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Research paper

A mathematical view on dynamics of a prey predator exemplary with C-M functional reaction and diffusion

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Abstract

In this article, we considered an ecosystem with Crowley-Martin functional response. We verified mathematical peculiarities of the model like boundedness, and positive invariance. Analysis of local and global stabilities of the system is also verified. Hopf bifurcation analysis is also carried out by finding the nature of the periodic solution around interior steady state using Taylor's series. We also studied the diffusion analysis. Numerical simulations are carried out with the help of MATLB.

Keywords: *Prey-Predator; Stability; Diffusion; HOPF Bifurcation.*

1. Introduction:

To know the dynamical behavior of an ecological system, it is required to have many mathematical tools. For studying the relationships between prey and predator, mathematical tools are very useful. There are several renewable resources in the environment. Some of them are agriculture, forestry and fishery etc. Out of these, fishery is one important resource and which can be studied by many scientists by using modelling technique. The general prey predator model [1-3] is represented as $m'(t) = mh_1(m) - nh_2(m)$ and $n'(t) = qnh_1(m) - dn$ where m and n are densities of prey and predator populations. $h_1(m)$ is the per capita growth rate of the prey in the absence of predator; q, $(0 < q < 1)$ is a conversion coefficient and d is mortality rate of predator. $h_2(m)$ is the functional response. If we assume that the consumption rate of prey by a predator is proportional to the amount of prey available, then $h_2(m) = am$, this is nothing but the law of linear mass action. If we assume that predator's handling time for prey is zero in the law of linear mass action, then it has been observed that the feeding rate of predator increases with prey density and, in conclusion, it attains a saturated bound. This is type of interaction is described by the Holling type II functional response, which is represented by $h_2(m) = (am)/(b+m)$; $a > 0, b > 0$. If we assume that an increase of predator density then it implies that the decrease in feeding rate of predator due to mutual interference among individual of predators then the predators do not interfere among themselves. Crowley and Martin [4] anticipated a predator dependent functional response of the form $h_2(m,n) = (a_1 m)/(1 + am + bn + abmn)$, where a_1, a, b are positive parameters. The investigational remarks assume that decrease in feeding rate of predators per unit consumer is due to joint meddling among predators. This is a function of both the species due to predator intrusion. Therefore, the effects of predator interference on feeding rate remain important all the time whether an individual predator is handling or searching for a prey at a given instant of time [5]. Depending on a and b, we come across the following circumstances as : (i) when $a > 0$, $b = 0$, the Crowley–Martin type of functional response becomes Michaelis–Menten functional response;(ii) when $a = 0$, $b > 0$, which is a saturation response; and (iii) when $a = 0$, $b = 0$, the Crowley–Martin type of functional response becomes a linear mass action function response. Spatial summaries change the chronological dynamics and steadiness properties of populace attentiveness at a diversity of spatial scales, their consequences must be integrated in ecological duplications that do not indicate space unambiguously. And the spatial segment of bio collaborations has been renowned as a substantial feature in how ecological civilizations are moulded [6-7]. In a heterogeneous system, there is an identification that the effects of convective collaborations on the firmness of two species are deliberated [11], [8].The significance of diffusion on the spatiotemporal prey predator model studied by many authors [9], [10]. Recently, the significance of self and cross-diffusions in diffusion systems carried out by environmentalists. Balram Dubey et.al.[12], motivated us to do this present work.

2. Mathematical model formulation

We consider an ecosystem with Crowley-Martin functional response between prey and predator species. It is also assuming that the prey and predator species interacting themselves. Also we neglect the natural death rate of prey species. The following equations represents mathematical model of anticipated system.

$$\begin{cases} x'(t) = \alpha_0 x - \beta_0 x^2 - \dfrac{\beta_1 xy}{1 + a_1 x + b_1 y + a_1 b_1 xy} \\ y'(t) = \dfrac{\beta_2 xy}{1 + a_1 x + b_1 y + a_1 b_1 xy} - \beta_3 y^2 - my \end{cases} \quad (1)$$

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with initial conditions $x(0)=0, y(0)=0$ (2)

3. Positive invariance and boundedness

Viability of positivity studies that aim to quantitatively and rationally uncover the asset of the anticipated ideal in the given location. Biologically, positiveness assures that the population never become negative and population always survive. The subsequent theorems guarantee that the positivity and boundedness of the structure (1).

