International Journal of Engineering & Technology, 7 (3.19) (2018) 85-91

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

Msthematical Model of Operational Competition

Eugeny P. Kolpak[1] , Ekaterina V. Gorynya[2]

Saint Petersburg State University, Faculty of Applied Mathematics and Control Process, Universitetskaya nab., 7/9, Saint Petersburg, Russian Federation

Abstract

They developed a mathematical model of operational competition on a linear range. The model is represented by an initial-boundary problem for the system of evolution equations. It is shown that the competition on the recovered trophic resource does not lead to the disappearance of one of the populations. The estimation of small population propagation rates in the range is given. They obtained the conditions for the existence of an autowave solution on an infinite straight line. The solution of nonlinear differential equations is based on numerical method use.

Keywords: *population, competition, waves, trophic resource, stability, differential equations.*

1. Introduction

The dynamic theory of populations in the second half of the twentieth century was formed as a scientific trend, bringing together biologists, ecologists and mathematicians. This is explained by the need to take into account a large number of factors in mathematical models affecting the life of populations [1]. This is birth and death, migration, the influence of neighboring populations, the presence of a trophic resource, anthropogenic pressure, etc. All processes are non-linear ones, characterized by threshold values, and by a large number of factors affecting them. Current mathematical models of population dynamics, as a rule, take into account only the main factors that determine the evolution of populations [2], [3], [4]. The main division of models is carried out by the nature of interaction between populations. Researchers are attracted most of all [4] by the interaction according to "predator-prey" and "parasite-master" principle. A significantly less interest is in the modeling of competition between populations. As a rule, interference and not operational competition is simulated [2].

The competition models originally developed for biological populations, were also used to model the interaction between economic actors [5], [6]. In population dynamics, a population is "a collection of individuals of a certain species, inhabiting a certain territory for a long time (of a large number of generations) within which any crossing is equally probable" [7]. The main characteristics of a population: number, density, fertility, etc. [2]. An economic entity with production funds and the personnel producing goods or services can also be regarded as a population. At that, its characteristics can be the volumes of products sold, the lifetime of a specific product, the speed of production area increase, etc. The models of interaction between subjects can be built on the same principles which are used in the interaction models of biological populations. The relations between economic entities are regulated by the agreements between them and state laws that limit an aggressive influence of different subjects on each other. Therefore, competition is the main type of modern relationships between economic subjects, which is taken into account in mathematical models.

2. Volterra Model

One of the first competition models was proposed by Volterra [8] as a model of species challenging the same food. For the case of two populations, the number of which N_1 and N_2, living in the same territory, feeding on the same food resource, is represented by a system of two differential equations

$$\begin{aligned} \frac{dN_1}{dt} &= N_1\left(a_1 - c_1F\left(N_1, N_2\right)\right), \\ \frac{dN_2}{dt} &= N_2\left(a_2 - c_2F\left(N_1, N_2\right)\right). \end{aligned} \tag{1}$$

in which the function $F = F\left(N_1, N_2\right)$ determines the rate of food "eating".

It follows from the first integral of this system of equations

$$\frac{N_1^{a_1}}{N_2^{a_2}} = \left.\frac{N_1^{a_1}}{N_2^{a_2}}\right|_{t=0} \exp\left(\frac{a_1}{c_1} - \frac{a_2}{c_2}\right)t$$

that one of the functions, depending on the sign of difference $\frac{a_1}{c_1} - \frac{a_2}{c_2}$ tends to zero, and the second one to infinity. That is, in this model, if it is extended to n species, the species that has the greatest value a_i / c_i ($i = 1,2,...,n$) survives, and the rest die [8]. Thus, the principle of competitive exclusion was formulated. In accordance with this principle, the number of species must be decreased, the energy needs for offspring reproduction of which are greater than among competing species. It is believed [3] that the experiments set by Gause in 1930-ies [9] confirm this

