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Research paper

Study on Feebly Local Function

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Abstract

A new type of local function in ideal topological spaces was submitted with some theorems and relations between the new type of local function and other types

Keywords: *Ideals, ideal topological spaces, local function, feebly local function, feebly compatible space.*

1. Introduction

The Ideal topology is a topological space$(X,\mathfrak{X})$ and ideal $\mathfrak{B}$ was introduced by Kuratawiski in [5] and denoted by$(X,\mathfrak{X},\mathfrak{B})$after that some authors like Njastad O. [8] and D.Jankovic and T. P. Hamelett [2] give us some studies about this new type of topological spaces , also E.Hayashi [3] and Ahmaad Al-omari and Takashi Noiri [1] and M. Khan , T. Noiri [7] study some types of local functions via ideal

In a topological space $(X,\mathfrak{X})$ semi-open set was introduced by N.Levine [6], thefamily of all semi-open sets in X is represented by the format SO(X). In this paper we will introduce the local function with respect to feebly open set and give a briefly study about this type

2. Preliminaries

Definition 2-1: [2]
Let $(X,\mathfrak{X})$ be a topological space and $\mathfrak{B}$ be a non-empty assemblage of subsets ofXsuch that :

1. $Q\in\mathfrak{B}$ and $Z\subseteq A$ implies $Z\in\mathfrak{B}$,
2. $Q\in\mathfrak{B}$ and $Z\in\mathfrak{B}$ implies$Q\cup Z\in\mathfrak{B}$.

Then it is called ideal on the space X

Definition 2-2:-[2]
For a subset $Q\subseteq X$ in the space $(X,\mathfrak{X},\mathfrak{B})$a set operator $(\ )^* : P(X) \to P(X)$, such that
$Q^*(\mathfrak{X},\mathfrak{B}) = \{x \in X|\ Q\cap W \notin \mathfrak{B}$ for every $W\in X_O\}$, is called the local function of Q with respect to $\mathfrak{B}$ and $\mathfrak{X}$ where $X_O=\{W \in X_O : x\in W\}$and X_O is the set of all open set in the topological space

Definition 2-3 :- [7]
Let $(X,\mathfrak{X},\mathfrak{B})$ be an ideal topological space and U a subset of X . Then
$U_*(\mathfrak{X},\mathfrak{B}) = \{x\in X|\ U\cap W\notin\mathfrak{B}$ (for every $W\in X_{SO})\}$is called the semi– local function of U with respect to $\mathfrak{X}$ and $\mathfrak{B}$, where$X_{SO} = \{W \in SO(X)|x\in W\}$.

Definition 2-4:- [7]
In the space $(X,\mathfrak{X},\mathfrak{B})$ the topology $\mathfrak{X}$ is compatible with the ideal $\mathfrak{B}$, denoted $\mathfrak{X}\sim\mathfrak{B}$, if the conditions exist for every $U\subseteq X$: if for every $x\in U$ there exists a $W\in\mathfrak{X}(x)$ such that $W\cap U\in\mathfrak{B}$, then $U\in\mathfrak{B}$, where $\mathfrak{X}(x)$ is the open neighborhood system at x .

Definition 2-5:- [7]
In the space$(X,\mathfrak{X},\mathfrak{B})$there exist a semi – compatible between the topology$\mathfrak{X}$and the ideal$\mathfrak{B}$, and we can express it by $\mathfrak{X}\sim\sim\mathfrak{B}$, if the condition exist for every $U\subseteq X$: for every $x\in U$ there exists a $W\in SO(X,x)$ such that $W\cap U\in\mathfrak{B}$,then $U\in\mathfrak{B}$

Theorem2-6 :[2] let $((X,\mathfrak{X},\mathfrak{B}))$ be a space where $\mathfrak{B}$ is an ideal onX and let A and B be subsets of X , Then :-
1- $A\subseteq B$ then $A^*\subseteq B^*$
2- $A^* = cl(A^*)\subseteq cl(A)$ (A^*isa closed subset of cl(A))
3- $(A^*)^*\subseteq A^*$
4- $(A\cup B)^* = A^*\cup B^*$
5-$(A\cap B)^*\subseteq A^*\cap B^*$

