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Research paper

Fixed Point Theorems Under Caristi's Type Map on $C*$-Algebra Valued Fuzzy Soft Metric Space

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Abstract

In this paper, we present the extension of Caristi's fixed point theorems for mappings defined on $C*$-algebra-valued Fuzzy soft metric spaces. We establish the existence of simple proof of caristi's type fixed point theorems in $C*$-algebra-valued Fuzzy soft metric spaces and we give some examples which supports our main results.

Keywords:. *Bounded below function; Caristi's mapping; $C*$-algebra-valued Fuzzy soft metric; completeness; fixed point; Lower semi continuity.*

1. Introduction and Preliminaries

In 2001, Maji et al. [1] introduced the notion of a fuzzy soft set which unites a fuzzy set and a soft set and after that Thangaraj Beaula et al.[2] defined fuzzy soft metric space in terms of fuzzy soft points and established some results. Subsequently to improve many author established so many results on fuzzy soft metric spaces and its topological properties (see. e.g.[3]-[6]). The Caristi's fixed point results is known as one of the very attractive and valuable generalization of the Banach fixed point results for self-mappings on a complete metric spaces. In fact Caristi's fixed point results [7] is appealing extension of Banach contraction principle [8]. The proof of Caristi's fixed point results has been generalized and elongated in varies streams and techniques (see. e.g .[9]-[16]).

In this paper, we will keep on to study fixed point in the setting of $C*$- algebra valued fuzzy soft metric space. More squarely, we introduce the notion of lower semi continuity in the context of $C*$-algebra valued fuzzy soft metric spaces and established the Caristi's type fixed point results in context of $C*$- algebra valued fuzzy soft metric spaces.

Throughout our discussion, U refers to an initial universe, E the set of all parameters for U, C subset of parameter set E and P $(\tilde{U})$ the set of all fuzzy set of U. (U, E) means the universal set U and parameter set E, $\tilde{C}$ refer to a unital C*-algebra. Now we recollect some basic definitions, notations, and results on C*-algebras are available in ([17], [18]). The set $\widetilde{C_h} = \{\tilde{a} \in \tilde{C} : \tilde{a} = \widetilde{a^*}\}$. An element $\tilde{a} \in \tilde{C}$ is called a positive element, if $\tilde{a} = \widetilde{a^*}$ and it is denoted by $\tilde{O}_{\tilde{C}} \preceq \tilde{a}$ where $\tilde{O}_{\tilde{C}}$ is the zero element in $\tilde{C}$ and the spectrum of $\tilde{a}$ is $\sigma(\tilde{a}) \subseteq R(C)_+$ is the set of fuzzy soft real numbers. A fuzzy soft real number is a fuzzy set on the set of all soft real set *F: E* →*B*(*R*) where *R* is set of real numbers, *B*(*R*) is the collection of non-empty bounded subset of *R* and *E* taken as set of parameters.. The general partial ordering on $\widetilde{C_h}$ define by $\tilde{a} \preceq \tilde{b} \Leftrightarrow \tilde{O}_{\tilde{C}} \preceq \tilde{b} - \tilde{a}$. From now on, $\tilde{C}$ + and $\widetilde{C'}$ will denote the Set $\{\tilde{a} \in \tilde{C} : \tilde{O}_{\tilde{C}} \preceq \tilde{a}\}$ and $\{\tilde{a} \in \tilde{C} : \tilde{a}\tilde{b} = \widetilde{ba^*}\}$, respectively.

Definition 1.1 ([3]): A Fuzzy set A in U is characterized by a function with domain as U and values in [0; 1]. The collection of all fuzzy set U is P $(\tilde{U})$.

Definition 1.2 ([1]): A pair (F; E) is called a soft set (over U) if and only if F is a mapping of E into the set of all sub set of the set U.

In other words, the soft set is a parametrized family of sub set of the set U. Every set F (ε), $\varepsilon \in$E, from this family may be considered as the set of ε −element of the soft set (F; E), or as the set of ε −approximate elements of the soft set.

Definition 1.3 ([4]): Let C ⊆ E then the map $F_E : C \to P(\tilde{U})$, defined by $F_E(e) = \mu^e F_E$ (a fuzzy sub set of U), is called fuzzy soft set over (U; E) where, $\mu^e F_E = \tilde{0}$ if e ∈ E - C and $\mu^e F_E \neq \tilde{0}$ if e ∈ C. The set of all fuzzy soft set over (U; E) is denoted by FS (U, E).