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principle. However, there are many examples in nature about a "peaceful" existence for different populations on the same trophic resource [2], [10]. Besides, there is the competition between individual groups of individuals within the population itself [2]. The accounting for intraspecific and interspecific competition was carried out in the modified Volterra model [2], [3]

$$\frac{dN_1}{d\tau} = N_1(c_1 - a_{11}N_1 - a_{12}N_2),$$
$$\frac{dN_2}{d\tau} = N_2(c_2 - a_{21}N_1 - a_{22}N_2). \quad (2)$$

In these equations c_1, c_2 are the parameters characterizing the own rates of population number change, a_{12} and a_{21} – the parameters determining the rate of population decline due to interspecific competition, and a_{11} and a_{22} – за счет внутривидовой конкуренции. due to intraspecific competition. A nontrivial fixed point of equation system that has a physical meaning,

$$N_1 = \frac{c_1}{a_{11}}(1-\gamma_1)/(1-\gamma_1\gamma_2)$$
$$N_2 = \frac{c_2}{a_{22}}(1-\gamma_2)/(1-\gamma_1\gamma_2)$$

where

$$\gamma_1 = \frac{c_2}{c_1}\frac{a_{12}}{a_{22}},\ \gamma_2 = \frac{c_1}{c_2}\frac{a_{21}}{a_{11}},$$

will be stable if the inequalities [2] $\gamma_1 < 1$ and $\gamma_2 < 1$ are performed. These inequalities can be satisfied simultaneously at small values of the parameters a_{21} and a_{12}, or at large values of the parameters a_{11} and a_{22}. That is, with a small level of interspecific competition in comparison with the intraspecific one ($a_{12} \ll a_{22}$, $a_{21} \ll a_{11}$).

At $a_{12} = 0$ и $a_{21} = 0$ the system of equations (2) is divided into two unrelated equations representing the logistic models of two independent populations. The logistic equation for one population in the "canonical" form has the following form

$$\frac{du}{dt} = \mu\left(1 - \frac{u}{K}\right),$$

where u - population number, μ - the Malthusian parameter, K - the capacity of the medium [1].

On the basis of the model (2) the following models were proposed: the competition of populations in the presence of a predator [11], the competition of populations with an ecological niche [12], the competition of cellular populations [13], the competition of populations on a mobile resource [14] and other models [3].

3. Trophic Resource

In order to take into account the influence of the resource S on the population amount, it should be considered that the rate of population individual reproduction depends on the resource. The rate of resource change is the sum of its replenishment rate and the rate of its consumption by the population

$$\frac{dS}{dt} = F(S) - \gamma u a(S),$$

where $F(S)$ - the function describes an "own" kinetics of the resource change, and $\gamma u a(S)$ describes the rate of its destruction by a population, γ - the coefficient. It is assumed that the rate of resource consumption is proportional to the population amount.

A large volume of trophic resource should not affect the population amount, and when it is absent the growth rate of the population makes zero. Therefore, the following conditions are imposed on the function $a(S)$:

$$0 < a(S) < const,$$
$$a(0) = 0,$$

and $a(S) \to K_s = const$ at $S \to \infty$.

One of the variants of this dependence is the hyperbolic dependence:

$$a(S) = \frac{S}{b+S},$$

where b is a positive constant.

In this case, the logistic population model is represented by a system of two differential equations [15]

$$\frac{du}{dt} = \mu u\left(\frac{S}{b+S} - u\right),$$
$$\frac{dS}{dt} = \mu_S S\left(1 - \frac{S}{K}\right) - \gamma\mu\frac{uS}{b+S}, \quad (3)$$

where γ - the parameter, μ_S - the indicator of resource exponential growth with its small number, K - the maximum possible amount of the resource. If the resource is not replenished, then it should be assumed that $\mu_S = 0$.

The stationary point $u = 0$, $S = 1$ of the equation system (3) is unstable, since the eigenvalues of the Jacobi matrix in the right-hand side of the equations at this point

$$\lambda_1 = \mu\frac{1}{b+1} \text{ и } \lambda_2 = \mu_S$$

will be positive.

