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Research paper

Relationship the Fuzzy P-Ideal with Some Fuzzy Subset of BH-Algebra

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Abstract

This work involved some studying ,we study and prove some theorems and a propositions which determine the relationships among the notion of fuzzy p-ideal with the intersection, union, image of function, inverse function and with some other fuzzy subsets of BH-algebra, also we gave some properties of this ideal of a BH-algebra.

***Keywords**: BH-algebra, fuzzy p-ideal, levels subset.*

1. Introduction

In 2001, Q. Zhang, E. H. Roh and Y. B. Jun studied the fuzzy theory in BH-algebras [8]. In 2011, H.H. Abbass and H.M.A.sSaeed generalized the notion of a fuzzy closed ideal, fuzzy p-ideal and fuzzy implicative ideal to a BH-algebra and BC A-part [4]. In 2014, H. H. Abbass and S. A. Neamah introduced the notion of a fuzzy implicative ideal with respect to an element of a BH-algebra [5].

2. Preliminaries

In this section, is devoted to some basic ordinary concepts of BH-algebra, fuzzy ideal, fuzzy p-ideal and homomorphism in BH-algebra, we gave some basic concepts about the image of function, the inverse image of a BH-algebra with some remarks in fuzzy sences.

Definition (2.1): [9]A **BH-algebra** is a non empty set X with a constant 0 and a binary operation * satisfying the following conditions:

i. x * xx= 0, ∀ x∈X.
ii. x * y = 0 and y * x = 0 imply x = y, ∀ x, y∈ X.
iii. x *0 = x, ∀ x∈xX.

Remark (2.2): [10]Let X and Y be BH-algebras. A mapping f: X→Y is called a **homomorphism** if f(x*y)a= f(x)*f(y), ∀ x, y∈X. A homomorphism f is called a **monomorphism** (resp., **epimorphism**) if it is injective (resp**.,** surjective). A bijective homomorphism is called an isomorphism. Two BH-algebras X and Y are said to be **isomorphic**, written X ≅ Y, if there exists an isomorphism f: X→Y. For any homomorphism f: X →Y, the set {x∈X:f(x)=0'} is called the **kernel** of f, denoted by ker(f), and the set {f(x):x∈X} is called the **image** of f, denoted by Im(f). Notice that f(0)=0', ∀ homomorphism f.

Definition (2.3):[1] if $\{A_\alpha, \alpha\in\Lambda\}$ is a family of fuzzy sets in X, then :

$\bigcap_{i\in I} A_i(x)$ =inf { A_i (x), i∈ I}, ∀ x ∈X.

$\bigcup_{i\in I} A_i(x)$ = sup { A_i (x), i∈ I}, ∀ x ∈X. which are also **fuzzy sets** in X.

Definition (2.4): [2]Let X and Y be any two sets, A be any fuzzy set in X and f: X → Y be any function. The set $f^{-1}(y)$ = {x∈X | f(x) = y}, ∀y∈Y. The fuzzy set B in Y defined by $B(y) = \begin{cases} \sup\{A(x) \mid x\in f^{-1}(y)\}; & \text{if } f^{-1}(y)\neq\emptyset \\ 0; & \text{otherwise} \end{cases}$, ∀y∈Y, is called the **image** of A under f and is denoted by f(A).

Definition (2.5): [2]Let X and Y be any two sets, f: X → Y be any function and B be any fuzzy set in f(A). The fuzzy set A in X defined by: A(x)=B(f(x)), ∀x∈X is called the **preimage** of B under f and is denoted by $f^{-1}(B)$.

Definition(2.6):[6]A fuzzy subset A of a BH-algebra X is said to be a fuzzy ideal if and only if:

i. A(0) ≥ A(x),a∀ x∈X.
ii. A(x) ≥ min{A(x*y), A(y)}, ∀xx, y∈X.

Definition(2.7): [3] A nonempty subset I of a BH-algebra X is called a P-ideal of X if:

i. 0 ∈ I,
ii. (x*z)*(y*z) ∈ I and y ∈ I imply x∈ I, ∀ x, y, z ∈ X.