Theorem 1: All solutions of $(x(t), y(t))$ of the system (1) with the initial condition (2) is positive for all $t \geq 0$

Proof: From (1) it is observed that

$$\frac{dx}{x} = \left[(\alpha_0 - \beta_0 x) - \frac{\beta_1 y}{(1+a_1 x)(1+b_1 y)} \right] dt$$

$$= \phi_1(x, y)$$

where $\phi_1(x, y) = \left[(\alpha_0 - \beta_0 x\right] - \frac{\beta_1 y}{(1+a_1 x)(1+b_1 y)}$

Integrating in the region $[0, t]$ we get

$x(t) = x(0)\exp(\int \phi_1(x, y)dt) > 0$ for all t

From (1), it is observed that

$$\frac{dy}{y} = \left[\frac{\beta_2 x}{(1+a_1 x)(1+b_1 y)} - \beta_3 y - m \right] dt$$

$$= \phi_2(x, y)$$

where $\phi_2(x, y) =$

$$\left[\frac{\beta_2 x}{(1+a_1 x)(1+b_1 y)} - \beta_3 y - m \right]$$

Integrating in the region $[0, t]$ we get

$y(t) = y(0)\exp(\int \phi_2(x, y)dt) > 0$ for all t. Hence, all solutions starting from interior of the first octant (In R_+^2) remain positive in it for future time.

Theorem 2: All the non-negative solutions of the model system (1) that initiate in $\Re_+^2$ are uniformly bounded.

Proof: Let $(x(t), y(t))$ be any solution of the system (1)

Since, from (1) $\frac{dx}{dt} \leq x(\alpha_0 - \beta_0 x)$, we have

$$\lim_{t\to\infty} \sup x(t) \leq \frac{\alpha_0}{\beta_0}$$

Let $\xi = x + \frac{\beta_1}{\beta_2} y$. Then

$$\frac{d\xi}{dt} = \frac{dx}{dt} + \frac{\beta_1}{\beta_2}\frac{dy}{dt} \quad (*)$$

Substituting the equation (1) in equation (*), we get

$$\frac{d\xi}{dt} + \zeta\xi$$

$$= x(\alpha_0 - \beta_0 x) - \frac{\beta_1\beta_3}{\beta_2} y^2 - \frac{m\beta_1}{\beta_2} y + \zeta x + \frac{\zeta\beta_1}{\beta_2} y$$

$$\leq \mu$$

Applying Lemma on differential inequalities, we obtain

$0 \leq \xi(x, y) \leq (\mu/\zeta)\left(1 - e^{-\zeta t}\right) + \left(\xi(x(0), y(0))/e^{\zeta t}\right)$ and for $t \to \infty$ we have $0 \leq \xi(x, y) \leq (\mu/\zeta)$. Thus all solutions of system (1-2) enter into the region

$$\Gamma = \left\{ \begin{matrix} (x, y) \in R_+^2 : 0 \leq x \leq \frac{\alpha_0}{\beta_0}, \\ 0 \leq \xi \leq (\mu/\zeta) + \varepsilon, \forall \varepsilon > 0 \end{matrix} \right\}$$

4. Stability analysis

Our aim is check the stability of the system (1) at interior steady state point $A(x^{\$}, y^{\$})$ where $x^{\$}$ and $y^{\$}$ are the positive solutions of the system (1) with $x^* = x^*$ and

$$y^* = \frac{d + \sqrt{d^2 - 4x^*(\beta_0 - \alpha_0)}}{2\beta_3} \quad (3)$$

For y^* positive, it must be $\beta_0 < \alpha_0$ (4)

Whenever x^* is positive, y^* is positive provided equation (4) holds.