A nontrivial stationary point (S,u) is the root of equation system

$$\mu_S\left(1-\frac{S}{K}\right)-\gamma\mu\frac{S}{(b+S)^2}=0,\qquad u=\frac{S}{b+S}.$$

The left-hand side of the first equation as the function of the argument S on the interval $[0,K]$ is a decreasing function and it has opposite signs at the ends of the interval. Therefore, this system of equations will have a unique positive root in this interval.

The Jacobi matrix on the right-hand side of the equation (3) at this stationary point

$$J=\begin{pmatrix} -\mu u & \mu u\dfrac{b}{(b+S)^2} \\ -\gamma\mu\dfrac{S}{b+S} & -S\left(\mu_S+\gamma\mu\dfrac{ub}{(b+S)^2}\right)\end{pmatrix}$$

has the eigenvalues with negative real part. Therefore, this stationary point is stable.

4. Two Populations on Trophic Resource

According to the models (1) and (2) it is considered that resource competition occurs at a direct contact of individuals. And the resource itself is not present clearly. The competition model in which the resource S is present follows from (3) by adding the equation for the second population:

$$\frac{du_1}{dt}=\mu_1u_1\left(\frac{S}{b_1+S}-\frac{u_1}{K_1}\right),$$

$$\frac{du_2}{dt}=\mu_2u_2\left(\frac{S}{b_2+S}-\frac{u_2}{K_2}\right),$$

$$\frac{dS}{dt}=-\gamma_1\frac{S}{b_1+S}u_1-\gamma_2\frac{S}{b_2+S}u_2+\mu_SS\left(1-\frac{S}{K}\right),\qquad(4)$$

where u_1 and u_2 is the number of populations, S is the volume of trophic resource, K_1, K_2 and K - the capacities of media populations and trophic resource, μ_1, μ_2, μ_S, γ_1, γ_2, b_1 and b_2 - parameters.

Functions

$$f_1(S)=\frac{S}{b_1+S}\ \text{и}\ f_2(S)=\frac{S}{b_2+S}$$

characterize the rate of consumption of a common trophic resource by populations.

The system of equations (4) has 5 stationary points

1. $u_1=0,\ u_2=0,\ S=0$

In this stationary point, the Jacobi matrix of the right-hand side of equations (4) has the following eigenvalues: $\lambda_1=0,\ \lambda_2=0,$ $\lambda_3=\mu_3$. Since one of the eigenvalues is positive, this stationary point will be unstable one.

2. $u_1=0,\ u_2=0,\ S=K$.

This stationary point is unstable, since two eigenvalues

$$\lambda_1=\mu_1\frac{K}{b_1+K}\ \text{и}\ \lambda_2=\mu_2\frac{K}{b_2+K}$$

of Jacobi matrix are positive ones.

3. $u_2=0,\ u_1=K_1\dfrac{S}{b_1+S}$, and S is the equation root

$$-\gamma_1K_1\frac{S}{(b_1+S)^2}+\mu_3(1-S/K)=0.$$

The left-hand side of this equation has opposite signs at $S=0$ and at $S=K$. Therefore, the solution of this equation exists on the interval $[0,K]$.

At this stationary point, one of the eigenvalues of the Jacobi matrix is positive one: $\lambda_2=\mu_2\dfrac{K}{b_2+K}$. And, accordingly, this stationary point is unstable.

4. $u_1=0,\ u_2=K_2\dfrac{S}{b_2+S}$,

and S is the equation root

$$-\gamma_2K_2\frac{S}{(b_2+S)^2}+\mu_3(1-S/K)=0.$$

As in the previous case, this stationary point will be unstable one.

5. $u_1=K_1\dfrac{S}{b_1+S},\ u_2=K_2\dfrac{S}{b_2+S}$,

and S is found as the root of the equation

$$f(S)=-\gamma_1K_1\frac{S}{(b_1+S)^2}-\gamma_2K_2\frac{S}{(b_2+S)^2}+\mu_3(1-S/K)=0.$$

At $S=0$ the function $f(S)$ is positive one, at $S=K$ it is negative one. Since it is monotonically decreasing, it will have the root on the interval $[0,K]$.

