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Research paper

Steady Natural Convection Heat and Mass Transfer Due to a Horizontal Line Source in the Presence Of Magnetic Field and Chemical Reaction

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Abstract

Natural convective heat and mass transfer due to a horizontal line source at an adiabatic plate and plane plume flow are analyzed in the presence of magnetic field and chemical reaction. The case of an isothermal plate and a plate of varying temperature are also discussed. Certain qualitatively distinct behaviours of the transport parameters noticed are - for an adiabatic plate as well as an isothermal plate, the transport parameters have exhibited asymptotic behaviour with the Prandtl number and Schmidt number. For Plume flow centre line velocity has exhibited asymptotic behaviour with Prandtl number and Schmidt number. Transport parameters diminished but centre line velocity of the Plume increased with increasing magnetic field. Chemical reaction is seen to increase centre line velocity/Sherwood number and diminish skin friction/Nusselt number. Comparison of the results at an adiabatic plate with those at an isothermal plate and a plate of varying temperature is made.

Keywords: *Plume flow; Adiabatic plate; Natural convection; Double diffusion; Plate temperature variation.*

1. Introduction

The present work examines the effects of magnetic field, chemical reaction, Buoyancy ratio, Schmidt number and Prandtl number on natural convection flow arising from combined buoyancies of thermal and mass diffusion due to a horizontal line source. The transport in Plane plumes, transport at an adiabatic vertical plate, at an isothermal plate and at a plate of varying temperature are discussed.

Buoyancy induced flows due to a line thermal source and freely rising plane plumes are being discussed by many authors, some of the significant contributions being those of Spalding and Cruddace [1], Fujji [2,3], Gebhart, Pera and Schorr [4], Zimin and Lyakhov [5], Liburdy and Faeth [6], Jaluria and Gebhart [7], Gray [8] and Rao et.al. [9].

In view of several applications of convection flows due to multiple diffusion effects, several authors have discussed different aspects of such flows, some of the earliest being those of Somers [10], Mathers et.al. [11] and Wilcox [12].

One of such important works is the exhaustive analysis of Gebhart and Pera [13] where in the authors have shown the existence of similarity solutions for such flows, obtained solutions for many practical values of the Schmidt number and discussed in detail other aspects like aiding/opposing buoyancies and laminar stability.

Fujii [3] studied natural convection due to a horizontal line source and a point heat Source. It was followed by a study of natural convection plume flows due to a horizontal line source by Gebhart, Pera and Schorr [4]. Liburdy and Faeth [6] discussed theory of steady laminar thermal plume along an adiabatic vertical plate. Jaluria and Gebhart [7] have also analyzed buoyancy induced flows due to a line thermal source at an adiabatic vertical plate and compared their results with those for a plane plume. Gray [8] studied the effect of a transverse magnetic field on the transport at an adiabatic vertical plate with a horizontal line source.

Rao, Armaly and Chen [9] analyzed mixed convection plume flows at vertical adiabatic surfaces. Many different aspects of plume flows and flows at an adiabatic plate including those in porous media were discussed in references [14]-[19].

2. Formulation & Solution

A horizontal line heat source is assumed to lie along the lower edge of a semi infinite vertical plate in a quiescent electrically conducting fluid. A coordinate system is so chosen that the heat source lies along z-axis, x-axis vertically upwards along the plate and y-axis perpendicular to it. A magnetic field of strength $B_0(x)$ is applied perpendicular to the plate and chemical reaction can occur between the fluid and the concentration species. Assuming concentration levels of the species to be small, the equations governing the steady double diffusive natural convection boundary layer flow and heat transfer can be written as (refer Gebhart and Pera [13])

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(t-t_\infty)+g\beta^*(c-c_\infty)-\frac{\sigma B_0^2(x)u}{\rho} \tag{2}$$

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$$u\frac{\partial t}{\partial x}+v\frac{\partial t}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 t}{\partial y^2} \quad (3)$$

$$u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=D\frac{\partial^2 c}{\partial y^2}-K_1(x)(c-c_\infty) \quad (4)$$