The Jacobean matrix of the given system is in the form of $J = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$, where

$$A = -\beta_0 x + \left[\frac{\beta_1 a_1 xy}{[(1+a_1 x)^2(1+b_1 y)]} \right]$$

$$B = -\frac{\beta_1 x\left[1 + a_1 x\right]}{\left[(1+a_1 x)(1+b_1 y)^2\right]}$$

$$C = -\frac{\beta_2 y}{\left[(1+a_1 x)^2(1+b_1 y)\right]}$$

$$D = -\beta_3 y - \left[\frac{\beta_2 b_1 xy}{[(1+a_1 x)(1+b_1 y)^2]} \right]$$

The characteristic equation of the Jacobean matrix of the given system (1) is in the form of

$$\lambda^2-(A+D)\lambda+(AD-BC)=0 \tag{5}$$

For locally asymptotic stable, it is required to have

$\lambda_1+\lambda_2=A+D<0$ and

$\lambda_1\lambda_2=AD-BC>0$, provided $\frac{\beta_2}{\beta_1}>\frac{a_1}{b_1}$ and $\frac{x}{y}>\frac{\beta_1}{\beta_2}$ (6)

Theorem 3: The steady state $A(x^{\$},y^{\$})$ is globally asymptotically stable provided $y>\frac{1}{\beta_1}$ and $\frac{\beta_2}{\beta_1}<\frac{(1+a_1x^*)}{(1+b_1y^*)}$

Proof: Let us consider a Lyapunov function

$$V=\left[x-x^*-x^*\ln\left(\frac{x}{x^*}\right)\right]+l_1\left[y-y^*-y^*\ln\left(\frac{y}{y^*}\right)\right]$$

So, $V'(t)$

$$=(x-x^*)\left[\alpha_0-\beta_0x-\frac{\beta_1y}{(1+a_1x)(1+b_1y)}\right]+l_1(y-y^*)\left[\frac{\beta_2x}{(1+a_1x)(1+b_1y)}-\beta_3y-d\right]$$

By choosing, $l_1=\frac{1}{\beta_2}$, $V'(t)<0$ provided $y>\frac{1}{\beta_1}$ (7)

and $\frac{\beta_2}{\beta_1}<\frac{(1+a_1x^*)}{(1+b_1y^*)}$ (8)

Hence the theorem.

5. Hopf-Bifurcation analysis:

In this section, One of the cases of interest is the existence of a Hopf bifurcation, this happens when an equilibrium changes its stability letting the existence of a limit cycle around it. The Hopf bifurcation occurs when the Jacobian matrix has at an equilibrium E^*, a pair of pure imaginary eigenvalues, i.e., $Tr\left(J\left(E^*\right)\right)=0$ and $\det\left(J\left(E^*\right)\right)>0$. Let $E^*=\left(x^*,y^*\right)$ an equilibrium of system, then its Jacobian matrix is given by

$$\mathrm{J}\left(x^*,y^*\right)=\begin{pmatrix}\mathrm{a}_{10} & \mathrm{a}_{01}\\ b_{10} & b_{01}\end{pmatrix} \tag{9}$$

where $a_{10}=-\beta_0x+\left[\frac{\beta_1a_1xy}{[(1+a_1x)^2(1+b_1y)]}\right]$

$$a_{01}=-\frac{\beta_1x\left[1+a_1x\right]}{\left[(1+a_1x)(1+b_1y)^2\right]}$$

$$b_{10}=-\frac{\beta_2y}{\left[(1+a_1x)^2(1+b_1y)\right]}$$

$$b_{11}=-\beta_3y-\left[\frac{\beta_2b_1xy}{[(1+a_1x)(1+b_1y)^2]}\right]$$

To obtain a pair of pure imaginary eigenvalues of $J\left(x^*,y^*\right)$ we ask for $\delta=A+D=\delta^H$ and δ^H-BC >0. To ensure the existence of Hopf- bifurcation we need to verify the condition $\frac{d\left(Tr\left(E^*\right)\right)}{d\left(\delta\right)}\mid\delta=\delta^H=-1\neq0$. In order to discuss the stability of the limit cycle, we use a change of co-ordinates $u=x-x^*$, $v=y-y^*$ to transform the system (1) into

$$\frac{du}{dt}=\alpha_0(u+x^*)-\beta_0((u+x^*)^2)-\frac{\beta_1(u+x^*)(v+y^*)}{(1+a_1(u+x^*))(1+b_1(v+y^*))} \tag{10}$$

$$\frac{dv}{dt}=\frac{\beta_2(u+x^*)(v+y^*)}{(1+a_1(u+x^*))(1+b_1(v+y^*))}-\beta_3(v+y^*)^2-m(v+y^*) \tag{11}$$