In a stationary position $\dfrac{u_1}{u_2}=\dfrac{K_1}{K_2}\dfrac{b_2+S}{b_1+S}$. Hence, it is possible to increase the stationary value u_1 by increasing the capacity of the medium (K_1) or by increasing the resource costs for population reproduction (b_1 decrease).

5. Two Populations on Linear Area

During the modeling of evolution populations on the territory, the equations "diffusion-reaction" are used [3]. Taking into account (4), this model represents the system of partial differential equations in private derivatives

$$\frac{\partial u_1}{\partial t} = D_1 \frac{\partial^2 u_1}{\partial x^2} + \mu_1 u_1 \left(\frac{S}{b_1 + S} - \frac{u_1}{K_1} \right),$$

$$\frac{\partial u_2}{\partial t} = D_2 \frac{\partial^2 u_2}{\partial x^2} + \mu_2 u_2 \left(\frac{S}{b_2 + S} - \frac{u_2}{K_2} \right),$$

$$\frac{\partial S}{\partial t} = -\gamma_1 \frac{S}{b_1 + S} u_1 - \gamma_2 \frac{S}{b_2 + S} u_2 + \mu_S S(1 - S/K), \quad (5)$$

where D_1 and D_2 - the parameters characterizing the mobility of individuals.

Two variants of the initial conditions are considered.

First: at $t = 0$

$$u_1(0,x) = u_1^0 \delta(x - x_0),$$
$$u_2(0,x) = u_2^0 \delta(x - x_0) \quad S(0,x) = S_0 = const, \quad (6)$$

where $\delta(x - x_0)$ – Dirac delta function.

These conditions imply that at the initial moment of time the populations appeared at the point $x = x_0$ simultaneously.

Second: at $t = 0$

$$u_1(0,x) = u_1^0(x),\ u_2(0,x) = u_2^0(x).$$

The conditions of medium filling [3] are considered as boundary conditions

$$\left.\frac{\partial u_1}{\partial x}\right|_{x=0} = \left.\frac{\partial u_1}{\partial x}\right|_{x=l} = 0, \quad \left.\frac{\partial u_2}{\partial x}\right|_{x=0} = \left.\frac{\partial u_2}{\partial x}\right|_{x=l} = 0,$$
$$\left.\frac{\partial S}{\partial x}\right|_{x=0} = \left.\frac{\partial S}{\partial x}\right|_{x=l} = 0. \quad (7)$$

For the case of an infinite line, it is assumed that at $x \to \pm\infty$ $S \to S_0$, and $u \to 0$.

The system of equations (5) is solved in the following way with the boundary conditions (7)

$$u_1 = 0,\ u_2 = 0 \text{ and } S = K, \quad (8)$$

if it is assumed that at the initial moment of time $u_1(0,x) = 0$, $u_2(0,x) = 0$ and $S(0,x) = K$. The disturbances of this solution $u_1 = \delta u_1(t,x)$, $u_2 = \delta u_2(t,x)$ and $S = K + \delta S(t,x)$, where $\delta u_1(t,x)$, $\delta u_2(t,x)$ and $\delta S(t,x)$ are small values:

$$\delta u_1(t,x) \ll \frac{K}{b_1 + K} K_1,$$

$$\delta u_2(t,x) \ll \frac{K}{b_2 + K} K_2, \left|\delta S(t,x)/K\right| \ll 1,$$

must satisfy the following system of equations in the linear approximation ((5))

$$\frac{\partial \delta u_1}{\partial t} = D_1 \frac{\partial^2 \delta u_1}{\partial x^2} + \mu_1 \frac{S_0}{b_1 + S_0} \delta u_1,$$
$$\frac{\partial \delta u_2}{\partial t} = D_2 \frac{\partial^2 \delta u_2}{\partial x^2} + \mu_2 \frac{S_0}{b_2 + S_0} \delta u_2. \quad (9)$$