Theorem2-7 :[4] in the space $((X,\mathfrak{X},\mathfrak{B})$for any subsets U and V of X we have
1- If $U\subseteq V$ then $cl^*(U)\subseteq cl^*(V)$
2- $cl^*(U\cup V) = cl^*(U)\cup cl^*(V)$
3- $cl^*(U\cap V)\subseteq cl^*(U)\cap cl^*(V)$
4 - $cl^*(cl^*(U))\subseteq cl^*(V)$

Proposition 2-8:[9]- In any topological space every feebly open set is semi open set

3. Local Function via Feebly Open Set

Definition 3-1:
In the space$(X,\mathfrak{X},\mathfrak{B})$ if Q is a subset of X .then a set operator $(\ )^{*F} : P(X) \to P(X)$, called a feebly local function of Q with respect to $\mathfrak{B}$ and $\mathfrak{X}$, which is defined as follows: for Q is a subset of X$Q^{*F}(\mathfrak{X},\mathfrak{B}) = \{x\in X|Q\cap W\notin\mathfrak{B}$ forevery $W\in X_F\}$ where$X_F = \{W\in FO(X)|x\in W\}$

Remark:
$\mathfrak{X}^{*F}$ is a topology on X generated by the set {Q-Z : Q is feebly open set and Z is ideal set } or in this orm$\mathfrak{X}^{*F}$ ={Q in X : Cl^{*F}(X -Q)= X –Q}

Proposition3-2:
In the space $(X,\mathfrak{X},\mathfrak{B})$ every feebly local function is local function.
Proof:-
Since every open set is feebly open Then if $x\in Q^{*F}$then $W\cap Q\notin\mathfrak{B}$,For every $W\in X_F$,Then $W\cap Q\notin\mathfrak{B}$, for every $W\in X_O$, $x\in A^*$

And then $Q^{*^F} \subseteq Q$ and hence every feebly local function is local function.

Proposition3-3 :-[7]

Every semi local function is local function.

Proposition3-4 :-

In the space $(X,\mathfrak{X},\mathfrak{B})$ every semi local function is feebly local function

Proof :-

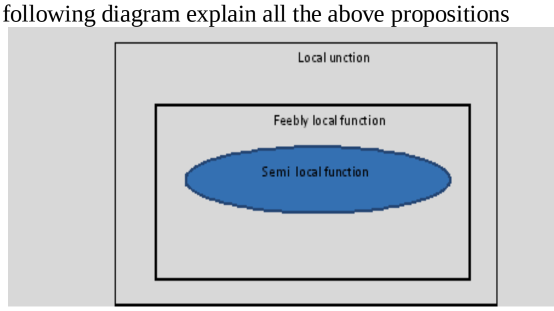
LetQ be a subset of X then if $x \in Q^{*^S}$ then $W \cap Q \notin \mathfrak{B}$ For every X_{SO} –W,Then $W \cap Q \notin \mathfrak{B}$, for every $W \in X_F$, $x \in Q^{*^F}$from that we get $Q^{*^S} \subseteq Q^{*^F}$And hence every semi local function is feebly local function .

Remark3-5

From the above proposition we have $Q^{*^S} \subseteq Q^{*^F} \subseteq Q^*$

Also we get that $t \subseteq t^* \subseteq t^{*S} \subseteq t^{*F}$

The following diagram explain all the above propositions



Theorem 3-6:-

In the space $(X,\mathfrak{X},\mathfrak{B})$ if Q,Z are two subsets of X then:

1- $\emptyset^{*^F}(\mathfrak{B}) = \emptyset$

2- If $Q \subseteq Z$ then $Q^{*^F}(\mathfrak{B}) \subseteq Z^{*^F}(\mathfrak{B})$ and then If $Q \subseteq Z$ then $Q^{*^F}(\mathfrak{B}) \subseteq Z^*(\mathfrak{B})$

3- $(Q \cup Z)^{*^F}(\mathfrak{B}) = Q^{*^F}(\mathfrak{B}) \cup Z^{*^F}(\mathfrak{B})$and then$(Q \cup Z)^{*^F}(\mathfrak{B}) = (Q \cup Z)^*(\mathfrak{B})$

