Time (V-BLAST).*

1. Introduction

The availability of radio spectrum is limited and needs more increment in the existing techniques to meet the required demand set by the International Telecommunication Union for next generation wireless system. These necessities the demand for the increase of spectral efficiency and data rate in the entire communication system by improving channel condition. The terminology "MIMO-multiple-input and multiple-output" refers to the multi-antenna array and employs multiple antennas both at the transmitter as well receiver to exploit multipath propagation. This increases the overall link quality of the system by providing the receiver the replicas of the transmitted signal. The link reliability gets enhanced and this helps us to overcome various issues like multipath fading, ISI, ICI etc. Diversity and various coding techniques offer low error rates and high signal to noise interference ratio. MIMO techniques are sub-divided into three main parts: spatial multiplexing, spatial diversity and beam forming. Spatial multiplexing techniques increase the data rate to parallel transmission whereas spatial diversity reduces the signal fading and hence reduces bit error rate. There are two types of space-time codes-space time block codes & space-time trellis codes. In this, our main focus is on orthogonal space-time block codes. In literature, various techniques have been discussed to improve the quality of MIMO wireless communication systems. Tarokh et al. [15] proposed two-dimensional coding in both spaces as well time to generate the correlation matrix between symbols generated from different transmitting antennas known as space time code. These codes are orthogonal to each other and can achieve full diversity with full transmission rate. Alamouti STBC can be modified to more than two transmitting antennas but will not provide full diversity and full rate [7][8].In order to achieve the full rate and full diversity code for more than two transmit antennas, Jafarkhani [12], introduced a generalized technique termed as Quasi-Orthogonal Space-Time Block Code structure for four transmitting antennas in which pairing of symbols was done, but all pairs of columns in transmission block are not orthogonal to each other .This provides full rate but with limited diversity. Choosing the symbols from the same constellation will actually maximize the minimum distance in the spatial and temporal constellation. The codes are termed as rotated Quasi-Orthogonal Space-Time Block Codes.

In this paper, the performance of various space time block codes with ML detection in terms of both SNR and BER is being reviewed. It's observed that the decoding complexity is increased for new codes at higher modulation techniques using ML decoder. This disadvantage of ML detector leads to the exponential growth of complexity proportional to the number of transmits antennas number of bits per symbol used in modulation techniques. The various decoding methods like V-BLAST and sphere decoder are proposed to reduce this exponentially growing complexity. The proposed detection techniques are not only simple but also enhance spectral efficiency of the entire multiple input multiple output system. Spectral efficiency as high as 40 bits/sec/Hz is being achieved by enabling interference suppression and successive interference cancellation in combination with V-BLAST. Sphere decoder further reduces the computational complexity of the system by determining the shortest vector and reduced basis of lattice to the given lattice. This detection technique becomes independent of the constellation size, so used efficiently used at higher modulation techniques (64 QAM) to obtain maximum SNR of 20 dB.

2. System model

In the proposed system model we design the detector, in which the modulated symbols are carried on the rotational quasi space time block coded antennas, whose symbol positions vary from block to block, the structure of the detection algorithm with respect to the transmit antenna order is irregular r, i.e ., the number of nodes generated by different root nodes is not fixed (upper triangular matrix will be different from lower triangular matrix).The proposed detection algorithm becomes different from the conventional algorithm with full antenna diversity where the number of node is generated by each root node depending on constellation size.

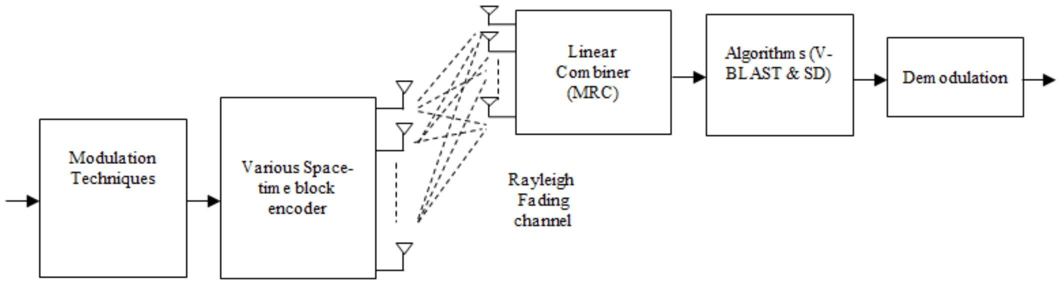

Fig. 1: Proposed System Model for Different STBCs.