The solution of the first equation in (9), satisfying the conditions ((8)) at $x = 0$ and $x = l$

$$\left.\frac{\partial \delta u_1}{\partial x}\right|_{x=0} = \left.\frac{\partial \delta u_1}{\partial x}\right|_{x=l},$$

is represented as a Fourier series

$$\delta u_1(x) = \sum_{k=0}^{\infty} A_k(t) \cos k\pi \frac{x}{l}.$$

The coefficients $A_k = A_k(t)$ must satisfy the following equations

$$\frac{dA_0}{dt} = \mu_1 \frac{S_0}{b_1 + S_0} A_0,$$
$$\frac{dA_k}{dt} = -D \left(\frac{k\pi}{l} \right)^2 A_k + \mu_1 \frac{S_0}{b_1 + S_0} A_k \qquad (k = 1,2,...). \quad (10)$$

The values $A_k(t = 0)$ are found from the following condition

$$\delta u_1^0(x) = \sum_{k=0}^{\infty} A_k(0) \cos k\pi \frac{x}{l},$$

where $\delta u_0(x) = \delta u(t = 0, x)$:

$$A_k(0) = \int_0^l \delta u_1^0(x) \cos k\pi \frac{x}{l} dx.$$

As follows from the first equation in (10) $A_0(t)$ will be an increasing function at any small positive value of $A_0(0)$. Similar results follow from the analysis of the second equation in (9). That is, the solution (8) will be unstable: the number of small competing populations should be increased.

For the case of the initial conditions (6) on an infinite straight line, the solution of equations (0) can be represented in the form [15]

$$\delta u_1 = \frac{u_1^0}{\sqrt{\pi D_1 t}} e^{\omega_1(t,x)},\ \delta u_2 = \frac{u_2^0}{\sqrt{\pi D_2 t}} e^{\omega_2(t,x)},$$

where

$$\omega_2(t,x) = \frac{1}{4D_1 t}\left(\sqrt{4\mu_1 D_1 \frac{S_0}{b_1+S_0}}t - x\right)\left(\sqrt{4\mu_1 D_1 \frac{S_0}{b_1+S_0}}t + x\right),$$

$$\omega_2(t,x) = \frac{1}{4D_2 t}\left(\sqrt{4\mu_2 D_2 \frac{S_0}{b_2+S_0}}t - x\right)\left(\sqrt{4\mu_2 D_2 \frac{S_0}{b_2+S_0}}t + x\right).$$

As follows from these expressions for $\omega_1(t,x)$ and $\omega_2(t,x)$, the number of both populations will be increased, and their propagation from the point $x=0$ on the straight line will occur with the following velocities in the first approximation

$$v_1 = \sqrt{4\mu_1 D_1 \frac{S_0}{b_1+S_0}} \text{ и } v_2 = \sqrt{4\mu_2 D_2 \frac{S_0}{b_2+S_0}}. \quad (11)$$

6. Autowave Solution

Nonlinear evolution equations with several stationary solutions can have autowave solutions [2], [16], [17]. For the case of the equation system (5), an autowave solution is sought in the form of functions of one argument $u_1 = u_1(x - v_1 t)$ and $u_2 = u_2(x - v_2 t)$.

For the case of a single population, such a solution must satisfy the following system of equations

$$\begin{aligned} D_1 \frac{d^2 u_1}{dz^2} + v\frac{du_1}{dz} + \mu_1 u_1\left(\frac{S}{b_1+S} - u_1\right) = 0, \\ v_1 \frac{dS}{dz} - \gamma_1 \frac{S}{b_1+S}u_1 + \mu_S S\left(1 - \frac{S}{K}\right) = 0. \end{aligned} \quad (12)$$

The solution of these equations must satisfy the following conditions [2], [15], [16]

$$\lim_{z\to\infty}\frac{du_1}{dz} = 0,\ \lim_{z\to\infty}\frac{dS}{dz} = S_0$$

and

$$\lim_{z\to-\infty}\frac{du_1}{dz} = u_1^*,\ \lim_{z\to-\infty}\frac{dS}{dz} = S^*,$$

where u_1^* and S^* are the roots of the equation system.

$$\gamma_1 \frac{1}{b_1+S^*}u_1^* + \mu_S\left(1 - \frac{S^*}{K}\right) = 0,$$

$$\frac{K}{b_1+K} - u_1^* = 0$$

These conditions mean that $u(z)$ and $S(z)$ should be increased at $z \to -\infty$, and at $z \to \infty$ the function $u(z)$ must be decreased, and $S(z)$ must be increased.