Definition 1.4 ([4]): The fuzzy soft set $F_\emptyset \in$ FS (U, E) is called null fuzzy soft set and it is denoted by Φ. Here $F_\emptyset(e) = \tilde{0}$ for every e ∈ E.

Definition 1.5 ([4]): Let $F_E \in$ FS (U, E) and F_E (e) = $\tilde{1}$ for all e ∈ E. Then F_E is called absolute fuzzy soft set and it is denoted by$\tilde{E}$.

2. Main Results

We begin this section by introducing the notion of lower semi continuity in the context of $C*$-algebra valued fuzzy soft metric space. And we proved that many of the known fixed point theorems can be deduced from caristi's mapping.

Definition 2.1: Let C ⊆ E and $\tilde{E}$.be the absolute fuzzy soft set. Let $\tilde{C}$ denote the C*-algebra. The C*-algebra valued fuzzy soft metric using fuzzy soft points is defined as a mapping $\widetilde{d_{C*}}: \tilde{E} \times \tilde{E} \to \tilde{C}$ satisfying the following conditions

(M0) $\tilde{O}_{\tilde{C}} \preceq \widetilde{d_{C^*}}(F_{e_1} ; F_{e_2})$ for all $F_{e_1}, F_{e_2} \in \tilde{E}$.
(M1) $\widetilde{d_{C^*}}(F_{e_1} ; F_{e_2}) = \tilde{O}_{\tilde{C}} \Leftrightarrow F_{e_1} = F_{e_2}$
(M2) $\widetilde{d_{C^*}}(F_{e_1} ; F_{e_2}) = \widetilde{d_{C^*}}(F_{e_2} ; F_{e_1})$
(M3) $\widetilde{d_{C^*}}(F_{e_1} ; F_{e_3}) \preceq \widetilde{d_{C^*}}(F_{e_1} ; F_{e_2}) + \widetilde{d_{C^*}}(F_{e_2}, F_{e_3})$ for all $F_{e_1}, F_{e_2}, F_{e_3} \in \tilde{E}$.
The fuzzy soft set $\tilde{E}$ with the C*-algebra valued fuzzy soft metric $\widetilde{d_{C^*}}$ is called the C*-algebra valued fuzzy soft metric space. It is denoted by $(\tilde{E}, \tilde{C}, \widetilde{d_{C^*}})$. It is obvious that C*-algebra valued fuzzy soft metric generalize the concept of fuzzy soft metric spaces, replacing the set of fuzzy soft real numbers by $\tilde{C}$+.

Definition 2.2: A sequence $\{F_{e_n}\}$ in a C*-algebra valued fuzzy soft metric space $(\tilde{E}, \tilde{C}, \widetilde{d_{C^*}})$ is said to converges to $F_{e'}$ in $\tilde{E}$ with respect to $\tilde{C}$. If $\| \widetilde{d_{C^*}}(F_{e_n}, F_{e'}) \| \overset{\|\|.\|_{\tilde{C}}}{\longrightarrow} \tilde{O}_{\tilde{C}}$ as n → ∞ that is for every $\tilde{O}_{\tilde{C}} \preceq \tilde{\epsilon}$ there exists $\tilde{O}_{\tilde{C}} \preceq \tilde{\delta}$ and a positive integer N = N($\tilde{\epsilon}$) such that $\| \widetilde{d_{C^*}}(F_{e_n}; F_{e'}) \| < \delta$ Implies that $\| \mu^a F_{e_n}(s) - \mu^a F_{e'}(s) \| < \tilde{\epsilon}$ whenever n ≥N. It is usually denoted as $\lim_{n\to\infty} F_{e_n} = F_{e'}$.

