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Research paper

A New Hamiltonian Chaotic System with Coexisting Chaotic Orbits and its Dynamical Analysis

Sundarapandian Vaidyanathan[1], *Aceng Sambas[2], Sen Zhang[3], Mohamad Afendee Mohamed[4] and Mustafa Mamat[4]

[1]*Research and Development Centre, Vel Tech University, Avadi, Chennai, India*
[2]*Department of Mechanical Engineering, Universitas Muhammadiyah Tasikmalaya, Indonesia*
[3]*School of Physics and Opotoelectric Engineering, Xiangtan University, Hunan, China*
[4]*Faculty of Informatics and Computing, Universiti Sultan Zainal Abidin, Malaysia*
[*]*acengs@umtas.ac.id*

Abstract

Hamiltonian chaotic systems are conservative chaotic systems which arise in many applications in Classical Mechanics. A famous Hamiltonian chaotic system is the Hénon-Heiles system (1964), which was modeled by Hénon and Heiles, describing the nonlinear motion of a star around a galactic centre with the motion restricted to a plane. In this research work, by modifying the dynamics of the Hénon-Heiles system (1964), we obtain a new Hamiltonian chaotic system with coexisting chaotic orbits. We describe the dynamical properties of the new Hamiltonian chaotic system .

Keywords: *Chaos, Chaotic systems, conservative systems, Hamiltonian systems, Lyapunov exponents.*

1. Introduction

Chaotic systems have applications in several areas of science and engineering ([1]-[2]). Some common applications of chaos theory can be mentioned such as finance systems ([3]-[6]), weather models ([7]-[9]), biological models ([10]-[13]), neural networks ([14]-[16]), chemical reactions ([17]-[20]), ecological models ([21]-[23]), oscillations ([24]-[30]), jerk systems ([31]-[34]), encryption ([35]-[38]), robotics ([39]-[40]), circuits ([41]-[52]), etc.

The finding of conservative chaotic systems is an active research topic in the chaos literature [53]. The conservative systems have the special property that they are *volume-conserving* along their flow [53]. For a chaotic system, if the sum of the Lyapunov exponents of the system is zero, then the system is conservative [53]. Classical examples of conservative chaotic systems are Nosé-Hoover system [54], Hénon-Heiles system [55], etc. Some recent examples of conservative chaotic systems are Vaidyanathan-Volos system [56], Vaidyanathan systems ([57]-[61]), etc. Modelling and stability of systems is an important topic in control literature ([62]-[66]).

Hamiltonian systems have received good attention in the research on nonlinear dynamical systems ([67]- [71]). In the chaos literature, many conservative chaotic systems can be derived by means of the following Hamilton's equations:

$$\begin{cases} \dot{x} &= \frac{\partial H}{\partial v} \\ \dot{v} &= -\frac{\partial H}{\partial x} \\ \dot{y} &= \frac{\partial H}{\partial u} \\ \dot{u} &= -\frac{\partial H}{\partial y} \end{cases} \tag{1}$$

where the Hamiltonian function $H(v,x,u,y)$ does not depend explicitly on the time, t. Also, for mechanical systems, the Hamiltonian function H usually represents the total energy of the mechanical system, which is the sum of the kinetic and potential energies of the system.
The Hamiltonian system (1) is conservative as the energy H is conserved along the motion of the dynamical system (1). Indeed, a simple calculation shows that

$$\dot{H} = \frac{\partial H}{\partial v}\dot{v} + \frac{\partial H}{\partial x}\dot{x} + \frac{\partial H}{\partial u}\dot{u} + \frac{\partial H}{\partial y}\dot{y} \tag{2}$$

Substituting from (1) into Eq. (2), we get

$$\dot{H} = \dot{x}\dot{v} - \dot{x}\dot{v} + \dot{y}\dot{u} - \dot{y}\dot{u} = 0 \tag{3}$$

Thus, the energy H is a constant along the flow of the Hamiltonian system (1). Hence, the Hamiltonian system (1) is always conservative.
It is well-known that there is no attractor for a Hamiltonian chaotic system. Also, for a Hamiltonian chaotic system, the flow of the system is time-reversible and incompressible. In addition, for a

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Hamiltonian chaotic system, the Lyapunov exponents occur as equal and opposite pairs with their sum equal to zero.
In this research work, we report the finding of a new Hamiltonian chaotic system, which is obtained by modifying the dynamics of the Hénon-Heiles chaotic system [55]. Our numerical simulations show that the two conservative chaotic systems have different phase plots. We describe the dynamical analysis of the new Hamiltonian chaotic system. A study of route to chaos helps to understand the complex properties of chaotic systems ([72]-[74]).
The organization structure of this paper is as follows. Section 2 describes the dynamics and phase plots of the Hénon-Heiles chaotic system [55]. Section 3 describes the modelling of the new Hamiltonian chaotic system. Section 4 pinpoints the dynamic properties of the new Hamiltonian chaotic system. Section 5 draws the main conclusions of the work.