In the neighborhood of the stationary point u_1^* and S^* small excitations δu_1 and δS of the equations (12) should meet the following equations

$$\begin{aligned} D_1 \frac{d^2\delta u_1}{dz^2} + v\frac{d\delta u_1}{dz} - \mu_1 u_1^* \delta u_1 = 0, \\ v_1 \frac{d\delta S}{dz} - \gamma_1 \frac{S^*}{b_1+S^*}\delta u_1 + \gamma_1 \frac{u_1^* S^*}{\left(b_1+S^*\right)^2}\delta S + \mu_S \frac{S^*}{K}\delta S = 0. \end{aligned} \quad (13)$$

The roots of the characteristic polynomial of the first equation have opposite signs. Therefore, we can develop the solution in the neighborhood of this point, on which the function $u(z)$ is decreased, and as follows from the second equation in (13) – the function $S(z)$ is also decreased.

In the neighborhood of the point $u=0$, $S=K$ small disturbances δu_1 and δS of the equations (12) should meet the following equations:

$$\begin{aligned} D_1 \frac{d^2\delta u_1}{dz^2} + v\frac{du_1}{dz} + \mu_1 \frac{K}{b_1+K}\delta u_1 = 0, \\ v_1 \frac{d\delta S}{dz} - \gamma_1 \frac{S}{b_1+S}\delta u_1 - \mu_S \delta S = 0. \end{aligned} \quad (14)$$

The roots of the characteristic polynomial of the first equation

$$\lambda_{1,2} = \frac{1}{2D_1}\left(-v_1 \pm \sqrt{v_1^2 - 4D_1\mu_1\frac{K}{b_1+K}}\right)$$

will be real, and negative if the following inequality is fulfilled

$$v_1 > \sqrt{4D_1\mu_1\frac{K}{b_1+K}}. \quad (15)$$

In this case, in the neighborhood of the point $u=0$, $S=K$ one can develop a solution with the decreasing function $u(z)$, and, as follows from the second equation in (14), with the increasing function $S(z)$.

The rates of population propagation must be different on an infinite line as follows from (11) and (15). Then the autowave solution is represented in the form $u_1 = u_1(x - v_1 t)$, $u_2 = u_2(x - v_2 t)$, and, accordingly, the first two equations in (5) take the following form:

$$D_1\frac{d^2u_1}{dz_1^2}-v_1\frac{du_1}{dz_1}+\mu_1u_1\left(\frac{S}{b_1+S}-\frac{u_1}{K_1}\right)=0,$$

$$D_2\frac{d^2u_2}{dz_2^2}-v_2\frac{du_2}{dz_2}+\mu_2u_2\left(\frac{S}{b_2+S}-\frac{u_2}{K_2}\right)=0,$$

where $z_1=x-v_1t$, $z_2=x-v_2t$.

The solution of these equations must satisfy the following conditions.

1. At $z_{1,2}=-\infty$

$$u_1=K_1\frac{S}{b_1+S},\ u_2=K_2\frac{S}{b_2+S},$$

and S is found as the equation root

$$-\gamma_1K_1\frac{S}{(b_1+S)^2}-\gamma_2K_2\frac{S}{(b_2+S)^2}+\mu_3(1-S/K)=0$$

;

2. At $z_{1,2}=\infty$

$$u_1=0,\ u_2=0,\ S=K.$$

In the neighborhood of the first stationary point, the perturbations δu_1 and δu_2 satisfy the following equations

$$D_1\frac{d^2\delta u_1}{dz_1^2}+v_1\frac{d\delta u_1}{dz_1}-\mu_1u_1^*\delta u_1=0,$$

$$D_2\frac{d^2\delta u_2}{dz_2^2}+v_2\frac{d\delta u_2}{dz_2}-\mu_2u_2^*\delta u_2=0.$$

The characteristic polynomials of both equations have the roots of opposite signs, therefore one can construct a solution in a neighborhood of this stationary point on which the functions $u_1(z)$ and $u_2(z)$ will be decreased with the increase of $z_{1,2}$.