Introducing a stream function, a similarity variable and certain non-dimensional functions through the relations

$$u=\frac{\partial \psi}{\partial y},\quad v=-\frac{\partial \psi}{\partial x},\quad \psi=\upsilon G f,\quad \theta=\frac{t-t_\infty}{t_w-t_\infty},\quad \phi=\frac{c-c_\infty}{c_w-c_\infty},$$

$$\eta=\frac{y\,G}{4x} \text{ where } G=4\left[\frac{g\beta(t_w-t_\infty)x^3}{4\upsilon^2}\right]^{1/4}$$

and, in view of the heat source at the bottom edge of the plate assuming that, $t_w-t_\infty=a\,x^n$, $c_w-c_\infty=a\,x^n$, the governing equations get transformed into the following system of non-linear ordinary differential equations

$$f'''+(n+3)ff''-2(n+1)f'^2+\theta+Br\phi-Mp\,f'=0 \quad (5)$$

$$\theta''+(n+3)\,Pr\,f\,\theta'-4nPr\,f'\theta=0 \quad (6)$$

$$\phi''+(n+3)Sc\,f\,\phi'-4nSc\,f'\phi-Sc\gamma\phi=0 \quad (7)$$

Here f, θ, ϕ are the non-dimensional stream function, temperature and concentration, a dash(') denotes derivative with respect to η, n is the index of the plate temperature, a is constant,

$$Pr=\frac{\mu c_p}{k},\qquad Sc=\frac{\upsilon}{D},\qquad Cr=\frac{2K}{\sqrt{g\beta a}},\qquad Mp=\frac{2\sigma B^2\sqrt{\beta}}{\rho\sqrt{ga}},$$

$Br=\dfrac{\beta^*(c_W-c_\infty)}{\beta(t_W-t_\infty)}$ are the parameters and the other symbols υ, c_p, k, β, σ, g, β^* denote respectively the kinematic coefficient of viscosity, specific heat, thermal conductivity, volumetric coefficient of expansion, electrical conductivity for the fluid, acceleration due to gravity and volumetric coefficient of expansion for the species. Subscripts ∞, w indicate the ambient condition and that at the plate respectively. It may be noted that for similarity solution to exist, the magnetic field $B_0(x)$ is taken as $Bx^{-2/5}$ (refer Gray [8]) and chemical reaction rate $K_1(x)$ is taken as $K\,x^{(n-1)/2}$.

As pointed out by Gray [8], this type of variations for $B_0(x)$ may not be practically possible. Similarly, a variation for $K_1(x)$ taken here may not be practically possible.

Following Gebhart & Pera[13], three types of boundary conditions are considered.

1. For an adiabatic vertical plate, since there is no flow of heat across the plate, the conditions will be:
   $f(0)=0,\ f'(0)=0,\ \theta'(0)=0,\ \phi(0)=1$ and

as $\eta\to\infty:\ f'\to 0,\ \theta\to 0,\ \phi\to 0$ (8)

2. For freely rising Plume flow, since there will be no flow of heat across the centre line and zero shear stress there, the conditions will be:
   $f(0)=0,\ f''(0)=0,\ \theta'(0)=0,\ \phi'(0)=0$ and

as $\eta\to\infty: f'\to 0,\ \theta\to 0,\ \phi\to 0$ (9)

3. For an isothermal plate or plate with varying temperature the conditions will be:
   $f(0)=0,\ f'(0)=0,\ \theta(0)=1,\ \phi(0)=1$ and

as $\eta\to\infty:\ f'\to 0,\ \theta\to 0,\ \phi\to 0$ (10)

2.1 Parameters of the Problem

In the present analysis, *Pr* is the Prandtl number, *Sc* is the Schmidt number, *Cr* is the chemical reaction parameter, *Mp* is the magnetic parameter or the Lykoudis number, *Br* is the buoyancy ratio parameter and *n* is the index of the plate temperature.

Based on the fact that the energy convected across any horizontal plane should be independent of *x*, *n* assumes the value '-0.6' for an adiabatic plate or for a Plume flow. For an isothermal plate *n* becomes zero (refer Gebhart & pera [13] and Carey & Mollendorf [20]).

Smaller values of *Sc* can indicate high diffusion of the species while larger values can indicate low diffusion of the species and small values of *Pr* can indicate higher thermal diffusivity as compared to momentum diffusivity. Positive values of *Br* indicate assisting buoyancies and negative values indicate opposing buoyancies and they can occur only for specific combination of fluid and species.