Definition 2.3: A sequence $\{F_{e_n}\}$ in a C*-algebra valued fuzzy soft metric space $(\tilde{E}, \tilde{C}, \widetilde{d_{C^*}})$ is said to be Cauchy sequence. If to every $\tilde{O}_{\tilde{C}} \preceq \tilde{\epsilon}$ there exists $\tilde{O}_{\tilde{C}} \preceq \tilde{\delta}$ and a positive integer N = N ($\tilde{\epsilon}$) such that $\| \widetilde{d_{C^*}}(F_{e_n}, F_{e_m}) \| < \delta$ implies $\| \mu^a F_{e_n}(s) - \mu^a F_{e_m}(s) \| < \tilde{\epsilon}$ whenever n, m ≥N. That is $\| \widetilde{d_{C^*}}(F_{e_n}; F_{e_m}) \| \overset{\|\|.\|_{\tilde{C}}}{\longrightarrow} \tilde{O}_{\tilde{C}}$ as n, m → ∞ .
If every Cauchy sequence is convergent in $\tilde{E}$, then $(\tilde{E}, \tilde{C}, \widetilde{d_{C^*}})$ is called a complete C*-algebra valued fuzzy soft metric space. It is obvious that any Banach space must be a complete C*--algebra valued fuzzy soft metric space. Moreover, C*-algebra valued fuzzy soft metric space generalized fuzzy soft normed linear spaces and fuzzy soft metric spaces.

Example 2.4: Let C ⊆R+ and E ⊆R+, let $\tilde{E}$ be absolute fuzzy soft set and $\tilde{C} = M_2$(R(C)+), define $\widetilde{d_{C^*}}: \tilde{E} \times \tilde{E} \to \tilde{C}$ by
$\widetilde{d_{C^*}}(F_{e_1} ; F_{e_2}) = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$ where
i = inf{| $\mu^a F_{e_1}(s)$ - $\mu^a F_{e_2}(s)$ | / s∈ C }. Then $(\tilde{E}, \tilde{C}, \widetilde{d_{C^*}})$ is complete C*-algebra valued fuzzy soft metric space by completeness of R.

Definition 2.5: Let $(\tilde{E}, \tilde{C}, \widetilde{d_{C^*}})$ be a C*--algebra valued fuzzy soft metric space. Let Γ: $\tilde{E} \to \tilde{C}$ be a mapping, we say that Γ is lower semi continuous at F_{e_0} with respect to $\tilde{C}$. If

$$\|\Gamma(F_{e_0})\| \leq \lim_{F_e \to F_{e_0}} inf \|\Gamma(F_e)\|$$

Example 2.6: Let E={e_1, e_2, e_3}, U={a, b, c, d} and C, D are two subsets of E, where C={e_1, e_2, e_3}, D={e_1, e_2},define fuzzy soft set as (F_E, C)=

$$\begin{Bmatrix} e_1 = \{a_{0.1}, b_{0.3}, c_{0.4}, d_{0.6}\}, \\ e_2 = \{a_{0.3}, b_{0.4}, c_{0.6}, d_{0.8}\}, \\ e_3 = \{a_{0.6}, b_{0.7}, c_{0.8}, d_{0.9}\} \end{Bmatrix}$$

(G_E, D)=

$$\begin{Bmatrix} e_1 = \{a_{0.4}, b_{0.5}, c_{0.2}, d_{0.6}\}, \\ e_2 = \{a_{0.5}, b_{0.6}, c_{0.3}, d_{0.7}\} \end{Bmatrix}$$

F_{e_1} =μF_{e_1}={$a_{0.1}, b_{0.3}, c_{0.4}, d_{0.6}$}, F_{e_2} =μF_{e_2}={$a_{0.3}, b_{0.4}, c_{0.6}, d_{0.8}$},

F_{e_3} =μF_{e_3}=$a_{0.6}, b_{0.7}, c_{0.8}, d_{0.9}$} and
G_{e_1} =μG_{e_1}={$a_{0.4}, b_{0.5}, c_{0.2}, d_{0.6}$}, G_{e_2} =μG_{e_2}={$a_{0.5}, b_{0.6}, c_{0.3}, d_{0.7}$}
Then FSC(F_E)={$F_{e_1}, F_{e_2}, F_{e_3}, G_{e_1}, G_{e_2}$}, let $\tilde{E}$ be a absolute fuzzy soft set. That $\tilde{E}(e) = \tilde{1}$ for all e∈E and $\tilde{C}$ = M_2(R(C)+) be the a C*--algebra with $\|(F_{e_1}, F_{e_2})\| = \sqrt{|F_{e_1}|^2 + |F_{e_2}|^2}$. Define an order ≤ on $\tilde{C}$ as follows $(F_{e_1}, G_{e_1}) \leq (F_{e_2}, G_{e_2}) \Leftrightarrow F_{e_1} \leq F_{e_2}, G_{e_1} \leq G_{e_2}$,
Where ≤ is the usual order on the element of **R**. It is easy to verify that ≤ is a partial order on $\tilde{C}$+. Consider $\widetilde{d_{C^*}}: \tilde{E} \times \tilde{E} \to \tilde{C}$ by
$\widetilde{d_{C^*}}(F_{e_1} ; F_{e_2}) = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$ where
i = inf{| $\mu^a F_{e_1}(s)$ - $\mu^a F_{e_2}(s)$ | / s ∈ C }. Then obviously, $(\tilde{E}, \tilde{C}, \widetilde{d_{C^*}})$ is complete C*-algebra valued fuzzy soft metric space. Define a mapping