Using the Taylor Expansion around (0, 0) then the system above is re written as

$$\frac{du}{dt}=a_{10}u+a_{01}v+a_{20}u^2+a_{11}uv+a_{30}u^3+a_{21}u^2v+Q_1((x,y))$$

$$\frac{dv}{dt}=b_{10}u+b_{01}v+b_{20}u^2+b_{02}v^2+b_{11}uv+b_{30}u^3+b_{21}u^2v+b_{12}uv^{^2}+Q_2(x,y) \tag{12}$$

where a_{10}, a_{01}, b_{10}, b_{01} are given by the Jacobian matrix $J\left(E^*\right)$ in (9), Q_1,Q_2 are polynomials in x^i,y^j with $i+j\geq4$ and

$$a_{20}=-\beta_0+\frac{\beta_1a_1y}{(1+b_1y)}\left(\frac{1-a_1x}{\left(1+a_1x\right)^3}\right)$$

$$a_{21}=\frac{\beta_1a_1(1-a_1x)}{\left(1+a_1x\right)^3}\left[\frac{1}{\left(1+b_1y\right)^2}\right]$$

$$a_{30}=\frac{\beta_1 a_1 y}{1+b_1 y}\left[\frac{2a_1{}^2x-4a_1}{\left(1+a_1x\right)^4}\right]$$

$$a_{11}=\frac{\beta_1 a_1 x}{\left(1+a_1x\right)^2}\left[\frac{1}{\left(1+b_1y\right)^2}\right]$$

$$b_{11}=\frac{-\beta_2 y b_1}{\left(1+a_1x\right)^2\left(1+b_1y\right)^2}$$

$$b_{20}=\frac{2\beta_2 y a_1}{\left(1+a_1x\right)^3\left(1+b_1y\right)}$$

$$b_{30}=\frac{-6\beta_2 a_1{}^2 y}{1+b_1 y}\left[\frac{1}{\left(1+a_1x\right)^4}\right]$$

$$b_{02}=-\beta_3-\frac{\beta_2 b_1 x}{(1+a_1x)}\left(\frac{1-b_1y)}{(1+b_1y)^3}\right)$$

$$b_{12}=\frac{-\beta_2(1-b_1y)}{(1+b_1y)^3}\left[\frac{1}{\left(1+a_1x\right)^2}\right]$$

Therefore using matrix notation, system (12) can be expressed as

$$\begin{pmatrix}\frac{du}{dt}\\ \frac{dv}{dt}\end{pmatrix}=J\left(E^*\right)\begin{pmatrix}u\\ v\end{pmatrix}+L\left(u,v\right)$$

with

$$L=\begin{pmatrix}a_{20}u^2+a_{11}uv+a_{30}u^3+a_{21}u^2v+Q_1\left(x,y\right)\\ b_{20}u^2+b_{11}uv+b_{02}v^2+b_{30}u^3+b_{21}u^2v\\ +b_{12}uv^2+Q_2\left(x,y\right)\end{pmatrix}$$

At $\delta=\delta^H$, matrix $J\left(E^*\right)$ has a pair of pure imaginary eigenvalues, so $a_{10}=b_{01}$. Let $w=\sqrt{\det\left(J\left(E^*\right)\right)}>0$, we make the change of coordinates $u=Y_2 v,\ v=wY_1-\frac{\delta}{a_{12}}Y_2$ obtaining the following equivalent system

$$\begin{pmatrix}\frac{dY_1}{dt}\\ \frac{dY_2}{dt}\end{pmatrix}=\begin{pmatrix}0 & -w\\ w & 0\end{pmatrix}+\begin{pmatrix}F\left(Y_1,Y_2\right)+Q_3\\ G\left(Y_1,Y_2\right)+Q_3\end{pmatrix}$$