2.2 Solution of the Problem

The equations (5) to (7) for f, θ and ϕ subjected to the boundary conditions (8) or (9) or (10) are solved by Nachtsheim- Swigert scheme (refer [21]). In fact solutions are obtained by leaving out the conditions $f''\to 0,\ \theta'\to 0$, and $\phi'\to 0$ as $\eta\to\infty$ and an error is defined as $\{error=\ (f'_\infty)^2+(\theta_\infty)^2+(\phi_\infty)^2\}$. Solutions are obtained for some practical as well as for some hypothetical values of the parameters (*Pr*: 0.7 to 500; *Sc*: 0.1 to 500; *Cr*: 0, 0.1, 0.2, 0.3, 0.5; *Mp*: 0, 0.5, 1; *Br*: -0.1, -0.5, 0.1, 0.2, 0.3 and 0.5 to 1000; *n*: -0.6, -0.2 and 0) such that error will be less than 10^{-16}. It may be noted that much smaller values of *Pr* that correspond to liquid metals are not taken in to consideration and very high values of *Br* considered here are, of course, hypothetical. Certain qualitatively and quantitatively interesting results are presented in the form of figures 1 to 18.

3. Discussion of the results

Solutions are presented in three cases - Flow at an adiabatic plate, Plane Plume flow, flow at an isothermal plate and flow at a plate of varying temperature. In this report, conventional plots of fluid velocity, fluid temperature and concentration are not presented. In the case of adiabatic plate, since $\theta'(0)=0$, plots of skin friction (shear stress at the plate) and Sherwood number only are presented. In case of Plume flows, since $f''(0)=0$, $\theta'(0)=0$, and $\phi'(0)=0$, plots of velocity at the center line of the plume are only presented. However, in the case of isothermal plate or plate of varying temperature, plots of skin friction, Nusselt number and Sherwood number are presented. Observations pertaining to the maximum velocity are also reported. For sake of brevity mathematical expressions for skin friction, Nusselt number and Sherwood number are not presented here.

3.1 Adiabatic Plate

The plots of skin friction and Sherwood number of this case (n = -0.6, $\theta'(0)=0$) are presented in figures 1 to 6. When the plate is adiabatic, in case of assisting buoyancies, skin friction initially diminishes and approaches a constant value asymptotically as

Prandtl number increases (refer fig.1). For assisting buoyancies, similar behavior of shear stress is noticed with the Schmidt number also (refer fig.3). On the other hand, the Sherwood number initially increases with Prandtl number but approaches a constant value as *Pr* further increases (refer fig.2). In the case of opposing buoyancies also skin friction has asymptotic variation with Schmidt number except that it initially increases with increasing *Sc* (refer fig.3). For assisting buoyancies, as *Br* increases skin friction increases but Sherwood number diminishes (refer figs.5, 6). Magnetic field diminishes skin friction and Sherwood number; chemical reaction diminishes skin friction and increases Sherwood number (refer figs.4,5and 6).

3.2 Plane Plume Flow

Plots of velocity at the centre line of the plume (i.e., when n = -0.6, $f''(0)=0$, $\theta'(0)=0$ and $\phi'(0)=0$) are presented in figures 7 to 10. In this case solutions are obtained for assisting as well as opposing buoyancies and for a range of buoyancy ratios when *Pr* equals *Sc*. In case of assisting buoyancies velocity diminishes initially and approaches a constant value asymptotically with *Sc* while for opposing buoyancies it increases initially and approaches a constant value (refer figs.7, 8). Similar behaviour is observed for the centre line velocity with the Prandtl number also (refer fig.9). From these figures as well as from the numerical results it is observed that magnetic field diminishes and chemical reaction increases centre line velocity. When *Pr* equals *Sc*, i.e., when thermal diffusivity is of the same order of magnitude as mass diffusivity, centre line velocity of the Plume increases, almost exponentially, as buoyancy ratio (*Br*) increases, but, diminishes with increasing values of *Pr* (or *Sc*) (refer fig.10). The increasing tendency of centre line velocity is more pronounced for small values of *Pr* and *Sc* (refer fig.10).

