



# Wrapped Lomax Distribution :a New Circular Probability Model

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## Abstract

Lomax Distribution (Pareto Type IV) is fitted for a life time random variable which can be studied for the data belongs to Actuarial science, medical diagnosis and Queuing theory etc. In the time of day events observed in cycles like hourly, daily, weekly, monthly or yearly are in circular distribution. By adopting the technique of wrapping an attempt is made to identify a new circular probability model originate as Wrapped Lomax Distribution. The concept of circular model is introduced and strategy of wrapping is given for Lomax Distribution. Wrapped Lomax Distribution PDF and CDF are derived, their graphs are also studied. The trigonometric moments and characteristic function of Wrapped Lomax Distribution are obtained and their graphs are also depicted. The characteristics like mean, variance, skewness, kurtosis and circular standard deviation for various values of location and scale parameters are derived in this paper.

**Keywords:** characteristics, circular probability model, trigonometric moments, wrapped Lomax distribution, wrapping.

## 1. Introduction

The Lomax distribution is incorporated to the data belongs to medical diagnosis is used to the determination of disease. The information required for medical diagnosis is typically collected from the past history and physical examination of the patient in his day to day of events observed in weekly or monthly or yearly cycles. These observations are in the form of 2-dimensional directional data come into the topic ‘CIRCULAR STATISTICS’.

Several wrapped versions for different life testing models are introduced by Dattatreya Rao *et al.* [1] and [2], Mardia and Jupp *et al.* [3] and [4], Rao Jammalamadaka *et al.* [5] and [6] elaborately discussed the concepts of Directional Statistics. Some other researchers worked out in this direction about circular and Lomax models include Giriya S.V.S. *et al.* [7] and [8], P. Srinivasa Subrahmanyam *et al.* [9], Fisher N.I. *et al.* [10], Ramabhadra Sarma *et al.* [11], Subba Rao *et al.* [12], [13] and [14].

By the technique of wrapping, an attempt is made to provoke a new circular probability model originate as Wrapped Lomax distribution. To assess the characteristics of population of the new circular probability model by the trigonometric moments obtained from the characteristic function of Wrapped Lomax distribution by converting a linear variable to its modulo  $2\pi$

Consider a function  $h$  is defined as,  $h : [0, 2\pi) \rightarrow \mathcal{R}$  is the probability density function of a circular distribution then it satisfies the following properties

$$h(\theta) \geq 0, \forall \theta \quad (1.1)$$

$$\int_0^{2\pi} h(\theta) d\theta = 1 \quad (1.2)$$

$$h(\theta) = h(\theta + 2k\pi) \quad (1.3)$$

(i.e.  $h$  is periodic), for any integer  $k$

(Mardia [3] and Jammalamadaka, Sengupta [5])

The research contribution in this paper is presented in the following manner.

The strategy of wrapping of a probability model is discussed in section 2. The procedure of wrapping, characteristic function and trigonometric moments for Wrapped Lomax model are given in section 3 of this paper. In section 4 the graphs of linear representation, circular representation and characteristic function of Wrapped Lomax model are depicted. The characteristics like mean, variance, trigonometric moments and circular S.D. etc. are calculated for different shape and scale parameters are derived in section 5 of this paper. The conclusion and further scope of study are mentioned in section 6.

## 2. Strategy of Wrapping

Let  $Y_w$  be a circular r.v. defined by the modulo  $2\pi$  reduction corresponding to ar.v.  $Y$  defined on  $\mathcal{R}$  then

$$Y_w = Y(\text{mod } 2\pi) \quad (2.1)$$

$$Y_w(\theta) = Y(\theta + 2k\pi), k \in \mathcal{Z} \quad (2.2)$$

The wrapped circular probability density function  $h(\theta)$  corresponding to the density function  $f$  of a linear v. Y is defined as

$$h(\theta) = \sum_{k=0}^{\infty} f(\theta + 2k\pi), \theta \in [0, 2\pi) \tag{2.3}$$

Total probability of circular probability distribution is located on the unit circle  $\{(\cos \theta, \sin \theta) / 0 \leq \theta < 2\pi\}$  in the plane which satisfies the properties (2.1) through (2.3).