with Q_3,Q_4 functions in $Y_1^iY_2^j$ for $i+j\geq 4$ and

$$F=\left(-\frac{a_{21}\delta}{a_{01}}+a_{30}\right)Y_2^3+\frac{a_{21}Y_1Y_2^2}{a_{01}}+\left(a_{20}-\frac{a_{11}\delta}{a_{01}}\right)Y_2^2+\frac{a_{11}wY_1Y_2}{a_{01}} \tag{13}$$

$$G=\left(-\frac{b_{21}\delta}{a_{01}}+\frac{b_{12}\delta^2}{a_{01}^2}+b_{30}\right)Y_2^3+\left(\frac{b_{21}w}{a_{01}}-\frac{2b_{12}\delta\omega}{a_{01}^2}\right)Y_1Y_2^2+\left(-\frac{b_{11}}{a_{01}}+b_{20}+\frac{b_{02}\delta^2}{a_{01}^2}\right)Y_2^2+\frac{b_{12}\omega^2Y_1^2Y_2}{a_{01}^2}+\left(\frac{b_{11}w}{a_{01}}-\frac{2b_{02}\omega\delta}{a_{01}^2}\right)Y_1Y_2+\frac{b_{02}\omega^2Y_1^2}{a_{01}^2} \tag{14}$$

Using theorem from (5), we define the following coefficient

$$\eta:=\frac{a_{21}\omega}{8a_{01}}+\frac{b_{12}\omega^2}{8a_{01}^2}-\frac{3b_{21}\delta}{8a_{01}}+\frac{3b_{12}\delta^2}{8a_{01}^2}+\frac{3b_{30}}{8}+\frac{1}{16\omega}\left(\begin{array}{l}\frac{a_{11}\omega}{a_{01}}\left(2a_{20}-\frac{2a_{11}\delta}{a_{01}}\right)-\\ \left(\frac{b_{11}\omega}{b_{01}}-\frac{2b_{02}\delta\omega}{a_{01}^2}\right)\left(\begin{array}{l}\frac{2b_{02}\omega^2}{a_{01}^2}-\frac{2b_{11}\delta}{a_{01}}\\ +\frac{2b_{02}\delta^2}{a_{01}^2}+2b_{20}\end{array}\right)\end{array}\right)+\frac{1}{16\omega}\left(\left(2a_{20}-\frac{2a_{11}}{a_{01}}\right)\left(\begin{array}{l}-\frac{2b_{11}\delta}{a_{01}}+\frac{2b_{02}\delta^2}{a_{01}^2}\\ +2b_{20}\end{array}\right)\right) \tag{15}$$

Theorem 4:

Suppose the system (1) has an interior equilibrium point E^* which satisfy and assume $\eta\neq 0$ with η defined in (15), then system undergoes a Hopf bifurcation around E^*, which implies the existence of periodic solution on around E^*. Moreover, the periodic solution are stable cycle if $\eta>0$ and repelling if $\eta<0$

6. Mathematical model with diffusion: Discussion of diffusive instability

In 1952, Turing anticipated the diffusion concept on the foundation of famous renowned rules. Some ideas have been applied to the

concept of environmental system to comprehend the dynamics of eco dealings. For progress of environment with diffusion, it is assumed that the species are experiencing with spatial dispersion. Now a days, the study of prey predator systems with diffusion, is developing noticeably. In this, investigation of local and global steadiness of an ecological system is carried out under the impact of diffusion. One dimensional diffusive ecological system which is analogous to the structure (1) will be categorised by the system of partial differential equations.

$$\begin{cases} x'(t) = \alpha_0 x - \beta_0 x^2 - \dfrac{\beta_1 xy}{1 + a_1 x + b_1 y + a_1 b_1 xy} + D_1 \dfrac{\partial^2 x}{\partial s^2} \\ y'(t) = \dfrac{\beta_2 xy}{1 + a_1 x + b_1 y + a_1 b_1 xy} - \beta_3 y^2 - my + D_1 \dfrac{\partial^2 y}{\partial s^2} \end{cases} \tag{16}$$