$$\Gamma: \tilde{E} \to \tilde{C} \text{ by } \Gamma(F_e) = \begin{cases} \left(\frac{F_e(a)}{2},\ 0\right), & if\ F_e(a) \geq 0 \\ (1, 0), & Otherwise \end{cases}$$

Then it is easy to verify that Γ is lower semi continuous at $F_{e_0} = 0$.

Definition 2.7: Let $(\tilde{E}, \tilde{C}, \widetilde{d_{C^*}})$ be a C*--algebra valued fuzzy soft metric space. Let T: $\tilde{E} \to \tilde{E}$ be a self-mapping, we say that T is C*--algebra valued fuzzy soft contractive mapping on $\tilde{E}$. If there exists $\tilde{a} \in \tilde{C}$ with $\|\tilde{a}\|$<1 such that
$\widetilde{d_{C^*}}(TF_{e_1}; TF_{e_2}) \preceq \tilde{a}^* \widetilde{d_{C^*}}(F_{e_1}, F_{e_2})\tilde{a}$ for all $F_{e_1}, F_{e_2} \in \tilde{E}$

Theorem 2.8: Let $(\tilde{E}, \tilde{C}, \widetilde{d_{C^*}})$ be a complete C*--algebra valued fuzzy soft metric space and suppose Γ: $\tilde{E} \to \tilde{C}$ be lower semi continuous and bounded below function. Let the self-mapping T: $\tilde{E} \to \tilde{E}$ satisfies for all $F_{e_1} \in \tilde{E}$

$$\widetilde{d_{C^*}}(F_{e_1}; TF_{e_1}) \preceq \Gamma(F_{e_1}) - \Gamma(TF_{e_1}) \quad (1)$$

Then T has a fixed point in $\tilde{E}$.

Proof: for any $F_{e_1} \in \tilde{E}$, define the set valued mapping $\tilde{S}: \tilde{E} \to 2^{\tilde{E}}$ $(The\ power\ set\ of\ \tilde{E})$ by
$\tilde{S}(F_{e_1})=\{F_{e_2} \in \tilde{E} /\ \widetilde{d_{C^*}}(F_{e_1}; F_{e_2}) \preceq \Gamma(F_{e_1}) - \Gamma(F_{e_2})\}$.
Clearly, $F_{e_1} \in \tilde{S}(F_{e_1})$ and hence $\tilde{S}(F_{e_1}) \neq \emptyset$ for all $F_{e_1} \in \tilde{E}$.
We claim that for each $F_{e_2} \in \tilde{E}$, we have $\Gamma(F_{e_2}) \preceq \Gamma(F_{e_1})$ and $\tilde{S}(F_{e_2}) \subseteq \tilde{S}(F_{e_1})$.
Let $F_{e_2} \in \tilde{S}(F_{e_1})$ be given. Then $\widetilde{d_{C^*}}(F_{e_1}; F_{e_2}) \preceq \Gamma(F_{e_1}) - \Gamma(F_{e_2})$. So we have $\Gamma(F_{e_2}) \preceq \Gamma(F_{e_1})$. since $\tilde{S}(F_{e_2}) \neq \emptyset$, let $F_{e_3} \in \tilde{S}(F_{e_2})$. Thus, $\widetilde{d_{C^*}}(F_{e_2}; F_{e_3}) \preceq \Gamma(F_{e_2}) - \Gamma(F_{e_3})$. It follows that $\Gamma(F_{e_3}) \preceq \Gamma(F_{e_2}) \preceq \Gamma(F_{e_1})$. Hence

$$\widetilde{d_{C^*}}(F_{e_1}; F_{e_3}) \preceq \widetilde{d_{C^*}}(F_{e_1}; F_{e_2}) + \widetilde{d_{C^*}}(F_{e_2}; F_{e_3}) \preceq \Gamma(F_{e_1}) - \Gamma(F_{e_3})$$