Definition (2.8): [3] A fuzzy set A of a BH-algebra X is called a **fuzzy p-ideal** of X if it satisfies:

i. A(0) ≥ A(x), ∀ x∈X.
ii. A(x) ≥ min {A((x *z) *(y *z)),aA(y)}, ∀ x, y, z ∈ X.

Proposition(2.9):[5]Let X be a BH-algebra. Then every fuzzy p-ideal of X is fuzzy ideal of X.

Definition (2.10):[7] Let μ be a fuzzy set in X, ∀α∈ [0, 1], thesetμ_α= {x∈X, μ(x) ≥α } is called a**level subset of A.** Note that,μ_α is asubset of X in the ordinary sense.

3. The Relationship the Fuzzy P-Ideal with Other Notions and Other Subsets in Fuzzy Sences

Proposition (3.1): Let $\{A_\alpha \mid \alpha\in\lambda\}$ be a family of fuzzy p-ideals of a BH-algebra X. Then $\bigcap_{\alpha\in\lambda} A_\alpha$ is a fuzzy p-ideal of X.

Proof: Let $\{A_\alpha \mid \alpha\in\lambda\}$ be a family of fuzzy p-ideals of X.

i. Let $x\in X$. Then $\bigcap_{\alpha\in\lambda} A(0) = \inf\{A_\alpha(0) \mid \alpha\in\lambda\}) \geq \inf\{A_\alpha(x) \mid \alpha\in\lambda\} = \bigcap_{\alpha\in\lambda} A_\alpha(x)$

[Since A_α is a fuzzy p-ideals of X, $\forall\alpha\in\lambda$. By definition (1.8)(i)]

$\Rightarrow \bigcap_{\alpha\in\lambda} A(0) \geq \bigcap_{\alpha\in\lambda} A_\alpha(x)$

ii. Let x, y, z $\in$X. Then, we have $\bigcap_{\alpha\in\lambda} A_\alpha(x) = \inf\{A_\alpha(x) \mid \alpha\in\lambda\}$

$\geq \inf\{\min\{A_\alpha((x*z)*(y*x)), A_\alpha(y) \mid \alpha\in\lambda\}\}$

[Since A_α is a fuzzy p-ideal of X, $\forall\alpha\in\lambda$. By definition (1.8)(ii)]]

$= \min\{\inf\{A_\alpha((x*z)*(y*z)), A_\alpha(y) \mid \alpha\in\lambda\}\}$

$= \min\{\inf\{A((x*z)*(y*z)) \mid \alpha\in\lambda\}, \inf\{A_\alpha(y) \mid \alpha\in\lambda\}\}$

$= \min\{\bigcap_{\alpha\in\lambda} A((x*z)*(y*z)) \mid \in\lambda\}, \bigcap_{\alpha\in\lambda} A_\alpha(y) \mid \alpha\in\lambda\}\}$

$\Rightarrow \bigcap_{\alpha\in\lambda} A_\alpha(x) \geq \min\{\bigcap_{\alpha\in\lambda} A((x*z)*(y*z)) \mid \alpha\in\lambda\}, \bigcap_{\alpha\in\lambda} A_\alpha(y) \mid \in\lambda\}\}$

Therefore, $\bigcap_{\alpha\in\lambda} A_\alpha$ is a fuzzy p-ideal of X.

Proposition (3.2): Let $\{A_\alpha \mid \alpha\in\lambda\}$ be a chain of fuzzy p-ideals of a BH-algebra X. Then $\bigcup_{\alpha\in\lambda} A_\alpha$ is a fuzzy p-ideal of X.

Proof: Let $\{A_\alpha \mid \alpha\in\lambda\}$ be a chain of fuzzy implicative ideal of X.

i. Let $x\in X$. Then $\bigcup_{\alpha\in\lambda} A(0) = \sup\{A_\alpha(0) \mid \alpha\in\lambda\}) \geq \sup\{A_\alpha(x) \mid \alpha\in\lambda\} = \bigcup_{\alpha\in\lambda} A_\alpha(x)$

[Since A_α is a fuzzy p-ideal of X, $\forall\alpha\in\lambda$. By definition(1.8)(i)]