### 3. Wrapped Lomax Distribution

The two parameter Lomax distribution with  $\alpha, \sigma$  as shape and scale parameters is defined as

$$f(x) = \frac{\alpha}{\sigma} \left(1 + \frac{x}{\sigma}\right)^{-(\alpha+1)}, x \geq 0 \tag{3.1}$$

Where  $\alpha > 0$  and  $\sigma > 0$

The cdf of the Lomax distribution is known as

$$F(x) = 1 - \left(1 + \frac{x}{\sigma}\right)^{-\alpha}, x \geq 0 \tag{3.2}$$

By applying the wrapping technique to equation (3.1), we get the corresponding circular Wrapped Lomax Distribution (WLD).

The pdf of wrapped Lomax model is

$$h(\theta) = \sum_{k=0}^{\infty} f(\theta + 2k\pi), \theta \in [0, 2\pi) \tag{3.3}$$

$$= \sum_{k=0}^{\infty} \frac{\alpha}{\sigma} \left(1 + \frac{\theta + 2k\pi}{\sigma}\right)^{-(\alpha+1)}$$

The cdf of wrapped Lomax model is

$$H(\theta) = \sum_{k=0}^{\infty} \left(1 + \frac{2k\pi}{\sigma}\right)^{-\alpha} - \left(1 + \frac{\theta + 2k\pi}{\sigma}\right)^{-\alpha} \tag{3.4}$$

Also, it is observed that the series is convergent.

If  $H(\theta)$  denotes the cdf of the r.v.,  $\theta$  then the characteristic function of the circular model is given by

$$\phi_{\theta}(t) = E(e^{it\theta}) = \int_0^{2\pi} e^{it\theta} dH(\theta) = \rho_t e^{i\mu_t}, t \in Z \tag{3.5}$$

Clearly,  $\phi(t) \neq 0, e^{2i\pi} = 1$

(Mardia [3]). i.e.  $\phi(t)$  is defined for only integer values of  $t$ .

Also the characteristic function for the wrapped distribution is

$$\phi(p) = \phi_p$$

$$\phi_{\theta}(p) = E(e^{ip\theta}) = \int_0^{2\pi} e^{ip\theta} dF(\theta) = \rho_p e^{i\mu_p}, p \in Z$$

,also  $\phi_0 = 1, \overline{\phi_p} = \phi_{-p}$

### 3.1. The Characteristic Function of Lomax Distribution

The characteristic function of Lomax distribution is

$$\phi(t) = \int_0^{\infty} e^{itx} \frac{\alpha}{\sigma} \left(1 + \frac{x}{\sigma}\right)^{-(\alpha+1)} dx \tag{3.6}$$

### 3.2. Trigonometric Moments

The value of characteristic function  $\phi_p$  at an integer  $p$  also called  $p^{th}$  trigonometric moment of  $\theta$ . The real and imaginary parts of  $\phi_p$  are trigonometric moments  $\alpha_p$  and  $\beta_p$  respectively and defined as  $\alpha_p = E(\cos p\theta)$  and  $\beta_p = E(\sin p\theta)$ , where  $p \in Z$ .

## 4. Graphs

The graph for probability density function of Wrapped Lomax Distribution (Linear Representation) is plotted in Figure 1.

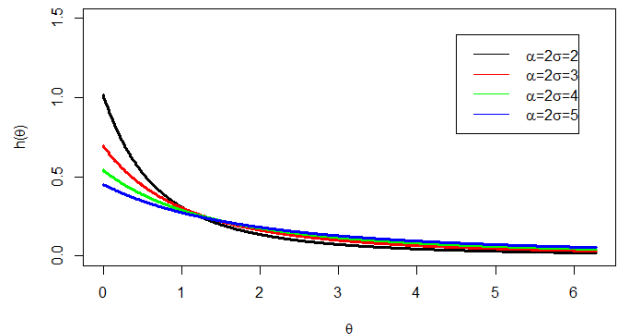


Figure 1: PDF of Wrapped Lomax distribution (Linear Representation)

The graph for probability density function of Wrapped Lomax distribution (Circular Representation) is plotted in Figure 2.