3.3 Isothermal Plate / Plate of Varying Temperature

3.3.1 Isothermal plate

Plots of skin friction, Nusselt number and Sherwood number of the isothermal case are presented in figures 11 to 15. In this case, skin friction and Sherwood number diminish initially with increasing *Pr* and approach a constant value for further increase of *Pr* (refer figs.11,13). Nusselt number also has asymptotic behaviour with Pr except that it increases initially with increasing *Pr* (refer fig.12). Also skin friction and Nusselt number diminish initially with increasing values of *Sc* and assume a constant value with further increase of *Sc*(refer fig.15). However, Sherwood number increases with increasing values of *Sc*(refer fig.14). It may be noted that these afore said behaviours are the same for both assisting and opposing buoyancies. All the three transport parameters diminish with increasing intensity of the magnetic field (refer figs.11,12,13). Enhanced chemical reaction diminishes skin friction as well as Nusselt number but increases Sherwood number (refer figs.11,12,13).

Further, for both assisting and opposing buoyancies, variations in $f''(0)$ are high up to *Pr* = 20; while variations in ' $-\theta'(0)$ ' are high up to *Pr* = 100(for assisting buoyancies) and *Pr* = 40(for opposing buoyancies) (refer figures 11 and 12). Also from figure 14, variations in ' $-\phi'(0)$ 'can be seen to be quite significant up to *Sc* = 100.

3.3.2 Plate of Varying Temperature

Sample plots of this case are presented in figures 16, 17 and 18. When index of plate temperature is '-0.2', the variations in the transport parameters are seen to be similar to those of the isothermal plate except that the numerical values differ between the two cases (compare figs.16,17,18 with 11,12,13 respectively).

Unlike in the isothermal case, for assisting buoyancies, here, variations in $f''(0)$ are significant up to *Pr* = 50; variations in ' $-\theta'(0)$ ' are high up to *Pr* = 100 and those in ' $-\phi'(0)$ ' are high up to *Sc* = 100.

In case of isothermal plate and plate of varying temperature, variations in Nusselt number are very high with changes in *Pr* while those in Sherwood number are very high with changes in *Cr* and *Sc* (refer to figures 12,17 for Nusselt number and figures 13,14,18 for Sherwood number). The afore said behaviours of the transport parameters with *Pr*, *Sc*, *Cr* are due to the fact that changes in *Pr* are associated with thermal diffusion while changes in *Sc* and *Cr* are associated with mass diffusion.

3.4 Maximum Velocities

From the numerical computations made, in the isothermal case it is observed that maximum velocities diminish with increasing Prandtl number, Schmidt number and Magnetic parameter. Similar trend is observed in the case of a plate of varying temperature also. However, maximum velocities increase with increasing values of *Cr* and *Br*.

Conclusions

Some significant conclusions of our analysis are:

(i) For an adiabatic plate as well as an isothermal plate, the transport parameters have exhibited asymptotic behaviour with the Prandtl number and Schmidt number.

(ii) For Plume flow centre line velocity has exhibited asymptotic behaviour with Prandtl number and Schmidt number.

(iii) Transport parameters diminished but centre line velocity of the Plume increased with increasing magnetic field.

(iv) Chemical reaction is seen to increase centre line velocity / Sherwood number and diminish skin friction /Nusselt number.

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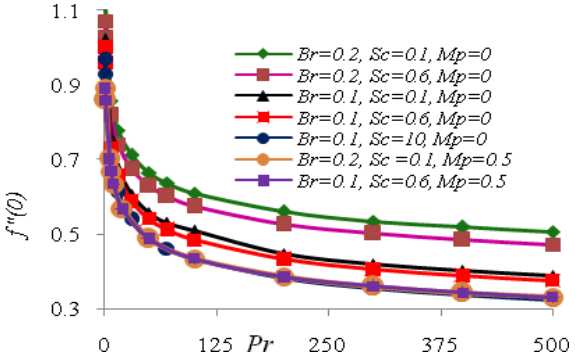

Figure.1: Variation of Skin friction $f''(0)$ with *Pr* for *Cr* = 0 and n = -0.6 (adiabatic plate)