$x_i(s,t), i = 1,2$ are the concentrations of prey and predator respectively at a position s at the time t ; D_1 , D_2 are prey predator diffusive coefficients. To analyse the diffusive instability, consider the effect of minor inhomogeneous perturbation of the common steady coexisting state E^* and the perturbations that are amplified by the combined force of reaction and diffusion in the system of Eq. (16) are considerable. Let $n_i(s,t), i = 1,2$ are the perturbations given to the general steady state equilibrium point E^* where n_1, n_2 adequately minor quantities are

$$\begin{aligned} x_1(s,t) &= x_1^* + n_1(s,t) \\ x_2(s,t) &= x_2^* + n_1(s,t) \end{aligned} \tag{17}$$

By substituting (17) in the system (16) we get

$$\begin{aligned} \frac{\partial n_1}{\partial t} &= c_{11} n_1 + c_{12} n_2 + D_1 \frac{\partial^2 n_1}{\partial s^2} \\ \frac{\partial n_2}{\partial t} &= c_{21} n_1 + c_{22} n_2 + D_2 \frac{\partial^2 n_2}{\partial s^2} \end{aligned} \tag{18}$$

where $c_{11} = \alpha_0 - 2\beta_0 x^* - \beta_1 y^* (1 + b_1 y^*)^{-1} (1 + a_1 x^*)^{-2}$

$c_{12} = -\beta_1 x^* (1 + a_1 x^*)^{-1} (1 + b_1 y^*)^{-2}$

$c_{21} = \beta_2 \, y^* (1 + b_1 y^*)^{-1} (1 + a_1 \, x^*)^{-2},$

$c_{22} = \beta_2 x^* (1 + a_1 x^*)^{-1} (1 + b_1 y^*)^{-2} - 2\beta_3 y^* - m$

Clearly, (18) are linear in time & space variables. Let the solution of (18) is in the form $n_i(s,t) = \alpha_i e^{\lambda_i t} \cos(Ms)$ where $i = 1,2$. λ_i is the growing rate of perturbation at time t ; a_1 , a_2 are the amplitudes at time t and $M = (m\pi)/d$. The characteristic equation is given by

$$\begin{aligned} &\lambda^2 + \lambda \left\{ \left(D_1^2 + D_1^2 \right) M^2 - \left(c_{11} + c_{22} \right) \right\} \\ &+ \left(c_{11} - M^2 D_1 \right) \left(c_{22} - M^2 D_2 \right) - c_{21} c_{12} = 0 \\ &\lambda^2 + \lambda T \left(M^2 \right) + G \left(M^2 \right) = 0 \end{aligned} \tag{19}$$

where $G(M^2) = (c_{11} - M^2 D_1)(c_{22} - M^2 D_2) - c_{21} c_{12}$;

$L(M^2) = (D_1^2 + D_1^2) M^2 - (c_{11} + c_{22})$

Theorem 5. The interior equilibrium point under the influence of diffusion is stable whenever $G(M^2) > 0$ and $L(M^2) > 0$ also stability holds. When $\Delta G(M^2) < 0 \ \forall \ M^2$, when $\Delta G(M^2) > 0 \ \forall \ M^2 \notin [M_\alpha^2, M_\beta^2]$, when $\Delta G(M^2) = 0 \ \forall \ M^2 \neq M_\gamma^2$ where M_α^2, M_β^2 are zeros of $G(M^2)$ and $G(M^2) = D_1 D_2 (M^2 - M_\gamma^2)^2$.

Proof: $G(M^2) > 0$ *and* $L(M^2) > 0$ imply that the eigenvalues of (13) are both negative and hence the interior equilibrium under the effect of diffusion is stable further discriminant of $G(M^2)$, $\Delta G(M^2)$ is negative and then $G(M^2)$ takes positive values for all values of M^2.

(i) When $\Delta G(M^2) > 0$, there exist two zeros of $\Delta(M^2)$ say M_α^2 and M_β^2 $(0 < M_\alpha^2 < M_\beta^2)$ and $G(M^2) \leq 0, M^2 \in [M_\alpha^2, M_\beta^2]$

(ii) when $\Delta G(M^2) = 0$ then there exist M_γ^2 such that $G(M^2) = D_1 D_2 (M^2 - M_\gamma^2)^2$ and $G(M^2) > 0 \ \forall \ M^2 \neq M_\gamma^2$

Theorem 6: When $G(M^2) < 0$ or when $G(M^2) > 0$ and $L(M^2) < 0$. The interior equilibrium point under the effect of diffusion is unstable.