$$R = \frac{N}{T} \text{ [symbol/channel used]} \quad (1)$$

At the receiver side, the symbol stream $\{\tilde{x}_i\}_{i=1}^{N}$ is estimated by using the received signals $\left\{y_j^{(t)}\right\}_{j=1}^{N_r}$, $t = 1,2,\ldots,T$. Let $H_{ji}^{(t)}$ denote the Rayleigh-distributed channel gain from the i_{th} transmit antenna to the j_{th} receive antenna over the t_{th} symbol period ($i = 1, 2, \ldots N_t$, $j=1,2,\ldots,N_r$, and $t=1,2,\ldots,T$).If we assume that the channel gains do not change during T symbol periods, the symbol time index can be omitted. Furthermore, as long as the transmit antennas and receive antennas are spaced sufficiently apart, N_r x N_t fading gains $H_{ji}^{(t)}$ can be assumed to be statistically independent. If $x_i^{(t)}$ is the transmitted signal from the i_{th} transmit antenna during t_{th} symbol period, the received signal at the j_{th} receive antenna during t_{th} symbol period is

$$y_j^{(t)} = \sqrt{\frac{E_x}{N_0 N_t}} \left[H_{j1}^{(t)} H_{j2}^{(t)} \ldots H_{jNt}^{(t)}\right] \begin{bmatrix} x_1^{(t)} \\ x_2^{(t)} \\ \vdots \\ x_{Nt}^{(t)} \end{bmatrix} + n_j^{(t)} \quad (2)$$

where, $n_j^{(t)}$ is the noise process at the j_{th} receive antenna during t_{th} symbol period, which is modeled as the AWGN noise of unit variance, and E_x is the average energy of each transmitted signal. Meanwhile, the total transmitted power is constrained as

$$\sum_{i=1}^{N_t} \left|x_i^{(t)}\right|^2 = N_t,\ t = 1,2,\ldots\ldots T \quad (3)$$

2.1. Singular value decomposition (SVD)

If, x represents the transmitted signal, n is the noise at the receiver, H represents the channel matrix, then the received vector can be expressed as,

$$y = Hx + n \quad (4)$$

The matrix $H \in C^{n_r \times n_t}$ has a singular value decomposition (SVD), represented as,

$$H = U \Sigma V^H \quad (5)$$

Where, $U \in C^{n_r \times n_r}$ and $V \in C^{n_t \times n_t}$ are unitary matrices, and $\Sigma \in C^{n_r \times n_t}$ is a rectangularmatrix, whose diagonal elements are non-negative real numbers and whose off-diagonalelements are zero. The diagonal elements of Σ are the singular values of the matrix H, denoting them by $\sigma_1, \sigma_2, \ldots\ldots, \sigma_{Nmin}$, where, $N_{min} \triangleq min(N_t, N_r)$. In fact, assume that $\sigma_1 \geq \sigma_2 \geq, \ldots\ldots, \geq \sigma_{Nmin}$, that is, the diagonal elements of Σ, are the ordered singular valuesof the matrix H. The rank of H corresponds to the number of non-zero singular values (i.e. rank (H) $\leq N_{min}$.In case of $N_{min} = N_t$, SVD in Equation (5) can also be expressed as

$$H = U \Sigma V^H$$

$$= \underbrace{\left[U_{N_{min}} U_{N_r - N_{min}}\right]}_{U} \underbrace{\begin{bmatrix} \Sigma_{N_{min}} \\ 0_{N_r - N_{min}} \end{bmatrix}}_{\Sigma} V^H$$

$$= U_{N_{min}} \Sigma_{N_{min}} V^H \quad (6)$$

Where, $U_{N_{min}} \in C^{N_r \times N_{min}}$ is composed of N_{min} left-singular vectors corresponding to the maximum possible nonzero singular values, and, $\Sigma_{N_{min}} \in C^{N_{min} \times N_{min}}$ is now a square matrix.Since, N_{min} singular vectors in $U_{N_{min}}$ are of length N_R, there always exist $N_R - N_{min}$ singularvectors such that $U_{N_{min}} U_{N_r - N_{min}}$ is unitary. In case of $N_{min} = N_r$, SVD in

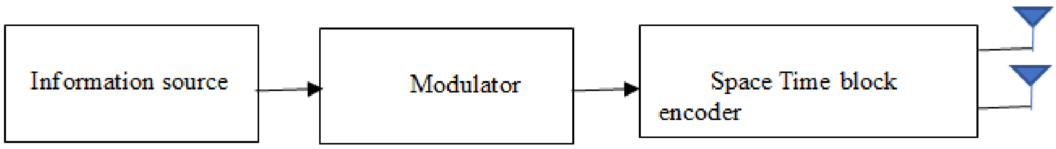

Fig. 2: Encoder for STBC.