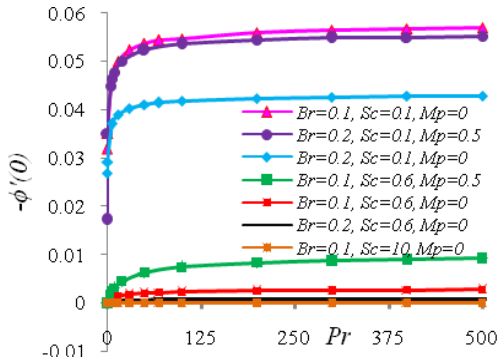

Figure 2: Variation of Sherwood number '$-\phi'(0)$' with *Pr* for *Cr* = 0 and *n* = -0.6 (adiabatic plate)

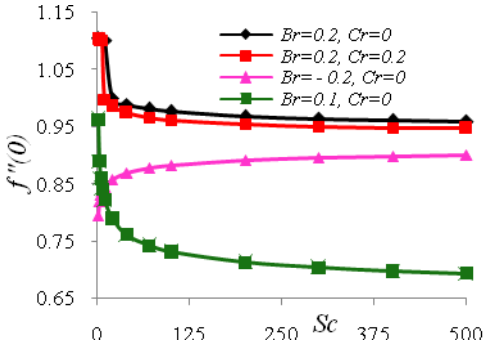

Figure 3: Variation of Skin friction with *Sc* for *Pr* =0.7, *Mp* = 0 and *n* = -0.6 (adiabatic plate)

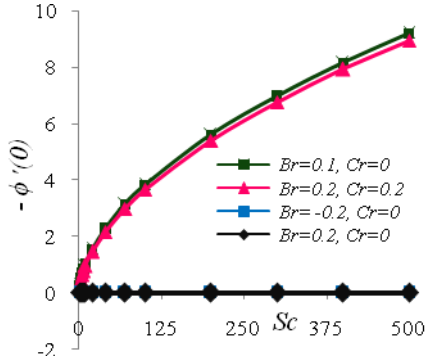

Figure.4: Plots of Sherwood number vs. *Sc* for *Pr* = 0.7, *Mp* = 0 and *n* = -0.6 (adiabatic plate)

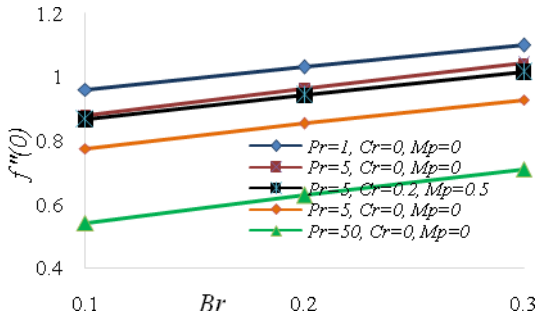

Figure.5: Effect of Buoyancy Ratio on Skin friction for *Sc* = 0.6 and *n* = -0.6 (adiabatic plate)

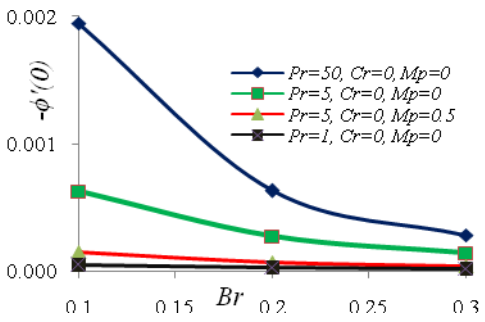

Figure.6: Effect of Buoyancy ratio on Sherwood number for *Sc* = 0.6 and *n* = -0.6 (adiabatic plate)

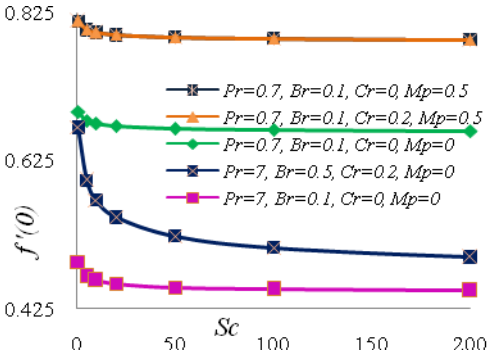

Figure 7: Plots of centre line velocity vs. *Sc* for *n* = -0.6 (plume flows, assisting buoyancies)