Proof: When $\Delta G(M^2) < 0$ at least one of the eigenvalues of (19) is positive. When and $L(M^2) < 0$ 0 both the eigenvalues of (19) is positive

7. Numerical simulations and remarks:

Example (1): For the set of parameters

$a_1 = 0.5;\ b_1 = 0.9;\ \beta_1 = 12;\ \alpha_0 = 2.8;\ \beta_2 = 8.4;$

$\beta_0 = 0.001;\ \beta_3 = 0.24;\ m_1 = 0.0005$; with initial values 10, 2

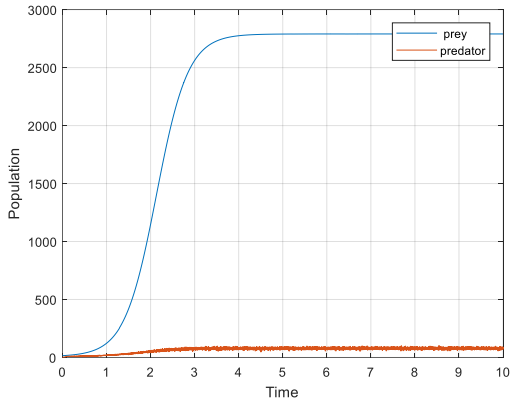


Figure-1

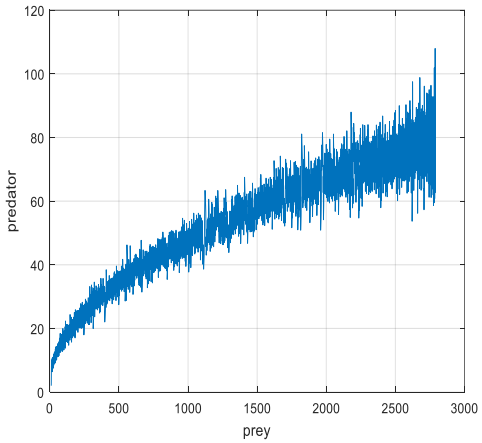


Figure-2

Example (2) For the set of parameters

$a_1 = 0.5;\ b_1 = 0.9;\ \beta_1 = 12;\ \alpha_0 = 2.8;\quad \beta_2 = 8.4;$

$\beta_0 = 0.001;\ \beta_3 = 0.24;\ m_1 = 0.0005;$ with initial values 15, 2

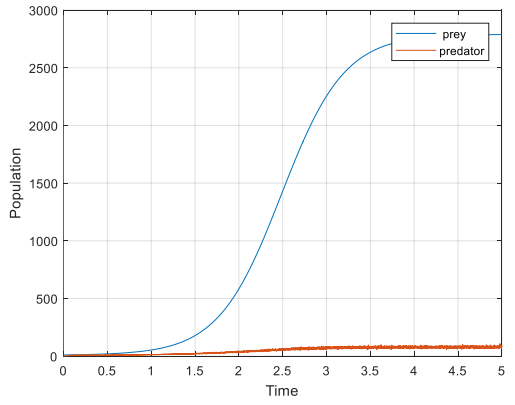


Figure-3

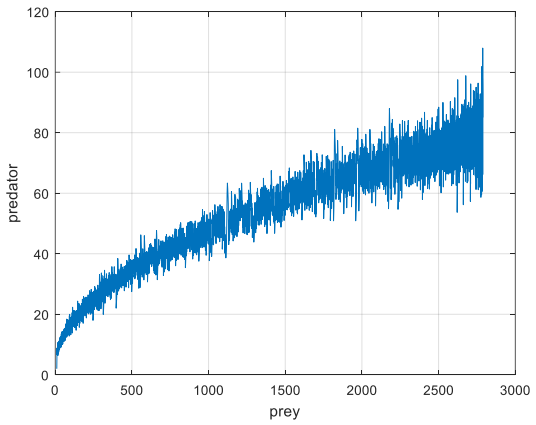


Figure-4

Example (3) For the set of parameters

$a_1 = 0.5;\ b_1 = 0.9;\ \beta_1 = 12;\ \alpha_0 = 2.8;\ \beta_2 = 8.4;$

$\beta_0 = 0.001;\quad \beta_3 = 0.24;\ m_1 = 0.0005;\ D_1 = 0.1;$

$D_2 = 0.2$ with initial values 15, 2

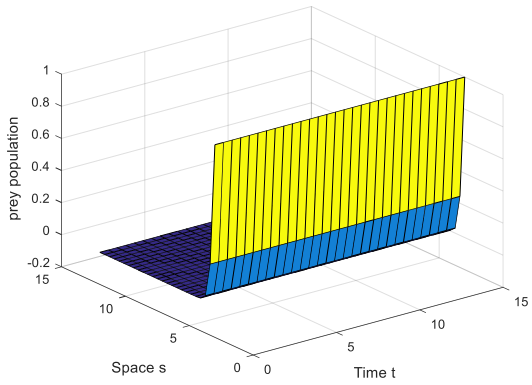


Figure-5

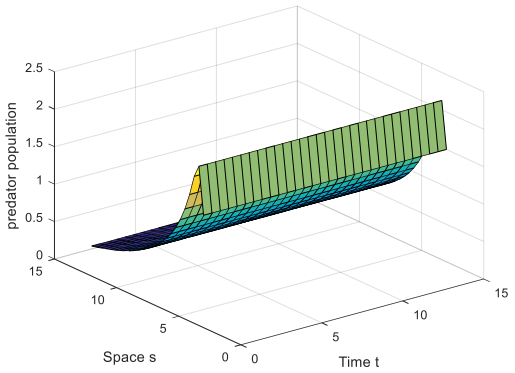


Figure-6

Example (4) For the set of parameters

$a_1 = 0.5;\ b_1 = 0.9;\ \beta_1 = 12;\ \alpha_0 = 2.8;\ \beta_2 = 8.4;$