Equation (5) can be expressed as

$$H = U \underbrace{\left[\Sigma_{N_{min}} 0_{N_r - N_{min}}\right]}_{\Sigma} \underbrace{\begin{bmatrix} V_{N_{min}}^H \\ V_{N_t - N_{min}}^H \end{bmatrix}}_{V^H} \quad (7)$$

$$= U \Sigma_{N_{min}} V_{N_{min}}^H \quad (8)$$

Where, $V_{N_{min}} \in C^{N_t \times N_{min}}$ is composed of N_{min} right-singular vectors. Given SVD of H, thefollowing eigen-decomposition holds:

$$HH^H = U \Sigma \Sigma^H U^H = Q \wedge Q^H \quad (9)$$

Where, $Q = U$ such that $Q^H Q = I_{N_r}$ and $\wedge \in C^{N_r \times N_r}$ is a diagonal matrix with its diagonal elements given as

$$\lambda_i = \begin{cases} \sigma_i^2, if\ i = 1,2,\dots,N_{min} \\ 0, if\ i = N_{min}+1,\dots,N_r \end{cases} \quad (10)$$

As the diagonal elements of Λ in Equation (9) are eigenvalues of$\{\lambda_i\}_{i=1}^{N_r}$, Equation (10) indicates that the squared singular values $\{\sigma_i^2\}$for H are the eigenvalues of the Hermitiansymmetric matrix HH^H, or similarly, of $H^H H$.

2.2. MIMO channel capacity

The capacity of a channel as the maximum data rate at which data transmitted from a transmitter, when passed across the channel, can be received at some receiver with no chance of error. If the transmitted and received data are taken as random variables, then the channel capacity offers to the maximum mutual information between them. The capacity C is

$$C = \max_{p(x)} I(X;Y) \quad (11)$$

Where the maximization is taken over all possible probability distributions p(x) of X. A data source with Gaussian probability distribution has the maximum entropy. Thus, to achieve a data rate close to the capacity, the data source should be Gaussian distributed. For a band-limited channel with noise being Gaussian and white, Shannon derived the normalized capacity (capacity per unit bandwidth) to be

$$C = log_2(1+\rho) \quad (12)$$

Where, ρ is the received SNR.
The basic idea in MIMO is that we can send different signals using the same bandwidth and still be able to decode correctly at the receiver. Thus, it is like we are creating a channel for each one of the transmitters. The capacity of each one of these channels is roughly equal to:

$$C_{MIMO} = B.log_2\left[1+\frac{N}{M}SNR\right]\ (bps/Hz) \quad (13)$$

Where, N is the number of receiving antennas and M is number of receiving antennas, B is bandwidth of channel. Roughly, with N≥ M, the capacity of MIMO channels is equal to,

$$C_{MIMO} = M.log_2[1+SNR]\ (bps/Hz) \quad (14)$$

Thus, we can get the linear increase in capacity of the MIMO channel with respect to the number of transmitting antennas. So, it is more beneficial to transmit data using many different low-powered channels than using one single, high-powered channel. The time-varying and randomly fading wireless channel, has the capacity of M x N MIMO system for known Channel is

$$C_{MIMO} = B.log_2\left|det\left[I_N + \frac{SNR}{M}.HH^*\right]\right| (bps/Hz) \quad (15)$$

We can see that the advantage of MIMO systems is significant in capacity. As an example, for a system $M = N\ and\ {HH^*}/{M} \to I_N$.Therefore, the capacity increases linearly with the number of transmit antennas.

$$C_{MIMO} = M.log_2[1+SNR]\ (bps/Hz) \quad (16)$$

MIMO offers best result when SNR and angular spread are large but for Small angular spread or presence of a dominant path (e.g. LOS) reduce MIMO performance.

3. Space time block codes

STBC's [8] have been proposed to realize the enhanced reliability of multi-antenna systems. This new paradigm uses the theory of orthogonal designs to design space time block codes. ST codes are of two types ST-Trellis and ST-Block codes. Tn ST -Trellis codes, the decoder complexity is quite high and it also increases exponentially with diversity and transmission rate. Alamouti proposed Orthogonal ST block codes (OSTBC) for MIMO systems with reduced decoder complexity. Alamouti coding, two information symbols $\boldsymbol{x_1}$ and $\boldsymbol{x_2}$ are sent through two different transmitting antennas 1 and 2 simultaneously at time instant t. Similarly, during next time instant (t+T) $-\boldsymbol{x_2^*}$ are sent through antennaland $\boldsymbol{x_1^*}$through antenna 2 respectively.
When transmitter has two antennas, Alamouti codes achieve the full diversity performance with a symbol rate of1 (rate-one) and simple linear processing under the assumption of no channel data information at the transmitter (CSlT) but perfect channel state information at the receiver (CSTR). Alamouti code provides the full diversity of 2 with two transmitting antenna with a rate of 1. For more reliable communication, Alamouti code can be further generalized for more than two transmitting antenna using the concept of orthogonal designs. But unfortunately, it doesn't provide any coding gain nor achieve a rate larger than 3/4 [33].