4- $(Q \cap Z)^{*^F}(\mathfrak{B}) \subseteq Q^{*^F}(\mathfrak{B}) \cap Z^{*^F}(\mathfrak{B})$and then$(Q \cap Z)^{*^F}(\mathfrak{B}) \subseteq Q^*(\mathfrak{B}) \cap Z^*(\mathfrak{B})$

5-$Q^{*^F}(\mathfrak{B}) = F - cl\left(Q^{*^F}(\mathfrak{B})\right) \subseteq F - cl(Q)$

6- $Q^{*^F}(\mathfrak{B}) - Z^{*^F} \subseteq (Q-Z)^{*^F}(\mathfrak{B}) \subseteq (Q-Z)^*(\mathfrak{B})$

7- $\left(Q^{*^F}\right)^{*^F}(\mathfrak{B}) \subseteq Q^{*^F}(\mathfrak{B}) \subseteq Q^*(\mathfrak{B})$

8- If $M \in \mathfrak{B}$ then $(Q \cup M)^{*^F}(\mathfrak{B}) = Q^{*^F}(\mathfrak{B}) = (Q-M)^{*^F}(\mathfrak{B}) \subseteq (Q-M)^*(\mathfrak{B})$

Proof :-

1- $\emptyset^{*F} = \{x \in X: W_x \cap \emptyset \notin \mathfrak{B}, \forall W_x \in X_F\} = \emptyset$

2- let $Q \subseteq Z$

$Q^{*F}(\mathfrak{B}) = \{x \in X: W_x \cap Q \notin \mathfrak{B}, \forall W_x \in X_F\}$

now let $x \in Q^{*F}(\mathfrak{B})$then $W_x \cap Q \notin \mathcal{B}, \forall W_x \in X_F$and then $W_x \cap Z \notin \mathfrak{B}$ There forx $\in Z^{*F}(\mathfrak{B})$ And by remark (3-5) we get the result

3- F – cl $\left(Q^{*F}(\mathfrak{B})\right)$ is the smallest feebly–closed set containing U^{*F}

Then $Q^{*F} \subseteq F - cl\,(Q^{*F})$, where F-cl mean the closure set with respect to feebly open set

Now let $x \in F - cl(Q^{*F})$ then$W_x \cap Q^{*F} \neq \emptyset$, $\forall W_x \in X_F$

Let $y \in W_x \cap Q^{*F}$Then $W_x \in X_F(y)$ and $y \in Q^{*F}$then $x \in Q^{*F}$

from that we get $F - cl\left(Q^{*F}(\mathfrak{B})\right) \subseteq Q^{*F}(\mathfrak{B})$ then F–cl $\left(Q^{*F}(\mathfrak{B})\right) = Q^{*F}(\mathfrak{B})$

Now if $x \in Q^{*F}$then $W_x \cap Q \neq \emptyset, \forall W_x \cap Q \neq \emptyset, \forall W_x \in X_F$

Then $x \in Fcl(U)$and by remark (3-5) we get the result[11]

$4 - (Q \cup Z)^{*F}(\mathfrak{B}) = \{x \in X: W_x \cap (Q \cup Z) \notin \mathfrak{B}, \forall W_x \in X_F\}$

$= \{x \in X: (W_x \cap Q) \cup (W_x \cap Z) \notin \mathfrak{B}, \forall W_x \in X_F\}$

$= \{x \in X: (W_x \cap Q) \notin \mathfrak{B} \text{ or } (W_x \cap Z) \notin \mathfrak{B}, \forall W_x \in X_F\}$

$= \{x \in X: (W_x \cap Q) \notin \mathfrak{B}, \forall W_x \in X_F\}$

or $\{x \in X, (W_x \cap Z) \notin \mathfrak{B}, \forall W_x \in X_F\} = Q^{*F} \cup Z^{*F}$

And by remark (3-5) we get the result

5-by (2) $(Q \cap Z)^{*F} \subseteq Q^{*F}, (Q \cap Z)^{*F} \subseteq Z^{*F}$then $(Q \cap Z)^{*F} \subseteq Q^{*F} \cap Z^{*F}$

6-$Q^{*F} - Z^{*F}(\mathfrak{B}) = \{x \in X: W_x \cap Q \notin \mathfrak{B}, \forall W_x \in X_F\}$