So $F_{e_3} \in \tilde{S}(F_{e_1})$. Therefore, we proved $\tilde{S}(F_{e_2}) \subseteq \tilde{S}(F_{e_1})$. We shall construct a sequence $\{F_{e_n}\}$ in $\tilde{E}$ by induction, stating with any point $F_{e_1} \in \tilde{E}$. Suppose that $F_{e_n} \in \tilde{E}$ is known. Then choose $F_{e_{n+1}} \in \tilde{S}(F_{e_n})$ such that

$$\Gamma(F_{e_{n+1}}) \preceq \inf_{F_{e_3} \in \tilde{S}(F_{e_n})} \Gamma(F_{e_3}) + \frac{1}{n} \text{ for n} \in N \quad (2)$$

for n∈ N, $F_{e_{n+1}} \in \tilde{S}(F_{e_n})$, we have

$$\widetilde{d_{C^*}}(F_{e_n}; F_{e_{n+1}}) \preceq \Gamma(F_{e_n}) - \Gamma(F_{e_{n+1}})$$

Implies that $\| \widetilde{d_{C^*}}(F_{e_n}; F_{e_{n+1}}) \| \leq \|\Gamma(F_{e_n}) - \Gamma(F_{e_{n+1}})\|$ (3)
So $\Gamma(F_{e_{n+1}}) \preceq \Gamma(F_{e_n}) \Rightarrow \|\Gamma(F_{e_{n+1}}) \leq \| \leq \|\Gamma(F_{e_n})\|$ for each $n \in N$, since Γ is bounded below function

$$\boldsymbol{\eta} = \lim_{n\to\infty} \|\Gamma(F_{e_n})\| = \inf_{n \in N} \|\Gamma(F_{e_n})\| \quad (4)$$

For m>n with n, m∈ N by (3) and (4), we obtain

$$\widetilde{d_{C^*}}(F_{e_n}; F_{e_m}) \preceq \sum_{i=n}^{m-1} \widetilde{d_{C^*}}(F_{e_i}; F_{e_{i+1}}) \preceq \Gamma(F_{e_n}) - \eta$$

$\Rightarrow \| \widetilde{d_{C^*}}(F_{e_n}; F_{e_m}) \| \leq \sum_{i=n}^{m-1} \| \widetilde{d_{C^*}}(F_{e_i}; F_{e_{i+1}}) \| \leq \|\Gamma(F_{e_n})\| - \eta$
Since $\lim_{n\to\infty} \|\Gamma(F_{e_n})\| = \boldsymbol{\eta}$, we get

$\lim_{n\to\infty} sup\{\| \widetilde{d_{C^*}}(F_{e_n}; F_{e_m}) \| /\ m > n\}=0$
Hence $\{F_{e_n}\}$ is Cauchy sequence in $\tilde{E}$, by the completeness of $\tilde{E}$.

There exists $F_e' \in \tilde{E}$ such that $F_{e_n} \to F_e'$ as n→ ∞. Since Γ is lower semi continuous, by (4), for all k∈ N, we get

$$\|\Gamma(F_e')\| \leq \liminf_{n\to\infty}\|\Gamma(F_{e_n})\| = \inf_{n\in N}\|\Gamma(F_{e_n})\| \leq \|\Gamma(F_{e_k})\| \quad (5)$$

Next, we prove $\bigcap_{n=1}^{\infty}\tilde{S}(F_{e_n})=\{F_e'\}$.

For m>n with n, $m \in N$ by (3) and (5), we get that

$$\widetilde{d_{C*}}(F_{e_n};\ F_{e_m}) \leq \sum_{i=n}^{m-1} \widetilde{d_{C*}}(F_{e_i};\ F_{e_{i+1}}) \leq \Gamma(F_{e_n}) - \Gamma(F_e')$$

$$\Rightarrow \|\widetilde{d_{C*}}(F_{e_n};\ F_{e_m})\| \leq \sum_{i=n}^{m-1}\|\widetilde{d_{C*}}(F_{e_i};\ F_{e_{i+1}})\| \leq \|\Gamma(F_{e_n}) - \Gamma(F_e')\| \quad (6)$$

Since $F_{e_m} \to F_e'$ as m→ ∞. The inequality (6) implies

$$\|\widetilde{d_{C*}}(F_{e_n};\ F_e')\| \leq \|\Gamma(F_{e_n}) - \Gamma(F_e')\| \quad \text{For all n} \in N \quad (7)$$

By (7), we know $F_e' \in \bigcap_{n=1}^{\infty}\tilde{S}(F_{e_n})$. Hence $\bigcap_{n=1}^{\infty}\tilde{S}(F_{e_n}) \neq \emptyset$ and $\tilde{S}(F_e')\subseteq\bigcap_{n=1}^{\infty}\tilde{S}(F_{e_n})$.