3.1. Quasi orthogonal space time block codes

Full- rate orthogonal designs with complex elements in its transmission matrix are impossible for more than two transmit antennas. The only example of a full-rate full-diversity complex STBC using orthogonal designs is Alamouti schemes [7].The generator matrix [7] of Alamouti code is given as

$$AorA_{12} = \begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix} \quad (17)$$

Here, the subscript 12 shows the indeterminate x_1 and x_2 in the transmission matrix. To design full rate codes with complex constellation, we consider codes with decoding pair of symbols [12] Such codes are called QOSTBC's as shown below

$$A_J = \begin{bmatrix} A_{12} & A_{34} \\ -A_{34}^* & A_{12}^* \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ x_3^* & -x_4^* & x_1^* & x_2^* \\ x_4 & -x_3 & -x_2 & x_1 \end{bmatrix} \quad (18)$$

we denote the ith column of above matrix by v_i, then for any intermediate variable we have x_1, x_2, x_3, x_4, we have

$$(v_1, v_2) = (v_1, v_3) = (v_2, v_4) = 0 \quad (19)$$

Where, in equation (19) symbols are inner product of each other independently. Therefore, the by v_1 and v_4 is orthogonal to v_2 and v_3 .So,it is named as "quasi-orthogonal" for the code.

3.2. Rotated quasi orthogonal space time block codes

It is impossible to achieve full rate and full diversity code if all symbols are chosen from same constellation. To provide full diversity, different constellations are used for different symbols. This can be done by rotating the symbols constellation [10] before transmission. Let we rotate symbols $x_3$and x_4. before transmission and their rotated versions can be represented as$\hat{x}_3$and$\hat{x}_4$.The generator matrix for such codes are given as

$$= \begin{bmatrix} x_1 & x_2 & \hat{x}_3 & \hat{x}_4 \\ -x_2^* & x_1^* & -\hat{x}_4^* & \hat{x}_3^* \\ \hat{x}_3^* & -\hat{x}_4^* & x_1^* & x_2^* \\ \hat{x}_4 & -\hat{x}_3 & -x_2 & x_1 \end{bmatrix} \quad (20)$$

Such codes will provide full diversity, full rate and simple pairwise decoding.

4. Decoding algorithm for V-BLAST system

One approach to a lower complexity design of the receiver is to use a “divide-and-conquer” strategy instead of decoding all symbols jointly. First, the algorithm decodes the strongest symbol. Then, cancelling the effects of this strongest symbol from all received signals, the algorithm detects the next strongest symbol. The algorithm continues by cancelling the effects of the detected symbol and the decoding of the next strongest symbol until all symbols are detected. The optimal detection order is from the strongest symbol to the weakest one. This is the original decoding algorithm [43] of V - BLAST present it only works if the number of receive antennas is more than the number of transmitantennas, that is M x N. Decoding Algorithm of V-BLAST includes three steps: ordering; interference cancellation and Interference nulling. Ordering: In decoding the first symbol, the interference from all other symbols is considered as noise. After finding the best candidate for the first symbol, the effects of this symbol in all of the receiver equations are canceled. Then, the second symbol is detected from the new sets of equations. The effects of the second detected symbol are canceled next to derive anew set of equations. The process continues until all symbols are detected. Of course, the order in which the symbols are detected will impact the final solution. Interference Cancellation: At stage n of the algorithm, when$c_n$is being detected, symbols$c_1, c_2, \dots, c_{n-1}$have beenalready detected. Let us assume a perfect decoder that is thedecoded symbols $\hat{c}_1, \hat{c}_2, \dots, \hat{c}_{n-1}$ are the same as thetransmitted symbols$c_1, c_2, \dots, c_{n-1}$.

One can subtract$\sum_{i=1}^{n-1} c_i H_i$from the received vector r toderive an equation that relates remaining undetected symbols to the received vector:

$$r_n = r - \sum_{i=1}^{n-1} c_i H_i + N,$$

$$r_n = \sum_{i=n}^{N} c_i H_i + N, n = 1,2, \dots \dots . N-1 \quad (21)$$

In fact, by using induction in addition to the convention $r_1 = r$,one can show that

$$r_{n+1} = r_n - c_n H_n, n = 1,2, \dots \dots . N-1 \quad (22)$$

Therefore, at the n_{th} stage of the algorithm after detecting then n_{th} symbol as $\hat{c}_n$, its effect is canceled from the equations by

$$r_{n+1} = r_n + \hat{c}_n H_n \quad (23)$$

Interference nulling: it detects c_n from r_n by first excluding the effects of undetected symbols. Detection is done by nulling the interference caused by symbols $c_{n+1}, c_{n+2}, \dots, c_N$. The ML receiver [45] performs optimum vector decoding by minimizing the error probability. The Maximal Likelihood receiver compares the received signals with all the possible transmitted signal vectors, modified by channel matrix H and estimates the transmitted symbol vector $\hat{C}$ [7], which is given as:

$$\hat{C} = \min_{\hat{C}} arg \, [\![y - C'H]\!]_F^2 \quad (24)$$

Where, $_F$ is the Frobenius norm.
Expanding the cost function using Frobenius norm given by

$$\hat{C} = \min_{\hat{C}} arg \left[Tr[(y - C'H)^H . (y - C'H)] \right] \quad (25)$$

$$\hat{C} = \min_{\hat{C}} arg \left[Tr[y^H . y + H^H . C'^H . C' . H - H^H . C'^H . y - y^H . C' . H] \right] \quad (26)$$