$\Rightarrow \bigcup_{\alpha\in\lambda} A(0) \geq \bigcup_{\alpha\in\lambda} A_\alpha(x)$

ii. Let x, y, z $\in$X. Then, we have $\bigcup_{\alpha\in\lambda} A_\alpha(x) = \sup\{A_\alpha(x) \mid \alpha\in\lambda\}$

$\geq \sup\{\min\{A_\alpha((x*z)*(y*z)), A_\alpha(y) \mid \alpha\in\lambda\}\}$

[Since A_α is a fuzzy p-ideal of X, $\forall\alpha\in\lambda$. By definition (1.8)(ii)]

$\geq \min\{\sup\{A_\alpha((x*z)*(y*z)), A_\alpha(y) \mid \alpha\in\lambda\}\}$ [Since A_α is a chain, $\alpha\in\lambda$]

$= \min\{\sup\{A_\alpha((x*z)*(y*x)) \mid \alpha\in\lambda\}, \sup\{A_\alpha(y) \mid \alpha\in\lambda\}\}$

$= \min\{\bigcup_{\alpha\in\lambda} A((x*z)*(y*z)), \bigcup_{\alpha\in\lambda} A_\alpha(y)\}$

$\Rightarrow \bigcup_{\alpha\in\lambda} A_\alpha(x) \geq \min\{\bigcup_{\alpha\in\lambda} A((x*z)*(y*x)), \bigcup_{\alpha\in\lambda} A_\alpha(y)\}$

Therefore, $\bigcup_{\alpha\in\lambda} A_\alpha$ is a fuzzy p-ideal of X.

Proposition (3.3): Let f: $(X,*,0)\to(Y,*',0')$ be a BH-epimorphism. If A is a fuzzyp-ideal of X, then f(A) is a fuzzy p-ideal of Y.

Proof: Let A be a fuzzy p-ideal of X. Then [11 and 12]

i. Let y $\in$Y. Then there exists x $\in$X.

$(f(A))(0') = \text{supp}\{A(x_1) \mid x_1\in f^{-1}(0')\}$

$= A(0) \geq \sup\{A(x) \mid x\in X\} \geq \sup\{A(x_1) \mid x = f^{-1}(y)\} = (f(A))(y)$

[Since A is a fuzzy p-ideal of X. By definition (1.8)(i)]

$\Rightarrow (f(A))(0') \geq (f(A))(y), \forall y\in Y$.

ii. Let $y_1, y_2, y_3 \in Y$. Then there exist $f(x_1)=y_1$, $f(x_2)=y_2$, $f(z)=y_3$ such that $x_1, x_2, z \in X$

$\Rightarrow (f(A))(y_1) = \sup\{A(x_1) \mid x \in f^{-1}(y_1)\}$

$\geq \sup\{A((x_1*z)*(x_2*z)), A(x_2) \mid (x_1*z)*(x_2*z)) \in f^{-1}((y_1*'y_3)*(y_2*'y_3))\}$ and $x_2 \in f^{-1}(y_2)\}\}$ [Since A is a fuzzy p-ideal of X. By definition (1.8)(ii)]

$\geq \min\{\sup\{A((x_1*z)*(x_2*z)) \mid (x_1*z)*(x_2*z) \in f^{-1}((y_1*'y_3)*(y_2*'y_3))\}$,

$\sup\{A(x_2) \mid x_2 \in f^{-1}(y_2)\}\}$

$= \min\{((f(A))(f((x_1*z)*(x_2*x_1)), (f(A))(f(x_2))\}$

$= \min\{((f(A))((f(x_1)*'f(z))(f(x_2)*'f(z))), (f(A))(f(x_2))\}$

$= \min\{(f(A))((y_1*'y_3)*(y_2*'y_3)), (f(A))(y_2)\}$

$\Rightarrow (f(A))(y_1) \geq m \min\{(f(A))((y_1*'y_3)*(y_2*'y_3)), (f(A))(y_2)\}$

Therefore, f (A) is a fuzzy p-ideal of Y.

Proposition (3.4): Let f: $(X, *, 0) \to (Y, *', 0')$ be a BH-homomorphism. If Bbbe a fuzzy p-ideal of Y, then $f^{-1}(B)$ is a fuzzy p-ideal of X.