2. Hénon-Heiles chaotic system

In this section, we discuss in detail the Hénon-Heiles system [55], which describes the nonlinear motion of a star around a galactic centre with the motion restricted to a plane.
The Hamiltonian function describing the Hénon-Heiles system is described as follows.

$$H(v,x,u,y) = \frac{1}{2}(v^2+u^2+x^2+y^2)+x^2y-\frac{1}{3}y^3 \tag{4}$$

Hénon-Heiles system [55] is described by the Hamilton's equations associated with the Hamiltonian function H defined by Eq. (4).
Thus, we obtain the Hénon-Heiles system as follows:

$$\begin{cases} \dot{x} = \frac{\partial H}{\partial v} = v \\ \dot{v} = -\frac{\partial H}{\partial x} = -x-2xy \\ \dot{y} = \frac{\partial H}{\partial u} = u \\ \dot{u} = -\frac{\partial H}{\partial y} = -y-x^2+y^2 \end{cases} \tag{5}$$

Equivalently, we can also express the system (5) as a set of two second-order ordinary differential equations in x and y as follows:

$$\begin{cases} \ddot{x} = -x-2xy \\ \ddot{y} = -y-x^2+y^2 \end{cases} \tag{6}$$

For the ease of understanding the properties of the Hénon-Heiles system, we consider it as a system of first-order differential equations given by Eq. (5).
The Hénon-Heiles system (5) is a conservative system since it is a Hamiltonian system.
Also, the Hénon-Heiles system (5) is a chaotic system which can be seen as follows.
We set $X = (x,v,y,u)$.
When $X(0) = (0.2,0,-0.2,0)$, the Lyapunov exponents of the Hénon-Heiles system (5) are calculated using Wolf's algorithm [75] for $T = 4000$ seconds as

$$LE_1 = 0.0012,\ \ LE_2 = 0,\ \ LE_3 = 0,\ \ LE_4 = -0.0012 \tag{7}$$

The Kaplan-Yorke dimension of the Hénon-Heiles system (5) is calculated as

$$D_{KY} = 3+\frac{LE_1+LE_2+LE_3}{|LE_4|} = 4 \tag{8}$$

The high value of D_{KY} shows the high complexity of the Hénon-Heiles system (5).
Figures 1-4 show the 2-D plots of the Hénon-Heiles system (5) for $X(0) = (0.2,0,-0.2,0)$.

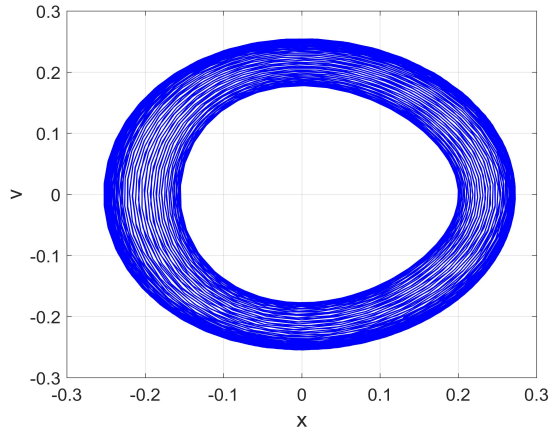


Figure 1: 2-D phase plot of the Hénon-Heiles system (5) in the (x,v) plane for $X(0) = (0.2,0,-0.2,0)$

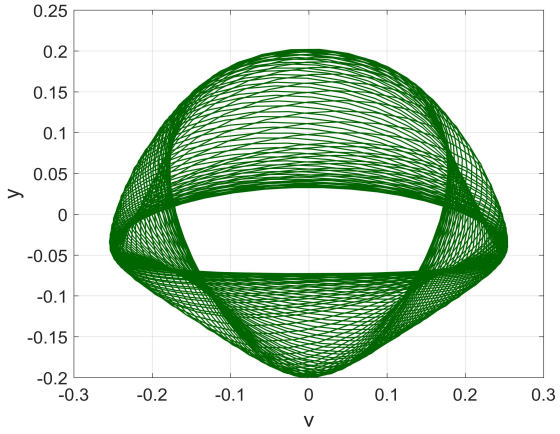


Figure 2: 2-D phase plot of the Hénon-Heiles system (5) in the (v,y) plane for $X(0) = (0.2,0,-0.2,0)$