Considering $r^H . r$ is independent of the transmitted codeword so can be rewritten as

$$\hat{C} = \min_{\hat{C}} arg \, [Tr - 2. Real(Tr[H^H . C'^H . y])] \quad (27)$$

Where $.^H$ is a Hermition operator. Although ML detection offers optimal error performance, it suffers from complexity issues.

5. Algorithm for sphere decoder

A serial to parallel demultiplexer generates N separate sub streams from the input. Each sub stream is processed separately and is transmitted from a different antenna. The processing may include temporal coding in addition to the depicted modulation. Denoting the transmitted 1 × N vector at each time slot by C, the 1 × M output vector r is

$$r = C.H + \mathcal{N} \quad (28)$$

Where H is the N × M channel matrix and $\mathcal{N}$ is the 1 × M noise matrix. Let us assume for the sake of simplicity that there is no temporal coding. The maximum likelihood decoding finds the codeword C that minimizes the Frobenius norm $\|r - C.H\|_F$. Using a full search to find the optimal codeword is computationally very demanding. If the modulation utilizes a constellation with 2b points to transmit b bits, the number of possibilities for C is 2^{bN} . For four transmit antennas and 16-QAM, b = 4, there are 65 536 possibilities, which is impractical in most cases. The space-time coding structures that we have considered so far make it possible to implement a simpler ML decoding. For example, symbols can be decoded lattice .A lattice, generated by a basis, consists of all linear combinations of the basis elements with integer coefficients. In fact, the problem of finding the closest point of a lattice to a given point appears in many other areas as well [46]. For example, lattice codes are used in vector quantization and source coding. For a study of closest point search methods refer to [47]. The idea of limiting the search in a given region that includes the optimal lattice point has been proposed for the shortest vector problem in [48] [49]. The “limiting region” can be a rectangular parallelepiped [50] [51] or a hyper-sphere [52]. In what follows, we consider the latter option, which is called sphere decoding, and explain the details of the algorithm. In general, one has to follow different steps in a sphere-decoding algorithm. First, the complex equations are converted to real equations. Second, the lattice structure of the noiseless vectors are defined. Third, the lattice points inside the sphere are enumerated. Finally, the closest point among the vectors inside the sphere to the received vector is found. We explain the details of each step in what follows. First, let us separate the real and imaginary parts of the received vector in (28) to define the following 1 × 2M signal.

$$(\Re\{r\}\Im\{r\}) = (\Re\{C\}\Im\{C\}). \begin{pmatrix} \Re\{H\} & \Im\{H\} \\ -\Im\{r\} & \Re\{H\} \end{pmatrix} + (\Re\{\mathcal{N}\}\Im\{\mathcal{N}\}) \quad (29)$$

The above equation includes only real numbers and can be written in terms of real matrices as

$$r' = C'.H' + \mathcal{N}' \quad (30)$$

Where, $r' = (\Re\{r\}\Im\{r\}), C' = (\Re\{C\}\Im\{C\}), \mathcal{N}' = (\Re\{\mathcal{N}\}\Im\{\mathcal{N}\})$

$$H' = \begin{pmatrix} \Re\{H\} & \Im\{H\} \\ -\Im\{r\} & \Re\{H\} \end{pmatrix} \quad (31)$$

Note that it is very unlikely to have a singular H'. If the transmitted constellation is QAM, the elements of the 1 × 2N vector C' are from the set of integers. Then, the received vectors without noise are represented by a lattice [53]. Let us define the following set of points that creates a lattice

$$\wedge = \{(X|X) = C'.H'\} \quad (32)$$

And assume N = M for the sake of simplicity. Then, the lattice ∧ is defined by its generator matrix H'. Since C' belongs to a cubic lattice , the lattice ∧ is a linear transform defined by the 2N × 2N generator matrix H' in the 2N dimensional space. Since the vector r' is a row vector, the Frobenius norm $\|r' - C'H'\|_F$ is the same as the Euclidean norm $\|r' - C'H'\|$. Therefore, the ML decoding is to solve the following minimization problem:

$$\min_{X\in\Lambda}\|r - X\| = \min_{y\in r-\Lambda}\|y\| \quad (33)$$

Where, $r - \Lambda$ is the transformed lattice in the 2N-dimensional space. The Gram matrix of H' by G $H'H'^T$, then we have

$$\|y\|^2 = b\cdot H'H'^T b^T = b\cdot G\cdot b^T \le D \quad (34)$$

The Gram matrix G is a positive definite matrix and therefore using Cholesky factorization it can be written as [54].