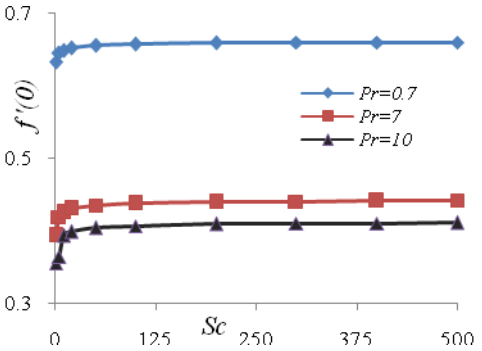

Figure.8: Plots of centre line velocity vs. *Sc* for *Br* = -0.1, *Mp* =0, *Cr* = 0 and *n* = -0.6 (plume flows, opposing buoyancies)

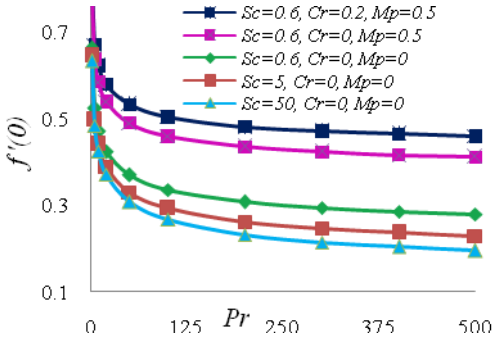

Figure 9. Plots of centre line velocity vs. *Pr* for *Br* = 0.1 and *n* = -0.6(plume flows, assisting buoyancies)

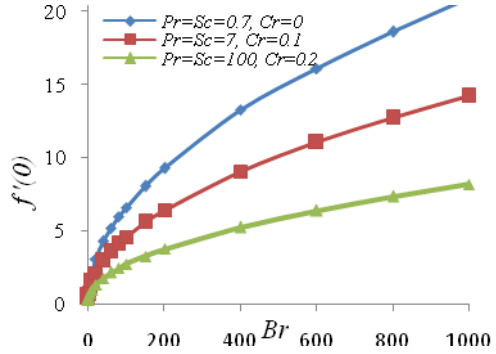

Figure.10: Plots of centre line velocity vs. *Br* when *Pr* = *Sc*, *Mp* = 0 and *n* = -0.6 (plume flows)

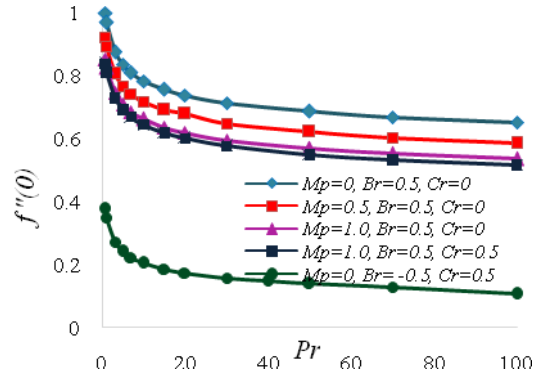

Figure.11: Plots of Skin friction vs. *Pr* for *Sc* = 0.1 and *n* = 0 (isothermal plate)

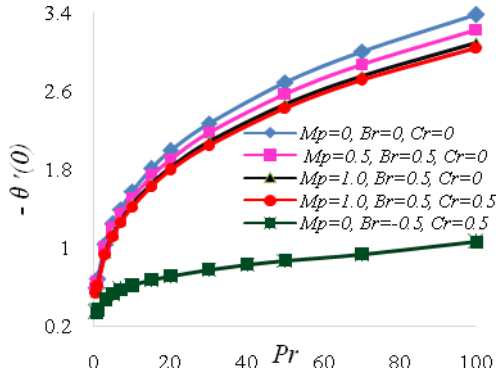

Figure.12: Plots of Nusselt number vs. *Pr* for *Sc* = 0.1 and *n* = 0 (isothermal plate)

