International Journal of Engineering & Technology, 7 (3.31) (2017) 150-152

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper



Wrapped Lomax Distribution :a New CircularProbability Model

R.Subba Rao[1]*, V.Ravindranath[2], A.V.Dattatreya Rao[3], G.Prasad[4], P.Ravi Kishore[5]

[1]*Department of MathematicsS. R. K. R.Engineering College, Bhimavaram, Andhra Pradesh, India – 534202*
[2] *Department of Mathematics, College of Engineering, JNT University, Kakinada, Andhra Pradesh, India – 533003*
[3] *DepartmentofStatistics, AcharyaNagarjuna University, Nagarjuna Nagar, Guntur,Andhra Pradesh, India – 522510.*
[4] *DepartmentofStatistics, Aditya Degree College, Rajahmundry, Andhra Pradesh, India – 533103.*
[5] *Department of Mathematics, Srinivasa Institute of Engineering and Technology, Cheyyeru, Andhra Pradesh, India – 533222.*
**Corresponding author E-mail:rsubbarao9@gmail.com*

Abstract

Lomax Distribution (Pareto Type IV) is fitted for a life time random variable which can be studied for the data belongs to Actuarialscience, medical diagnosis and Queuing theory etc. In the time of day events observed in cycles like hourly, daily, weekly, monthlyor yearly are in circular distribution. By adopting the technique of wrapping an attempt is made to identify a new circular probabilitymodel originate as Wrapped Lomax Distribution. The concept of circular model is introduced and strategy of wrapping is given forLomaxDistribution. Wrapped Lomax Distribution PDF and CDF are derived, their graphs are also studied. The trigonometric momentsand characteristic function of Wrapped Lomax Distribution are obtained and their graphs are also depicted. The characteristics likemean, variance, skewness, kurtosis and circular standard deviation for various values of location and scale parameters are derived in this paper.

Keywords:*characteristics, circular probability model , trigonometric moments , wrapped Lomax distribution , wrapping .*

1. Introduction

The Lomax distribution is incorporated to the data belongs tomedical diagnosis is used to the determination of disease. Theinformation required for medical diagnosis is typically collectedfrom the past history and physical examination of the patient in his day to day of events observed in weekly or monthly or yearly cycles. These observations are in the form of 2-dimensional directional data come into the topic 'CIRCULAR STATISTICS'.

Several wrapped versions for different life testing models areintroduced by Dattatreya Rao*et.al.*[1]and[2], Mardia and Jupp*et.al.*[3]and[4],RaoJammalamadaka*et.al.*[5]and[6] elaborately discussed the concepts of Directional Statistics. Some other researchers worked out in this direction about circular and Lomax models include Girija S.V.S. *et.al.*[7]and[8], P. SrinivasaSubrahmanyam*et.al.*[9], Fisher N.I. *et.al.[*10], RamabhadraSarma*et.al.*[11], Subba Rao*et.al.*[12],[13]and[14].

By the technique of wrapping, an attempt is made to provokea new circular probability model originate as Wrapped Lomaxdistribution. To assess the characteristics of population of the newcircular probability model by the trigonometric moments obtained fromthe characteristic function of Wrapped Lomax distribution byconverting a linear variable to its modulo 2π

Consider a function h is defined as, $h:[0,2\pi) \to R$ is the probabilitydensity function of a circular distribution then it satisfies thefollowing properties

$$h(\theta) \geq o, \forall \theta \quad (1.1)$$

$$\int_o^{2\pi} h(\theta)d\theta = 1 \quad (1.2)$$

$$h(\theta) = h(\theta + 2k\pi) \quad (1.3)$$

(i.e. h is periodic) , for any integer k

(Mardia[3] and Jammalamadaka, Sengupta[5])

The research contribution in this paper is presented in the following manner.

The strategy of wrapping of a probability model is discussed in section2. The procedure of wrapping, characteristic function and trigonometric moments forWrapped Lomax model are given in section3 of this paper. In section 4 the graphs of linear representation, circular representation and characteristic function of Wrapped Lomax model are depicted. The characteristics like mean,variance, trigonometric moments and circular S.D. etc. are calculated for different shape and scale parameters are derived in section5 of this paper.The conclusion and further scope of study are mentioned insection6.