$-\{x \in X: W_x \cap Z \notin \mathfrak{B}, \forall W_x \in X_F\}$

$= \{x \in X: W_x \cap (Q-Z) \notin \mathfrak{B}, \forall W_x \in X_F\} = (Q-Z)^{*F}$

And by remark (3-5) we get the result

7- by (3) $Q^{*F} \subseteq F - cl(Q)$Then $(Q^{*F})^{*F} \subseteq F - cl(Q^{*F}) = Q^{*F}$And then $(Q^{*F})^{*F} \subseteq Q^{*F}$

8-Let $M \in \mathfrak{B}$ then $M^{*F} = \emptyset$ and since $(Q \cup M)^{*F} = Q^{*F}$

$(Q \cup M)^{*F} = \{x \in X: W_x \cap (Q-M) \notin \mathfrak{B}, \forall W_x \in X_F\}$

$= \{x \in X: W_x \cap Q \notin \mathfrak{B}, \forall W_x \in X_F\} = Q^{*F}$

And by remark (3-5) we get the result

Remark 3-7 :

Since $cl^{*^F}(Q) = Q \cup Q^{*^F}$and by remark (3-5)we get that $cl^{*^F}(Q) \subseteq cl^*(Q)$

Corollary3-8:-

In the ideal topological space $(X,\mathfrak{X},\mathfrak{B})$for any subsets Q andZ of X we have

1- If $Q \subseteq Z$ then $cl^{*^F}(Q) \subseteq cl^{*^F}(Z)$ and then If $Q \subseteq Z$ then $cl^*(Q) \subseteq cl^*(Z)$

2- $cl^{*^F}(Q \cup Z) = cl^{*^F}(Q) \cup cl^{*^F}(Z)$and then$cl^{*^F}(Q \cup Z) = cl^*(Q \cup Z)$

3- $cl^{*^F}(Q \cap Z) \subseteq cl^{*^F}(Q) \cap cl^{*^F}(Z)$and then$cl^{*^F}(Q \cap Z)) \subseteq cl^*(Q) \cap cl^*(Z)$

4- $cl^{*^F}(cl^{*^F}(Q)) \subseteq cl^{*^F}(Q) \subseteq cl^*(Q)$

Proof:-

The proof exist by remark (3-5) and theorem (3-6)

Theorem 3-9:-

In the ideal topological space $(X,\mathfrak{X},\mathfrak{B})$ with the subset Q of X the following holds:

1- $(X-Z)^{*^F}(\mathfrak{B}) = X^{*^F}(\mathfrak{B})$if $Z \in \mathfrak{B}$

2- $(X-(Q-Z)^{*^F} = ((X-Q) \cup Z)^{*^F}, Z \in \mathfrak{B}$

Proof:-

(1) Let $x \in (X-M)^{*F}$ then $W_x \cap (X-M) \notin \mathfrak{B}$, $\forall W_x \in X_F$

Now $(W_x \cap X) - (W_x \cap M) \notin \mathfrak{B}$, $\forall W_x \in X_F$then$W_x \cap X \notin \mathfrak{B}$, $\forall W_x \in X_F$

From that we get$x \in X^{*F}$and this implies that$(X-M)^{*F} \subseteq X^{*F}$ … (1)

now

Let $x \in X^{*F}$ then $W_x \cap X \notin \mathfrak{B}, \forall W_x \in X_F$ and then $(W_x \cap X) - (W_x - M) \notin \mathfrak{B}, \forall W_x \in X_F$Therefore$W_x \cap (X-M) \notin \mathfrak{B}, \forall W_x \in X_F$From that we get $x \in (X-M)^{*F}$and this implies that $X^{*F} \subseteq (X-M)^{*F}$ … (2)

Form (1) & (2) we get the result.

(2) if $Z \in \mathfrak{B}$ then $(Q \cup Z)^{*F} = Q^{*F} = (Q-Z)^{*F}$

Let $Q = X - Q$ then$((X-Q) \cup Z)^{*F} = (X-Q)^{*F} = ((X-Q)-Z)^{*F}$

Definition 3-10 :-

In the space $(X,\mathfrak{X},\mathfrak{B})$ we say the topology $\mathfrak{X}$ and the ideal $\mathfrak{B}$ is feebly compatibleand we can express it by $\sim_F \mathfrak{B}$, if the condition exist , for every $U \subseteq X$:

And everyx $\in$ Uthere exists $W \in X_F$ such that $W \cap U \in \mathfrak{B}$, then $U \in \mathfrak{B}$,where X_F denotes the set of all feelby open sets of x.