$$G = Q\cdot Q^T \text{G} \quad (35)$$

Where Q is a 2N × 2N real lower triangular matrix with non-negative diagonal entries. Note that a matrix Q is lower triangular if Qi j = 0 whenever i < j and

$$b\,Q = \begin{pmatrix} \sum_{i=1}^{2N} b_i Q_{i1} \\ \vdots \\ b_{2N}Q_{2N,2N-1} + b_{2N-1}Q_{2N-1,2N-1} \\ b_{2N}Q_{2N,2N}\end{pmatrix}^T \quad (36)$$

Then, replacing (35) in (34) results in

$$\|y\|^2 = b\cdot Q\cdot Q^T\cdot b^T = \|b\cdot Q\|^2 \le D \quad (37)$$

Now, the problem is to find the limits of b2N , b2N−1, . . . , b1 from (37). Then, using the limits of bn and bn = an $- C'_n$, we find the limits of the possible integer values for C'_n.To reduce the complexity of the search, we usually use the lower triangular nature of Q to sequentially and recursively find the limits of b2N , b2N−1, . . . , b1.For example, considering only the last column of Q in (37), b2N is bounded by

$$\frac{-\sqrt{D}}{Q_{2N,2N}} \le b_{2N} \le \frac{\sqrt{D}}{Q_{2N,2N}} \quad (38)$$

Then, for C'_{2N}, we have

$$a_{2N} - \frac{-\sqrt{D}}{Q_{2N,2N}} \le b_{2N} \le a_{2N} + \frac{\sqrt{D}}{Q_{2N,2N}} \quad (39)$$

where [a] is the smallest integer greater than a and [a] is the greatest integer smaller than a. Finding the limited number of indices for C'_{2N} and the corresponding possible values of b2N ,we need to find the limits of the next symbol C'_{2N-1}. Then, the limits of C'_{2N} and C'_{2N-1} are utilized to find the limits of C'_{2N-2}. Sequentially, we can limit the possibilities for all symbols in the codeword. The main difference between different decoding algorithms is in the method that they limit the nth symbol using all available limits of the symbols from the previous steps. For example, to find the limits of C'_{2N-1} from those of C'_{2N} , one can upper-bound the sum of the squares of the 2N − 1st and 2Nth elements of (36) to satisfy the upper bound in (37). Using this method, we have

$$\left(b_{2N}Q_{2N,2N-1} + b_{2N-1}Q_{2N-1,2N-1}\right)^2 + \left(b_{2N}Q_{2N,2N}\right)^2 \le D \quad (40)$$

Then, using the limits of b2N from (38), we find the limits of b2N−1 from (40) and the corresponding limits for C'_{2N-1}. The sequence continues recursively as we find the bounds of C'_n for all values of n. For example, using the last 2N − n + 1 elements of (36) in (37) results in

$$\sum_{j=n}^{2N}\left(\sum_{i=j}^{2N} b_i Q_{ij}\right)^2 \le D \quad (41)$$

The bounds for bn, and consequently C'_n, are found using the above equation. Let us denote these bounds by Ln and Un, that is

$$L_n \le C'_n \le U_n,\, n = 2N, 2N-1, \ldots\ldots, 1 \quad (42)$$

Using (42), the sphere decoding algorithm calculates the limits of the possible indices inside the sphere for each received vector. Then, among all possible codewords inside the sphere, it finds the closest codeword to the received vector. Of course, the limits Ln and Un for the nth element of the codeword depend on the choice of the radius √D. For a very small value of D, it is possible that the sphere does not contain any lattice point. In this case, either the decoder reports an error or it increases the value of D and repeats the process. On the other hand, a large value of D results in a huge number of possibilities and a slow decoding process. In practice, the value of D may be selected according to the SNR or may be adjusted adaptively. The complexity of finding the above limits is polynomial with the dimension of the lattice [55]. After finding the limits, instead of a full search ML decoding that requires an exponential complexity, we may only search within the calculated limits.
Following algorithm gives the proposed algorithm for advance detection scheme in STBC systems.

```
Initialization
N(no. of symbols)
TX,RX,Eb_no_dB
Processing
For ii=1:lengh(,Eb_no_dB)
Ip=rand(1,N)>0.5
S=2*ip-1
Smod=mod(S)
Smod=reshape(Smod)
Addition of Noise
Y=AWGN(sum(h.*SMod,2)+10^(-,Eb_no_dB(ii)/20)
Receiver
hCof=zeros(2,2,N/Tx)
Define: hCof(1,1,:);hCof(2,2,:);hCof(2,1,:);hCof(1,2,:)
hDen=((hCof((1,1,:)+hCof(2,2,:)-(hCof(1,2,:).*hCof(2,1:)))
hDen=reshape(hDen);
hInv=hCof/.(hDen)
hMod=mod(hInv)
YMod=Kron(y,ones(1,2));
YHat=sum(reshape(hInv 2,N).Ymod,1);
Define k1=SQRT(10^2-2^2);
Output
SimBer=nErr/k*S*N)
```