2. Strategy of Wrapping

Let Y_w be a circularr.v. defined by the modulo 2πreduction corresponding to ar.v. Y defined on R then

$$Y_w = Y(\mathrm{mod}\, 2\pi) \quad (2.1)$$

$$Y_w(\theta) = Y(\theta + 2k\pi), k \in Z \quad (2.2)$$

Copyright © 2018Authors. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

The wrappedcircular probabilitydensity function $h(\theta)$ corresponding to the density function f of a linearr.v. Y is defined as

$$h(\theta)=\sum_{k=0}^{\infty} f(\theta+2k\pi), \theta\in[0,2\pi) \quad (2.3)$$

Total probability of circular probability distribution is located on the unit circle $\{(\cos\theta,\sin\theta)/0\le\theta<2\pi\}$ in the plane which satisfies the properties (2.1) through (2.3).

3. Wrapped Lomax Distribution

Thetwoparameter Lomax distribution with α, σ as shape and scaleparameters is defined as

$$f(x)=\frac{\alpha}{\sigma}\left(1+\frac{x}{\sigma}\right)^{-(\alpha+1)}, x\ge 0 \quad (3.1)$$

Where α >0 and σ >0

The cdf of the Lomax distribution is known as

$$F(x)=1-\left(1+\frac{x}{\sigma}\right)^{-\alpha}, x\ge 0 \quad (3.2)$$

By applying the wrapping technique to equation(3.1), we get the Correspondingcircular Wrapped Lomax Distribution (WLD).

The pdf of wrapped Lomax model is

$$h(\theta)=\sum_{k=0}^{\infty} f(\theta+2k\pi), \theta\in[0,2\pi)$$

$$=\sum_{k=0}^{\infty}\frac{\alpha}{\sigma}\left(1+\frac{\theta+2k\pi}{\sigma}\right)^{-(\alpha+1)} \quad (3.3)$$

The cdf of wrapped Lomax model is

$$H(\theta)=\sum_{k=0}^{\infty}\left(1+\frac{2k\pi}{\sigma}\right)^{-\alpha}-\left(1+\frac{\theta+2k\pi}{\sigma}\right)^{-\alpha} \quad (3.4)$$

Also, it is observed that the series is convergent.

If $H(\theta)$ denotes the cdf of the r.v., θ then the characteristic functionof the circular model is given by

$$\phi_\theta(t)=E\left(e^{it\theta}\right)=\int_0^{2\pi} e^{it\theta}dH(\theta)=\rho_t\, e^{i\mu_t}, t\in Z \quad (3.5)$$

Clearly, $\phi(t)\ne 0, e^{2i\pi}=1$

(Mardia [3]).i.e. $\phi(t)$ is defined for only integer values of t.
Also the characteristic function for the wrapped distribution is

$$\phi(p)=\phi_p$$

$$\phi_\theta(p)=E\left(e^{ip\theta}\right)=\int_0^{2\pi} e^{ip\theta}dF(\theta)=\rho_p e^{i\mu_p}, p\in Z$$

,also $\phi_0=1$, $\overline{\phi_p}=\phi_{-p}$

3.1. The Characteristic Function of Lomax Distribution

The characteristic function of Lomax distribution is

$$\phi(t)=\int_0^{\infty} e^{itx}\frac{\alpha}{\sigma}\left(1+\frac{x}{\sigma}\right)^{-(\alpha+1)}dx \quad (3.6)$$

3.2. Trigonometric Moments

The value of characteristic function ϕ_p at an integer p also called p^{th} trigonometric moment of θ. The real and imaginary parts of ϕ_p are trigonometric moments α_p and β_p respectively and defined as $\alpha_p=E(\cos p\theta)$ and $\beta_p=E(\sin p\theta)$, where $p\in Z$.

4. Graphs

The graph for probability density function of Wrapped Lomax Distribution(Linear Representation) is plotted in Figure 1.