For any $F_e'' \in \bigcap_{n=1}^{\infty}\tilde{S}(F_{e_n})$ by (2), we have

$$\widetilde{d_{C*}}(F_{e_n};\ F_e'') \leq \Gamma(F_{e_n}) - \Gamma(F_e'') \leq \Gamma(F_{e_n}) - \inf_{F_{e_3}\in\tilde{S}(F_{e_n})}\Gamma(F_{e_3}) \leq \Gamma(F_{e_n}) - \Gamma(F_{e_{n+1}}) + \frac{1}{n}$$

Therefore,

$$\|\widetilde{d_{C*}}(F_{e_n};\ F_e'')\| \leq \|\Gamma(F_{e_n}) - \Gamma(F_{e_{n+1}})\| + \frac{1}{n} \text{ for all n} \in N$$

Hence, $\lim_{n\to\infty}\|\widetilde{d_{C*}}(F_{e_n};\ F_e'')\|$=0, or equivalently, $F_{e_n} \to F_e''$ as n→ ∞. By the uniqueness of limit of a sequence, we have F_e'=F_e''.Hence we shown$\{F_e'\} = \bigcap_{n=1}^{\infty}\tilde{S}(F_{e_n})$.

Since $\tilde{S}(F_e') \neq \emptyset$ and $\tilde{S}(F_e')\subseteq\bigcap_{n=1}^{\infty}\tilde{S}(F_{e_n}) = \{F_e'\}$. We get $\tilde{S}(F_e')$=$\{F_e'\}$. On the other hand, by (1), we know T$F_e' \in \tilde{S}(F_e')$. Hence it must be TF_e'=F_e', that is T has a fixed point.

Example 2.9: Let E=C=[0, 1] and $\tilde{E}$ be a absolute fuzzy soft set, that $\tilde{E}$ (e) =$\tilde{1}$ for all e∈E and $\tilde{C}$ = M_2(R(C)+) be a C*-algebra with partial order as given in Example 2.6. Define$\widetilde{d_{C*}}$: $\tilde{E} \times \tilde{E} \to \tilde{C}$ by $\widetilde{d_{C*}}(F_{e_1};\ F_{e_2}) = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$ where$F_{e_1},\ F_{e_2} \in \tilde{E}$ and i = inf{| $\mu^a F_{e_1}(s)$ - $\mu^a F_{e_2}(s)$ | / s∈ C } and Γ: $\tilde{E} \to \widetilde{C_+}$ by $\Gamma(F_{e_1}) = \begin{bmatrix} F_{e_1} & 0 \\ 0 & F_{e_1} \end{bmatrix}$ be a continuous mapping and T: $\tilde{E} \to \tilde{E}$ be given as T$F_{e_1} = {F_{e_1}}^2$. Then it is easy to see that all the conditions of Theorem 2.8 are satisfied and T has a fixed point.

Corollary 2.1.1: Let $(\tilde{E}, \tilde{C}, \widetilde{d_{C*}})$ be a complete C*--algebra valued fuzzy soft metric space and suppose. Suppose the self-mapping T: $\tilde{E} \to \tilde{E}$ satisfies for all $F_{e_1}, F_{e_2} \in \tilde{E}$

$$\widetilde{d_{C*}}(F_{e_1}; F_{e_2}) \leq \Gamma(F_{e_1}, F_{e_2}) - \Gamma(\mathrm{T}F_{e_1}, TF_{e_2}) \quad (8)$$

Where Γ: $\tilde{E} \times \tilde{E} \to \tilde{C}$ is lower semi continuous function with respect to first variable. Then T has a unique fixed point in $\tilde{E}$.

Proof: for any$F_{e_1} \in \tilde{E}$, let we define F_{e_2}=,TF_{e_1} and $\Gamma(F_{e_1}) = \Gamma(F_{e_1}, T\ F_{e_1})$ then for each $F_{e_1} \in \tilde{E}$, we have $\widetilde{d_{C*}}(F_{e_1}; TF_{e_1}) \leq \Gamma(F_{e_1}) - \Gamma(\mathrm{T}F_{e_1})$, Since Γ is lower semi continuous function. Thus, we can applying Theorem 2.8 lead us to conclude the appropriate result.