Remark3-11 :- [7]

Compatible space satisfy the semi compatible of the space

Proposition3-12 :-[10]

Compatible space satisfy the feebly compatible of this space.

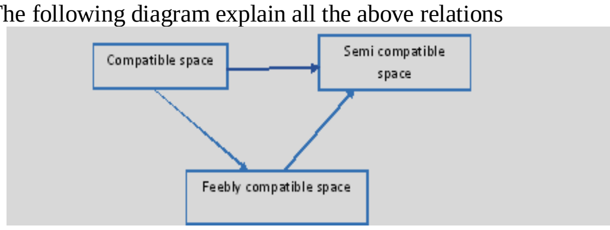
Proof: obvious

Proposition3-13 :-

Feebly compatible satisfy the semi compatible of this space.

Proof: obvious

The following diagram explain all the above relations



Theorem 3-14:-

In the ideal topological space $(X,\mathfrak{X},\mathfrak{B})$, $\mathfrak{X} \sim_F \mathfrak{B}$ if and only if for every $Q \subseteq X$,
$Q \cap Q^{*^F} = \varphi$ then$Q \in \mathfrak{B}$.

Proof:

Let$Q \subseteq X$, and let$x \in Q$,Since$Q \cap Q^{*^F} = \varphi$, then $x \notin Q^{*^F}$ for each x in Q
And for that there exist $W \in X_F$ such that$W \cap Q \in \mathfrak{B}$,Then$Q \in \mathfrak{B}$, from that we get $\mathfrak{X} \sim_F \mathfrak{B}$
Conversely, obvious

Definition 3-15 :-

A space $(X,\mathfrak{X})$ is known a semi –compact [4] if every cover of semi – open sets has a finite sub cover .

Lemma 3-16:[7]-

If $(X,\mathfrak{X})$ is a semi – compact space, then every subspace of X is semi – compact relative to X

Theorem 3-17:-

Let $(X,\mathfrak{X})$ be a semi – compact space and $\mathfrak{B}$ an ideal on X , then $\mathfrak{X} \sim_F \mathfrak{B}$.

Proof:-

If Q is a subset of X, and x be any point of X such that $x \in Q$, then there exists $W_X \in X_F$
Such that $Q \cap W_X \in \mathfrak{B}$,then, the collection of feebly–open sets $\{W_X | x \in Q\}$ is a cover ofQ and then
$\{W_X | x \in Q\}$is a cover of Q by semi – open sets of X .
Then ,by lemma [3-16] , Q is semi – compact relative to X, from that we get a finite number of points ,say ,$x_1, x_2, \dots, x_n$ in Q such that$Q \subset \cup_{i=1}^{n} W_{Xi}$,and then
$Q = Q \cap (\cup_{i=1}^{n} W_{xi}) = \cup_{i=1}^{n} (Q \cap W_{xi})$
But $(Q \cap W_{xi}) \in \mathfrak{B}$ for each i , then $Q \in \mathfrak{B}$, and therefore, $\mathfrak{X}$ and $\mathfrak{B}$ are Feebly – compatible

Definition 3-18: a subset Q of the space $(X,\mathfrak{X},\mathfrak{B})$ is ideal dense set if and only if x belong to Q^* for every x in X

Definition 3-20: a subset Q In the space $(X,\mathfrak{X},\mathfrak{B})$ is said to be feebly ideal dense set if and only if x belongs to Q^{*F} for every x in X

Remark 3-21: clearly that every feebly ideal dense set is ideal dense set

Definition3-22:

A non-empty topological space $(X,\mathfrak{X})$ with the ideal $\mathfrak{B}$ is called ideal resolvable space if X has two disjoint ideal dense sets

Definition 3-23: In the space $(X,\mathfrak{X},\mathfrak{B})$ we say that the space is feebly ideal resolvable space ifX has two disjoint feebly ideal dense sets

Remark3-24: clearly that every feebly ideal resolvable space is ideal resolvable space

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