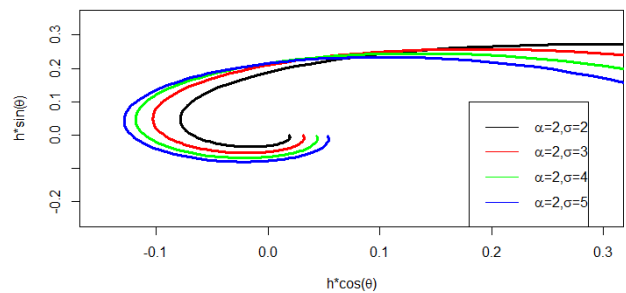


Figure 2: PDF of Wrapped Lomax distribution (Circular Representation)

The graph of the characteristic function of Wrapped Lomax distribution is plotted in Figure 3.

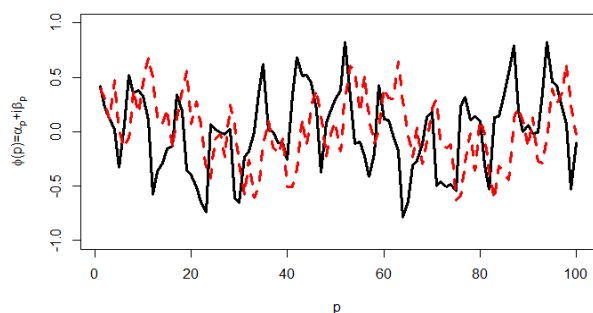


Figure 3: Characteristic function of Wrapped Lomax distribution

### 5. Characteristics of Wrapped Lomax Distribution

The characteristics of new circular probability model are derived through the trigonometric moments obtained from the characteristic function. The trigonometric moments which are real and imaginary parts of the characteristic function will adequate. Mardia [3] gave expressions of mean direction, resultant length, circular variance, circular standard deviation, central trigonometric moments, skewness and kurtosis for circular distributions. These characteristics for the Wrapped Lomax model are also based on their respective trigonometric moments. These can be expressed in terms of trigonometric moments.

The characteristic function of Lomax distribution is

$$\phi(t) = \int_0^\infty e^{itx} \frac{\alpha}{\sigma} \left(1 + \frac{x}{\sigma}\right)^{-(\alpha+1)} dx \quad (5.1)$$

For obtaining the trigonometric moments, the n-point GaussLaguerrequadrature formula for numerical integration as given in Rao et al. [15] is applied for equation (5.1).

For  $p \in \mathbb{Z}$ , the characteristic function of Wrapped Lomax distribution is hence given by

$$\phi(p) = \int_0^{2\pi} e^{i\theta} h(\theta) d\theta \quad (5.2)$$

The real and imaginary parts  $\alpha_p$  and  $\beta_p$  respectively are obtained from characteristic function of the Wrapped Lomax distribution are presented in Table 1.

**Table 1:** Characteristics of Wrapped Lomax Model

Characteristics		$\alpha = 2 \sigma = 2$	$\alpha = 2 \sigma = 3$	$\alpha = 2 \sigma = 4$	$\alpha = 2 \sigma = 5$
Mean		0.7597	0.9098	1.0181	1.105
Trigonometric Moments	$\alpha_1$	0.417725	0.278683	0.191763	0.133645
	$\alpha_2$	0.2237	0.146058	0.111317	0.0936708
	$\beta_1$	0.396827	0.358352	0.310878	0.265839
	$\beta_2$	0.29303	0.195502	0.119108	0.0604399
Resultant Length	$\rho$	0.576165	0.453961	0.365265	0.2975422
Variance	$V_0$	0.423835	0.546039	0.634735	0.7024578
	$\alpha_1^*$	0.576165	0.453961	0.3652645	0.2975422
Trigonometric moments	$\alpha_2^*$	0.304115	0.153511	0.0564874	-0.0073655
	$\beta_1^*$	0	0	0	0
	$\beta_2^*$	-0.20838	-0.189706	-0.152929	-0.1112338
	$\gamma_1^0$	-0.755196	-0.47016	-0.302414	-0.1889323
Kurtosis	$\gamma_2^0$	1.07948	0.372424	0.0960238	-0.0308103
Circular Standard Deviation	$\sigma_0$	1.0501	1.2568	1.4192	1.557
		1.4127	1.6795	1.9046	2.0947

### 6. Conclusion

From Table 1 we can observe that with increasing the value of scale parameter  $\sigma$ , keeping shape parameter  $\alpha = 2$ , the circular variance gradually increased, the distribution started shifting from negatively skewed to near symmetric and from platy kurtic to mesokurtic.

The further scope of this probability model is to study about the stereographic projection and developing the quality control techniques to the considered circular model.

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