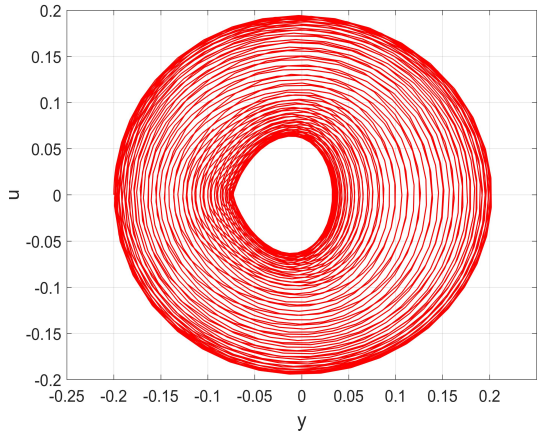


Figure 3: 2-D phase plot of the Hénon-Heiles system (5) in the (y,u) plane for $X(0) = (0.2,0,-0.2,0)$

3. A new Hamiltonian chaotic system

In this section, we introduce a new Hamiltonian conservative chaotic system.
For this purpose, we modify the Hamiltonian function (4) defining the Hénon-Heiles system (5) as follows:

$$H(v,x,u,y) = \frac{1}{2}(v^2+u^2+x^2+y^2)+x^2y+axy^2-\frac{1}{3}y^3-\frac{b}{5}y^5 \tag{9}$$

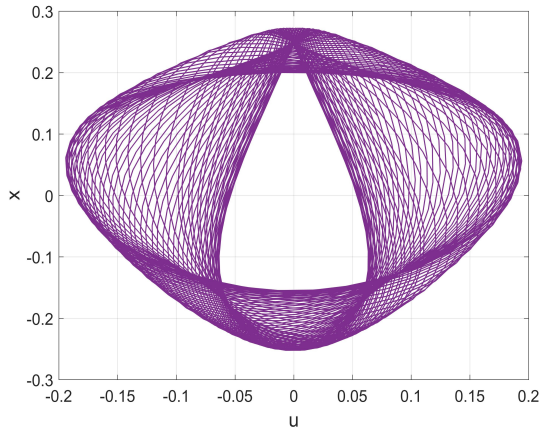


Figure 4: 2-D phase plot of the Hénon-Heiles system (5) in the (u,x) plane for $X(0) = (0.2, 0, -0.2, 0)$

where a and b are constant parameters.
Comparing the Hamiltionian functions (4) and (9), we note that we have obtained the new Hamiltonian function (9) by adding a cubic nonlinearity (ax^2y) and a quintic nonlinearity, ($-\frac{b}{5}y^5$).
When $a = b = 0$, the new Hamiltonian function (9) reduces to the Hamiltonian function (4) .
The Hamliton's equations associated with the new Hamiltonian function (9) are described by the following 4-D system of first-order ordinary differential equations:

$$\begin{cases} \dot{x} = \frac{\partial H}{\partial v} = v \\ \dot{v} = -\frac{\partial H}{\partial x} = -x - 2xy + ax^2 \\ \dot{y} = \frac{\partial H}{\partial u} = u \\ \dot{u} = -\frac{\partial H}{\partial y} = -y - x^2 + y^2 + by^4 \end{cases} \tag{10}$$

Equivalently, we can also express the system (10) as a set of two second-order ordinary differential equations in x and y as follows:

$$\begin{cases} \ddot{x} = -x - 2xy \\ \ddot{y} = -y - x^2 + y^2 \end{cases} \tag{11}$$

For the ease of understanding the properties of the new Hamiltonian system, we consider it as a system of first-order differential equations given by Eq. (10).
Using Wolf's algorithm [75], it can be easily seen that the new Hamiltonian dynamical system (10) is chaotic for $a \in [-2, 2]$ and $b \in [-2, 2]$. In particular, when we take $(a,b) = (1,1)$ and $X(0) = (0.2, 0, -0.2, 0)$, we obtain the Lyapunov exponents of the new Hamiltonian system (10) using Wolf's algorithm for $T = 4000$ seconds as

$$LE_1 = 0.0015, \ LE_2 = 0, \ LE_3 = 0, \ LE_4 = -0.0015 \tag{12}$$

Eq. (12) shows that the new Hamiltonian system (10) is both chaotic and conservative. Indeed, the Lyapunov exponents of the new Hamiltonian system (10) occur in opposite pairs and their sum is equal to zero.
As a consequence, the Kaplan-Yorke dimension of the new Hamiltonian chaotic system (10) is calculated as

$$D_{KY} = 3 + \frac{LE_1 + Le_2 + LE_3}{|LE_4|} = 4 \tag{13}$$

The high value of D_{KY} shows the high complexity of the new Hamiltonian chaotic system (10).