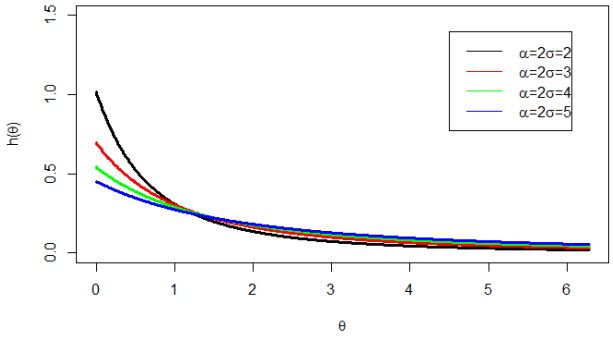

Figure 1: PDF of Wrapped Lomax distribution(LinearRepresentation)

The graph for probability density function of Wrapped Lomaxdistribution(CircularRepresentation) is plotted in Figure 2.

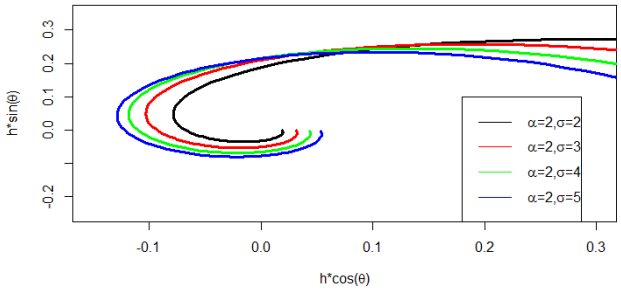

Figure 2: PDF of Wrapped Lomax distribution(Circular Representation)

The graph ofthe characteristic function ofWrapped Lomax distribution is plotted in Figure3.

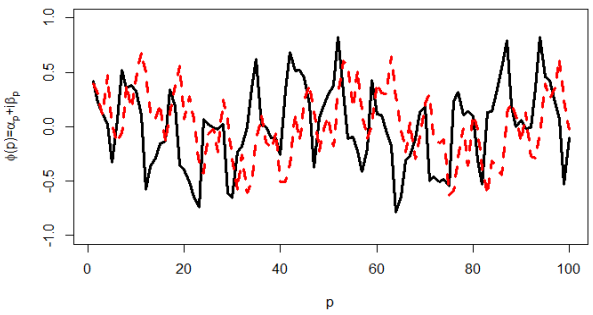

Figure 3: Characteristic function of Wrapped Lomax distribution

5. Characteristics of Wrapped Lomax Distribution

The characteristics of new circular probability model are derivedthrough the trigonometric moments obtained from the characteristic function. The trigonometric moments which are real and imaginary parts of the characteristic function will adequate. Mardia [3] gave expressions of mean direction, resultant length,circular variance, circular standard deviation, central trigonometricmoments, skewness and kurtosis for circular distributions. Thesecharacteristics for the Wrapped Lomax model are also based ontheir respective trigonometric moments. These can be expressed in terms of trigonometric moments.

The characteristic function of Lomax distribution is

$$\phi(t)=\int_0^{\infty} e^{itx}\frac{\alpha}{\sigma}\left(1+\frac{x}{\sigma}\right)^{-(\alpha+1)}dx \quad (5.1)$$

Forobtaining the trigonometric moments, the n-point GaussLaguerrequadrature formula for numerical integration as given inRao*et. al.* [15] is applied for equation (5.1).

For $p \in Z$, the characteristic function of Wrapped Lomax distributionis hence given by

$$\phi(p)=\int_0^{2\pi} e^{it\theta}h(\theta)d\theta \quad (5.2)$$

The real and imaginary parts α_p and β_p respectively are obtainedfrom characteristic function of the Wrapped Lomax distribution arepresented in Table1.