Let's assume that the speed of the first autowave is greater than the second one ($\mu_2D_2<\mu_1D_1$). Then, since the first population is ahead of the second one, the inequality (15) remains a necessary condition for the existence of an autowave in the area where it is alone.

The second population moves on a trophic resource S^* with a smaller volume than K, which is defined as the equation root

$$-\gamma_1K_1\frac{S}{(b_1+S)^2}+\mu_3(1-S/K)=0,$$

and, accordingly, for the second population, the autowave solution can exist if the following inequality is performed

$$v_2>2\sqrt{\mu_2D_2\frac{S^*}{b_2+S^*}}.$$

At that, since $S_1^*<S_0$, then $v_2<v_1$.

For the case of constants $\gamma=0.3$, $\mu_1=200$, $\mu_2=600$, $\mu_S=100$, $b_1=1.0$, $b_2=0.7$, $K_1=1$, $K_2=0.5$, $D_1=0.001$, $D_2=0.001$ the figure demonstrates the change of $u_1(x)$, $u_2(x)$ and $S(x)$ at the moment of time $t=0.4$ on the interval $[0,1]$. The following conditions were taken as the boundary ones:

at $x=0$: $u_1=K_1\frac{S_0}{b_1+K}$, $u_2=K_2\frac{S_0}{b_2+K}$, and the value S_0 was obtained as the root of the following equation

$$-\gamma_1K_1\frac{S}{(b_1+S)^2}-\gamma_2K_2\frac{S}{(b_2+S)^2}+\mu_3(1-S/K)=0$$

;

at $x=l$: $\frac{\partial u_1}{\partial x}=0$, $\frac{\partial u_2}{\partial x}=0$;

at $t=0$: $u_1=0$, $u_1=0$, $S=K$.

The "arrows" mark the direction of an autowave movement.

The solution was built in the programming environment of the mathematical package Matlab [18, 19], using the built-in function pdepe with the number of node points equal to 1500.

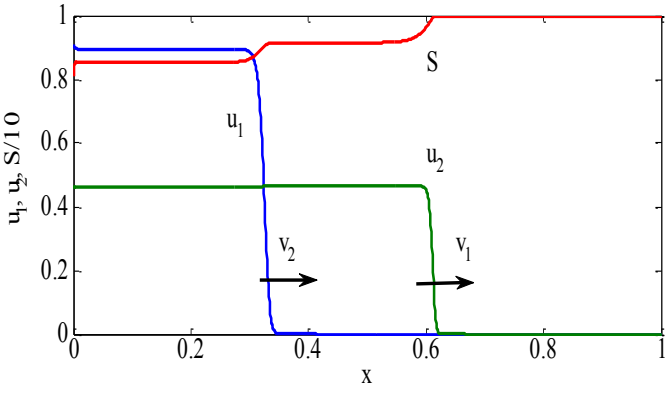

Fig. – Dependence graphs for the functions $u_1(x)$, $u_2(x)$ and $S(x)$ in the moment of time $t=0.4$ $\gamma=0.3$, $\mu_1=200$, $\mu_2=600$, $\mu_S=100$, $b_1=1.0$, $b_2=0.7$, $K_1=1$, $K_2=0.5$, $D_1=0.001$, $D_2=0.001$.

7. Summary

The model of operational conference allows the simultaneous existence of competing populations on a trophic resource. Unlike the Volterra model the largest population in this model is reached

by the population with a higher specific growth rate and with a greater consumption of trophic resource. The same conditions make it possible to outstrip competing populations in the development of new territories.

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