Proof: Let B be a fuzzy p-ideal of Y. To prove $f^{-1}(B)$ is afuzzy p-ideal of X.

i. Let x $\in$ X. Then $(f^{-1}(B))(0) = B(f(0)) = B(0') \geq B(f(x)) = (f^{-1}(B))(x)$

[Since B is a fuzzy p-ideal of Y. By definition (1.8)(i)]

$\Rightarrow (f^{-1}(B))(0) \geq (f^{-1}(B))(x), \forall x \in X$.

ii. Let x , y , z $\in$X. Then $(f^{-1}(B))(x) = B(f(x))$

$\geq \min\{B((f(x)*''f(z))*'(f(y)*'f(z)))), B(f(y))\}$ [By definition(1.8)(ii)]

$= \min\{B(f(((x*z)*(y*z))), BB(f(y))\}$

[Since f is a homomorphism]

$= \min\{(f^{-1}(B))(((x*z)*(y*z))), ff^{-1}(B)(y)\}$.

[Since$(f^{-1}(B))(x) = B(f(x))$]

$\Rightarrow (f^{-1}(B))(x) \geq \min\{(f^{-1}(B))(((x*z)*(y*z))), (f^{-1}(B))(z)\}$.

Thus $f^{-1}(B)$ is a fuzzy p-ideal of X.

Theorem (3.5): Let X be a BH-algebra and μ be a fuzzy ideal of X. Then μ is a fuzzy p-ideal of X if and only if the level subset μ_α is a p-ideal of X, $\forall\alpha\in[0, \mu(0)]$.

Proof: Let μ be a fuzzy p-ideal of X. We must prove that μ_α is p-ideal of X.

i. $\mu(0) \geq \mu(x), \forall x\in X$. [By definition (1.8)(i)]

$\Rightarrow \mu(0) \geq \alpha, \forall \alpha\in[0, \mu(0)]$. So $0 \in\mu_\alpha$.

ii. Let x, y, z $\in$X such that $(x*z)*(y*z)\in\mu_\alpha$ and $y\in\mu_\alpha$

$\Rightarrow \mu((x*z)*(y*z)) \geq \alpha$ and $\mu(y) \geq \alpha$ [By definition(1.10) of μ_α]

$\min\{\mu((x*z)*(y*z)), \mu(y)\} \geq \alpha$

But $\mu(x) \geq \min\{\mu((x*z)*(y*z)), \mu(y)\}$

[Since μ is a fuzzy p-ideal of X. By definition (1.8)(ii)]

$\Rightarrow \mu(x) \geq \alpha \Rightarrow x\in\mu_\alpha$ [By definition (1.10) of μ_α]

Therefore, μ_α is a p-ideal of X.

Conversely, Let μ_α be a p-ideal of X, $\forall\alpha\in[0, \mu(0)]$ and Let α= sup μ(x). To prove that μ is a fuzzy p-ideal of X.

i. $0\in\mu_\alpha$. [Since μ_α is a p-ideal of X]

$\Rightarrow \mu(0) \geq \alpha$.

$\Rightarrow \mu(0) \geq \mu(x), \forall x\in X$.

ii. Let x , y , z $\in$ X such that $\min\{\mu((x*z)*(y*z)), \mu(y)\} = \alpha$

$\Rightarrow \mu((x*z)*(y*z)) \geq \alpha$ and $\mu(y) \geq \alpha$

$\Rightarrow (x*z)*(y*z)\in\mu_\alpha$ and $y\in\mu_\alpha$

$\Rightarrow x\in\mu_\alpha$. [Since μ_α be a p-ideal of X.]

$\Rightarrow \mu(x) \geq \alpha$

$\Rightarrow \mu(x) \geq \min\{\mu((x*z)*(y*z)), \mu(y)\}$. Therefore, μ is a fuzzy p-ideal of X.

Corollary (3.6): Let X be a BH-algebra, Then A is a fuzzy p-ideal A of X if and only if the set X_A is a p-ideal of X, where $X_A = \{x \in X \mid A(x)=A(0)\}$

Proof: Let A be a fuzzy p-ideal of X. We must prove that X_A is a p-ideal of X.

i. If x=0, then A(x) =A(0). So $0 \in X_A$

ii. Let x, y, z $\in$ X such that $(x*z)*(y*z)) \in X_A$ and $y\in X_A$.