6. Simulation results and discussion

Figure 3 shows the BER vs SNR for Alamouti v-blast scheme. The figure reveals that when V blast is applied the simulated values show a very significant decrement in BER as compared to theoretical values. Simulated 64-QAM has highest BER while Simulated BPSK has lowest BER.However using V-blast significantly increases SNR at the receiver and there is a decrement in the BER at higher SNRs.At optimum SNR of 29 dB 64-QAM shows significant decrement in BER. The BER value is almost 0.01. Figure 4 shows BER vs SNR value for the proposed Sphere decoder. Again, when sphere decoder is applied the simulated value show a significant decrement in BER as compared to theoretical values. The BER curves show a gradual fall when compared to theoretical value.64-QAM modulation technique is having highest BER while BPSK having lowest BER. The SD helps in significantly reducing the BER at higher SNR values. At the optimum SNR of 20db the BER for 64-QAM modulation is very close to 0. 001.Thus a significant reduction in BER is obtained at higher modulation technique is achieved using Sphere Decoder. Figure 5 shows BER vs SNR for K shaped Sphere Decoder. The proposed K shaped sphere decoder modifies the conventional sphere decoder to reduce the BER for higher modulation techniques at higher SNRs as depicted from figure. At 20 dB SNR the BER value for 64 QAM is less than 0. 0001. Thus a significant decrement in BER is achieved for 64-QAM technique using k Sphere decoder. Figure 5 shows BER vs SNR for K shaped Sphere Decoder. The proposed K shaped sphere decoder modifies the conventional sphere decoder to reduce the BER for higher modulation techniques at higher SNRs as depicted from fig ure. At 20 dB SNR the BER value for 64 QAM is less than 0. 0001. Thus a significant decrement in BER is achieved for 64-QAM technique using k Sphere decoder.

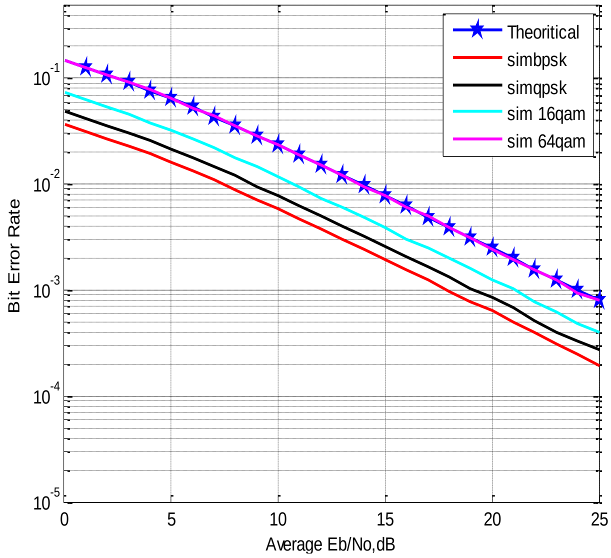

Fig. 3: BER vs. SNR for Alamounti V-Blast Scheme.

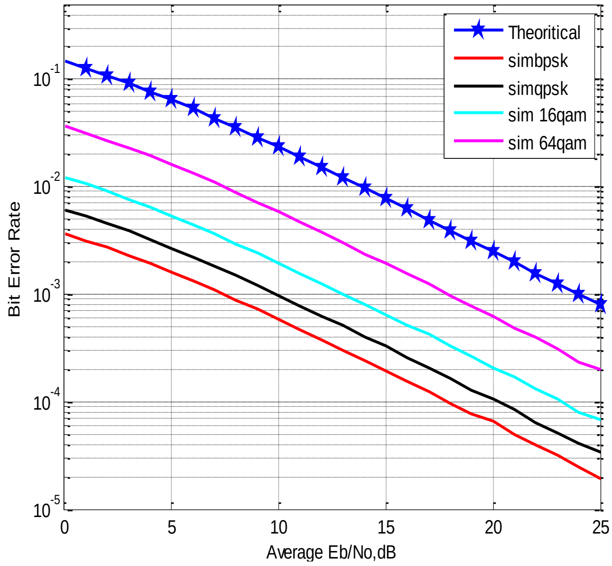

Fig. 4: BER vs. SNR for Proposed Sphere Decoder with All Modulation Techniques.