To see the uniqueness of fixed point, SupposeF_e', F_e'' are two distinct fixed points of T. Then we have

$$\tilde{O}_{\tilde{C}} \leq \widetilde{d_{C*}}(F_e', F_e'') \leq \Gamma(F_e', F_e'') - \Gamma(TF_e', TF_e'') \leq \Gamma(F_e', F_e'') - \Gamma(F_e', F_e'') \leq \tilde{O}_{\tilde{C}}$$

Therefore, we get F_e'= F_e''.

Corollary 2.1.2: Let $(\tilde{E}, \tilde{C}, \widetilde{d_{C*}})$ be a complete C*--algebra valued fuzzy soft metric space and suppose. Suppose the self-mapping T: $\tilde{E} \to \tilde{E}$ satisfies for all $F_{e_1}, F_{e_2} \in \tilde{E}$

$$\widetilde{d_{C*}}(TF_{e_1}; TF_{e_2}) \leq \tilde{\alpha}^*\widetilde{d_{C*}}(F_{e_1},\ F_{e_2})\tilde{\alpha} \quad (9)$$

Where $\tilde{\alpha} \in \tilde{C}$ with $\|\tilde{\alpha}\|$<1. Then T has a unique fixed point in$\tilde{E}$.

Proof: Let $\Gamma(F_{e_1}, F_{e_2})$=$(I - \tilde{\alpha}\tilde{\alpha}^*)^{-1}\widetilde{d_{C*}}(F_{e_1},\ F_{e_2})$.
Then (9) show that

$$(I - \tilde{\alpha}\tilde{\alpha}^*)\ \widetilde{d_{C*}}(F_{e_1}, F_{e_2}) \leq \widetilde{d_{C*}}(F_{e_1}; F_{e_2}) - \widetilde{d_{C*}}(TF_{e_1}; TF_{e_2})$$

Which means

$$\widetilde{d_{C*}}(F_{e_1}, F_{e_2}) \leq (I - \tilde{\alpha}\tilde{\alpha}^*)^{-1}\left(\widetilde{d_{C*}}(F_{e_1}; F_{e_2}) - \widetilde{d_{C*}}(TF_{e_1}; TF_{e_2})\right)$$

So we have $\widetilde{d_{C*}}(F_{e_1}; F_{e_2}) \leq \Gamma(F_{e_1}, F_{e_2}) - \Gamma(\mathrm{T}F_{e_1}, TF_{e_2})$.
Therefore, by applying Corollary 2.1.1 one can conclude that **T** has a unique fixed point in $\tilde{E}$.

Corollary 2.1.3: Let $(\tilde{E}, \tilde{C}, \widetilde{d_{C*}})$ be a complete C*--algebra valued fuzzy soft metric space and suppose. Suppose the self-mapping T: $\tilde{E} \to \tilde{E}$ satisfies for all $F_{e_1}, F_{e_2} \in \tilde{E}$

$$\widetilde{d_{C*}}(F_{e_1}; F_{e_2}) \leq \Gamma(F_{e_1}, F_{e_2}) - \Gamma(F_{e_2}, F_{e_3}). \quad (10)$$

Where Γ: $\tilde{E} \times \tilde{E} \to \tilde{C}$ is lower semi continuous function with respect to first variable. Then T has a unique fixed point in$\tilde{E}$.

Proof: for each $F_{e_1} \in \tilde{E}$, let we define F_{e_2}=,TF_{e_1} and F_{e_3}=,TF_{e_2}=$T^2F_{e_1}$ and $\Gamma(F_{e_1}) = \Gamma(F_{e_1}, T\ F_{e_1})$. Then for each $F_{e_1} \in \tilde{E}$ $\widetilde{d_{C*}}(F_{e_1}; TF_{e_1}) \leq \Gamma(F_{e_1}) - \Gamma(\mathrm{T}F_{e_1})$. Since Γ is lower semi continuous function. Thus, we can applying Theorem 2.8 lead us to conclude the appropriate result.

3. Conclusion

In the present research, we have presented unique fixed point results on various Caristi's type contractive conditions defined on $c*$-algebra valued fuzzy soft metric spaces and provided suitable examples that supports our main results.

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