$\Rightarrow$A((x*z)*(y*z))=A(0) and A(y)= A(0)
We have A(x) ≥ min{A((x*z)*(y*z)), A(y)} [Since A is a fuzzy p-ideal of X]
= min {A(0), A(0)}=A(0)
$\Rightarrow$ A(x) ≥ A(0).
$\Rightarrow$ A(x) = A(0). [Since A is a fuzzy p-ideal of X, A(0) ≥ A(x)]
$\Rightarrow$x∈ X_A. So, X_A is a p-ideal of X.
Conversely, Suppose that X_A be a p-ideal of X. We must show that A is a fuzzy p-ideal of X. Since X_A= A_α, α =A(0). Therefore, A is a fuzzy p- ideal of X.[By theorem (2.5)].
Proposition (3.7): Let X be a BH-algebra, A be a fuzzy subset of X defined by A(x)=$\begin{cases}\alpha_1 & ; \ x\in X_A \\ \alpha_2 & ; \ \text{otherwise}\end{cases}$, where $\alpha_1, \alpha_2 \in$ [0, 1] such that $\alpha_1 > \alpha_2$. Then A is a fuzzyp-ideal of X if and only if X_A is a p-ideal of X.
Proof: Let A be a fuzzy p-ideal of X. We should prove that X_A is a p-ideal of X.
i. A(0) =α_1.
$\Rightarrow$0 ∈X_A.[Since A(0) ≥ A(x) ; ∀x ∈X. By definition (1.8)(i)]
ii. Let x, y, z ∈ $X_A$such that (x*z)*(y*z) ∈ X_A and y ∈ X_A
$\Rightarrow$A((x*z))*(y*z))=A(0)=α_1 and A(y)=A(0)=α_1
$\Rightarrow$A(x) ≥min{A((x*z)*(y*z)), A(y)}=α_1
[Since A is a fuzzy p-ideal of X, by definition(1.8)(ii)]
$\Rightarrow$ A(x) =α_1
$\Rightarrow$x∈ X_A . Then X_A is a p-ideal of X.
Conversely, LetX_A be ap-ideal of X. To prove A is a fuzzy p-ideal of X.
i. Since 0 ∈ X_A, then A(0) =α_1.
$\Rightarrow$A(0) =α_1≥ A(x).
$\Rightarrow$A(0) ≥ A(x), ∀ x∈X
ii. Let x, y, z ∈ X. Then we have four cases:
Case1: If (x*z)*(y*z)∈ X_A and y∈ X_A
$\Rightarrow$x∈ X_A . [Since X_A is a p-ideal of X. by definition(1.7)(ii)]
$\Rightarrow$A((x*z))*(y*z))=α_1
$\Rightarrow$A(x) ≥ min{A((x*z)*(y*z)), A(y)}.
Case2: If (x*z)*(y*z)∈ X_A and y ∉X_A
$\Rightarrow$A((x*z)*(y*z))=$\alpha_1$and A(y)=α_2
$\Rightarrow$min{A((x*z)*(y*z)), A(y)}=α_2
$\Rightarrow$ A(x) ≥min{A((x*z)*(y*z)), A(y)}.
Case3: If (x*z)*(y*z)∉X_A and y∈ X_A
$\Rightarrow$A((x*z)*(y*z))=$\alpha_2$and A(y)=α_1
$\Rightarrow$min {A((x*z)*(y*z)), A(y))=α_2
$\Rightarrow$ A(x) ≥ min {A((x*z)*(y*z), A(y)}.
Case4: If ((x*z)*(y*z))∉ X_A and y∉X_A
$\Rightarrow$A((x*z)*(y*z))=$\alpha_2$and A(y)=α_2
$\Rightarrow$min{A((x*z)*(y*z)), A(y)}= α_2
$\Rightarrow$ A(x) ≥min{A((x*z)*(y*z)), A(y)}.
Therefore, A is a fuzzy p-ideal of X.