Figure 6 shows BER vs SNR curve for all modulation techniques using modified K1 Sphere decoder. The figure reveals that BER curve coincides for all modulation technique. Thus, all the modulation techniques show a significant decrease in BER. The BER cure becomes independent of all modulation techniques and shows a significant decrement at higher SNR values. At SNR of 18 dB the BER for all modulation techniques becomes close to 0.00001. Figure 7 shows the Spectral efficiency vs CDF for all decoding techniques. At 5-th percentile CDF level when no diversity is applied spectral efficiency is minimum. Thus, cell edge users exhibit no connectivity when no space coding is applied. The spectral efficiency of the cell edge users is improved when almounti 2*2 is applied.

The V blast scheme and Sphere decoder along with R-QOSTBC further increases the spectral efficiency for cell edge users. The R-QOSTBC with proposed sphere decoder has highest spectral efficiency.

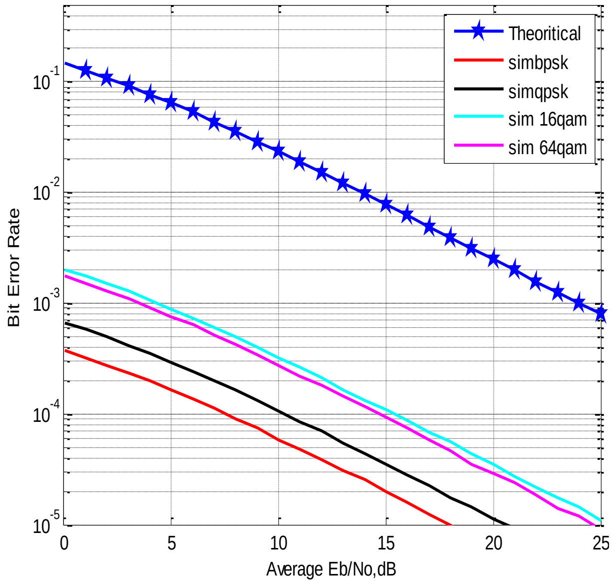

Fig. 5: BER vs. SNR for Proposed K Sphere Decoder with All Modulation Techniques.

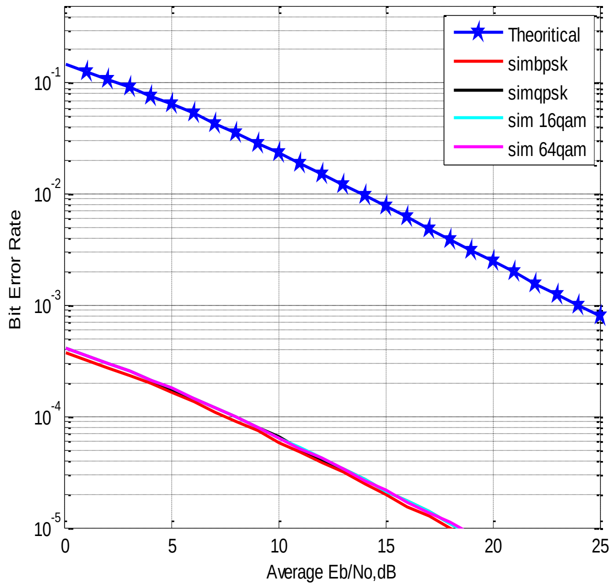

Fig. 6: BER vs. SNR for Proposed K1 Sphere Decoder with All Modulation Techniques.

7. Conclusion

In this paper we have evaluated the performance of various decoding techniques for various modulation schemes. A new k1 and k shaped Sphere decoder have been proposed to significantly reduce the BER. The proposed K1 SD reduces the BER to about 0.00001, at a optimal SNR of 18 dB, which is a significant improvement especially for 5G environment.

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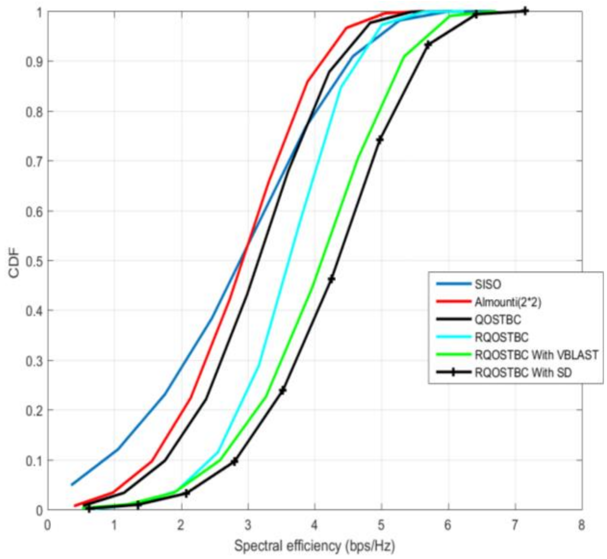

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