$\beta_0 = 0.001;\ \ \beta_3 = 0.24;\ \ m_1 = 0.0005;$

$D_1 = 0.001; D_2 = 0.002$ with initial values 15, 2

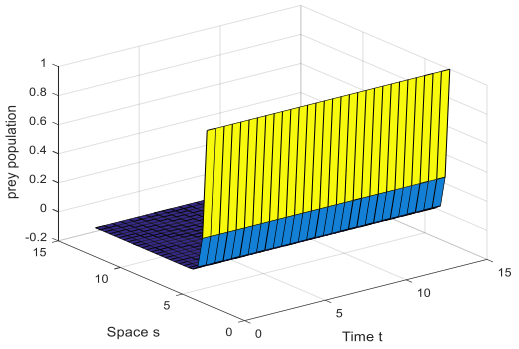


Figure-7

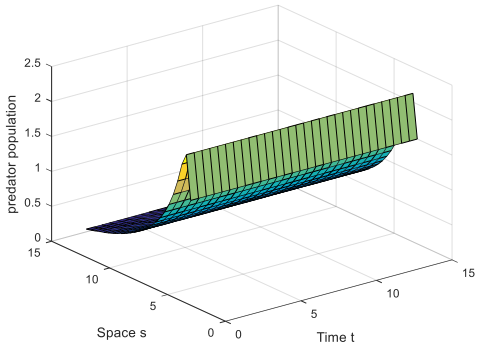


Figure-8

8. Observations/Remarks:

In this article, we proposed a C-M functional response oriented prey predator model. It is verified that the positive variance and boundedness of the system (1). Also, local and global stabilities are found with the help of R-H Criteria and a suitable Lyapunov functions respectively. It is also found that Hopf bifurcation occurs around interior steady state E^* using Taylor's series, whenever the existence of stable periodic solution around E^* provided the periodic solution is stable cycle if $\eta > 0$ and repelling if $\eta > 0$. It is observed that the figures (1-4) shows the variation of species against time and phase portrait diagrams for the parameters mentioned in examples 1 and 2. Again, from examples 3 and 4, it is see that the figures (5-8) shows that the variation of species against time and space variables with a suitable set of diffusion coefficients.

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