Proposition (3.8): Let X be a BH-algebra and let A be an fuzzy subset of X. Then A is a fuzzy p-ideal of X if and only if $A^\#$(x) = A(x) + 1- A(0) is a fuzzy p-ideal of X.
Proof: Let A be a fuzzy p-ideal of X. To prove $A^\#$ be a fuzzy p-ideal of X.
i. $A^\#$(0)= A(0)+1-A(0).
$\Rightarrow$ $A^\#$(0)=1.
$\Rightarrow$ $A^\#$ (0) ≥ $A^\#$(x), ∀x∈X.
ii. Let x, y, z ∈X. Then
$A^\#$(x))=A(x)+1-A(0) ≥ min{A((x*z)*(y*z)),A(y)}+1-A(0)
[Since A is a fuzzy p-ideal of X.]
= min {A((x*z)*(y*z))+1- A(0),A(y) +1-A(0)}
= min {$A^\#$ ((x*z))*(y*z)), $A^\#$(y)}
Then $A^\#$ is a fuzzy p-ideal of X.
Conversely, Let $A^\#$ be a fuzzy p-ideal of X. We must prove that A be a fuzzy p-ideal of X.
i. Let x∈X. Then we have A(0)=$A^\#$(0)-1+A(0)≥$A^\#$(x)-1+A(0)=A(x)
$\Rightarrow$A(0) ≥ A(x), ∀x∈X.
ii. Let x, y, z ∈X. Then since $A^\#$ is a fuzzy p-ideal of X, we have
A(x)=$A^\#$(x)-1+A(0)≥min{$A^\#$((x*z)*(y*z)),$A^\#$(y)}-1+A(0)
=min{ $A^\#$((x*z)*(y*z))-1+ A(0), $A^\#$(y) +1-A(0)}
=min{ A((x*z)*(y*z)), A (y)}
Then A is a fuzzy p-ideal of X.
Proposition(3.9): Let A be a fuzzy ideal of a BH-algebra X and w ∈X. If A satisfies the condition: ∀x, y∈ X, A(x) ≥ A ((x*z)*(y*z)) , then ↑A(w) = {x∈X| A(w) ≤A(x)} is a p-ideal of X.
Proof: Assume A be a fuzzy ideal of X. Then
i. A(0) ≥A(x), ∀x ∈X. [By definition (1.6)(i)]
$\Rightarrow$A(0)≥A(w) [Since w ∈X.] . Then 0 ∈↑A(w)
ii. Let x, y, z ∈ X such that (x*z)*(y*z) ∈ ↑A(w) and y∈↑A(w)
$\Rightarrow$A(w) ≤A((x*z)*(y*z)) and A(w) ≤A(y)
$\Rightarrow$A(w) ≤min{A((x*z)*(y*z)),A(y)} ≤ A((x*z)*(y*z))
[Since A is a fuzzy ideal of X. By definition (1.6)(ii)]
But A((x*z)*(y*z)) ≤A(x). [By the condition]
$\Rightarrow$A(w) ≤A(x).
$\Rightarrow$x∈↑A(w).Therefore, ↑A(w) is a p-ideal of X.
Proposition (3.10): Let X be a BH-algebra and w ∈X. If A is a fuzzy p-ideal of X, then ↑A(w) is a p-ideal of X.
Proof: Suppose that A be a fuzzy p-ideal of X. To prove that ↑A(w) is a p-ideal of X.
i. A(0) ≥A(x), ∀x ∈X. [By definition (1.8)(i)]
$\Rightarrow$A(0)≥A(w) [Since w ∈X.]
$\Rightarrow$ 0 ∈↑A(w).
ii. Let x, y, z ∈ X such that (x*z)*(y*z) ∈↑A(w) and y ∈↑A(w),
$\Rightarrow$A(w) ≤A((x*y)*(y*z)) and A(w) ≤A(y),
$\Rightarrow$A(w) ≤ min{ A(((x*z))*(y*z)) , A(y)} .
But min{A((x*z))*(y*z)), A(y)} ≤ A (x).
$\Rightarrow$A(w) ≤A(x).
$\Rightarrow$x∈↑A(w). Therefore, ↑A(w) is a p-ideal of X.

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