Table 1: Characteristics of Wrapped Lomax Model

Characteristics		α = 2 σ = 2	α = 2 σ = 3	α = 2 σ = 4	α = 2 σ = 5
Mean		0.7597	0.9098	1.0181	1.105
Trigonomietric Moments	α1	0.417725	0.278683	0.191763	0.133645
	α2	0.2237	0.146058	0.111317	0.0936708
	β1	0.396827	0.358352	0.310878	0.265839
	β2	0.29303	0.195502	0.119108	0.0604399
Resultant Length	ρ	0.576165	0.453961	0.365265	0.2975422
Varince	Vo	0.423835	0.546039	0.634735	0.7024578
Central Trigonometricmoments	α1*	0.576165	0.453961	0.3652645	0.2975422
	α2*	0.304115	0.153511	0.0564874	-0.0073655
	β1 *	0	0	0	0
	β2*	-0.20838	-0.189706	-0.152929	-0.1112338
Skewness	Υ_1^0	-0.755196	-0.47016	-0.302414	-0.1889323
Kurtosis	Υ_2^0	1.07948	0.372424	0.0960238	-0.0308103
Circular Standard Deviation	σ_0	1.0501 1.4127	1.2568 1.6795	1.4192 1.9046	1.557 2.0947

6. Conclusion

From Table 1 we can observe that with increasing the value of scale parameter σ, keeping shape parameter $\alpha = 2$, the circular variance gradually increased, the distribution started shifting from negatively skewed to near symmetric and from platy kurtic to mesokurtic.

The further scope of this probability model is to study aboutthe stereographic projection and developing the quality controltechniques to the considered circular model.

References

[1] DattatreyaRao A.V., Girija S.V.S., Devaraaj V.J., "On Characteristicsof Wrapped Gamma Distribution", *An International Journal (ESTIJ)*,ISSN: 2250-3498, Vol.3, No.2, (April 2013).

[2] DattatreyaRao, A.V., RamabhadraSarma, I. and Girija S.V.S., "OnWrapped Version of Some Life Testing Models", *Comm Statist,-Theor.Meth.* , 36, issue # 11, (2007), pp.2027-2035.

[3] Mardia K.V. and Jupp P.E., *Directional Statistics*, John Wiley,Chichester(2000).

[4] Mardia K.V., *Statistics of Directional Data*,Academic Press-NewYork(1972).

[5] Jammalamadaka S. Rao, Sen Gupta A., *Topics in Circular Statistics*,World Scientific Press, Singapore, (2001).

[6] Jammalamadaka S. Rao and Sen Gupta S., "Mathematical Techniquesfor Paleocurrent Analysis: Treatment of Directional Data",*Journal ofintl Assoc. Geol*, 4, (1972), pp. 235 –258.

[7] Girija S.V.S., Dattatreya Rao A.V. and Phani Y., "On Construction ofStereographic Semicircular Models", *Journal of Applied Probabilityand Statistics*. Vol. 8, No. 1,(2013), pp. 75-90.

[8] Girija S.V.S., Dattatreya Rao A.V. and Phani Y., "On Stereographic Lognormal Distribution ", *International Journal of Advances in APpliedSciences (IJAAS*), Vol.2, No.3, (2013), pp.125-132.

[9] SrinivasaSubrahmanyam P., Dattatreya Rao A.V., Girija S.V.S., "OnWrapping of Exponentiated Inverted Weibull Distribution*", InternationalJournal for Innovative Research in Science &Technolog*Vol. 3, Issue 11 , (April 2017),ISSN (online): 2349-6010,.

[10] Fisher, N.I., *Statistical Analysis of Circular Data*, CambridgeUniversityPress, (1993).

[11] RamabhadraSarma I., Dattatreya Rao A.V. and Girija S.V.S., "On-Characteristic Functions of Wrapped Lognormal and WeibullDistribution*",Journal of Statistical Computation and Simulation*, Vol. 81, No.5, (2011),pp. 579–589.

[12] Subba Rao R., Durga mamba A. N., Kantam R.R.L., " Reliability testplan for size biased lomax model", *International journal of scientificand Innovative mathematical research*, Vol. 2, Issue 12, (2015), pp1-8.4 International Journal of Engineering & Technology

[13] Subba Rao R., Kantam R.R.L., " Pareto Distribution –Some methodsof Estimation" ,*International Journal of Computationl MathematicalIdeas*, Vol. 2, Issue 1, (2010).

[14] SubbaRao R., Prasad G. and Kantam R.R.L. " Percentiles of RangeChart: Pareto Type Model", *International Journal of Advanced inApplied Science (IJAAS)*. Vol. 2. No. 3, (2013), pp. 141-144.

[15] Rao C.R. &Mitra S.K. ,*Formulae and Tables for Statistical Work*,Statistical Publishing Society,(1975).