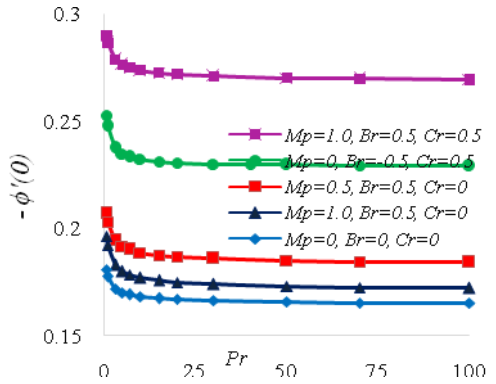

Figure.13: Plots of Sherwood number vs. *Pr* for *Sc* = 0.1 and *n* = 0 (isothermal plate)

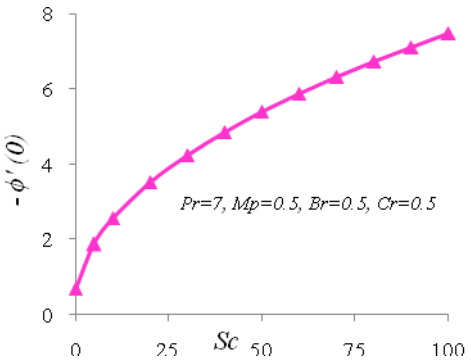

Figure.14: Variation of Sherwood number with *Sc* for *n* = 0 (isothermal plate)

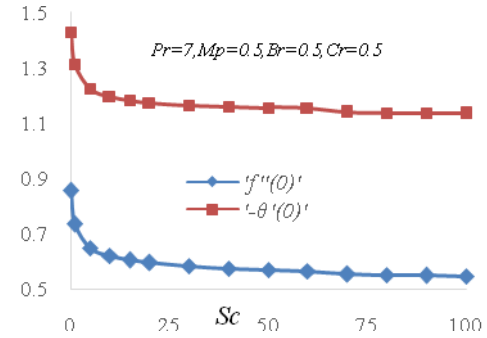

Figure.15: Variations of Skin friction and Nusselt number with *Sc* for *n* = 0 (isothermal plate)

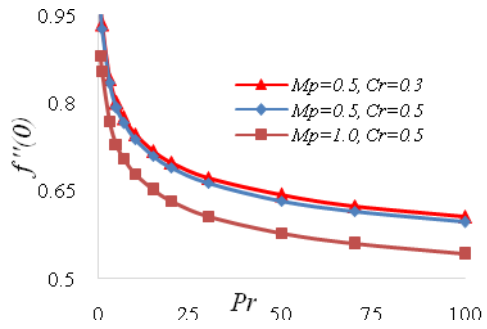

Figure.16: Plots of Skin friction vs. *Pr* for *Sc* = 0.1, *Br* = 0.5 and *n* = -0.2 (plate of varying temperature)

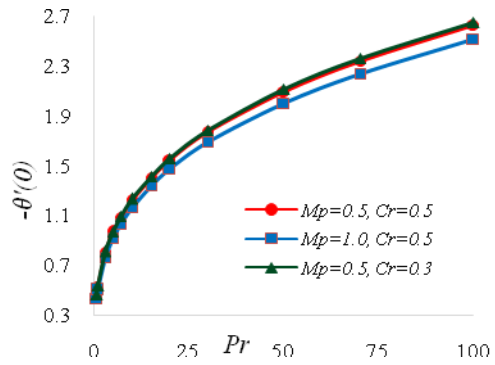

Figure. 17: Plots of Nusselt number vs. *Pr* for *Sc* = 0.1, *Br* = 0.5 and n = -0.2 (plate of varying temperature)

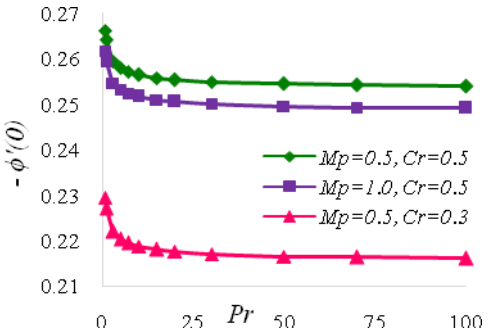

Figure.18: Plots of Sherwood number vs. *Pr* for *Sc* = 0.1, *Br* = 0.5 and *n* = -0.2 (plate of varying temperature)

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