Figures 5-8 show the 2-D plots of the new Hamiltonian chaotic system (10) for $(a,b) = (1,1)$ and $X(0) = (0.2, 0, -0.2, 0)$.
It is remarked that the 2-D phase plots of the new Hamiltonian chaotic system (10) are different from the 2-D phase plots of the Hénon-Heiles system (5) for the same initial state, $X(0) = (0.2, 0, -0.2, 0)$.

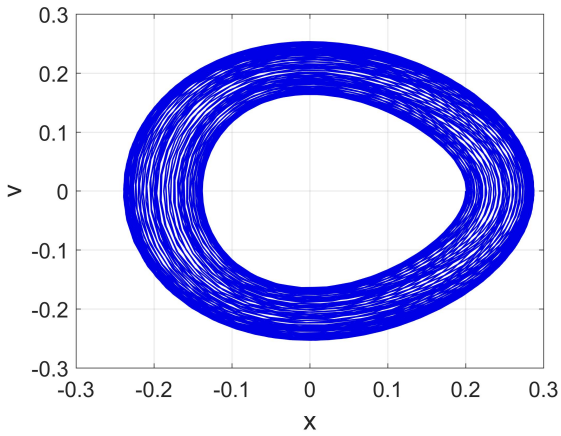


Figure 5: 2-D phase plot of the new Hamiltonian chaotic system (10) in the (x,v) plane for $(a,b) = (1,1)$ and $X(0) = (0.2, 0, -0.2, 0)$

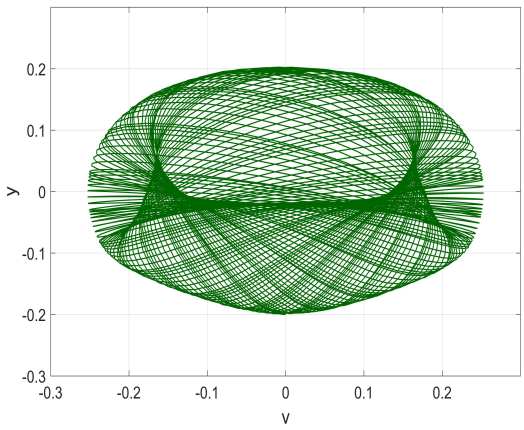


Figure 6: 2-D phase plot of the new Hamiltonian chaotic system (10) in the (v,y) plane for $(a,b) = (1,1)$ and $X(0) = (0.2, 0, -0.2, 0)$

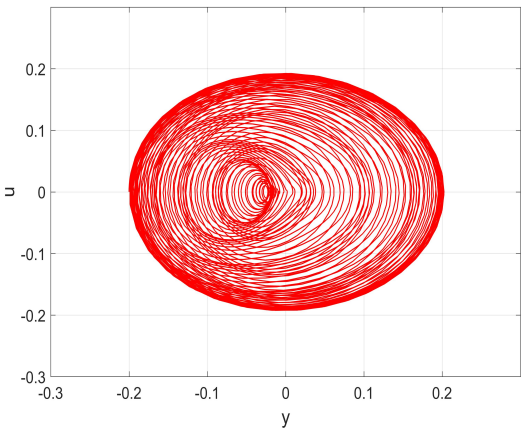


Figure 7: 2-D phase plot of the new Hamiltonian chaotic system (10) in the (y,u) plane for $(a,b) = (1,1)$ and $X(0) = (0.2, 0, -0.2, 0)$

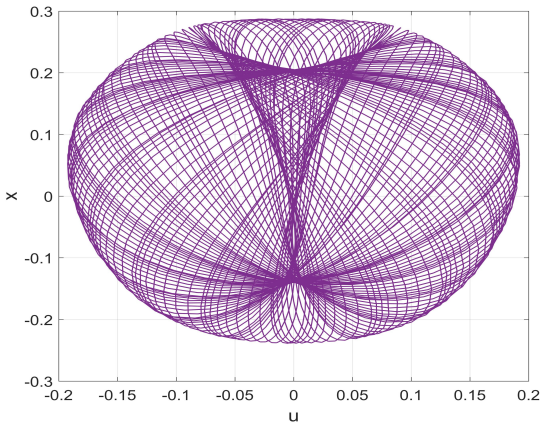


Figure 8: 2-D phase plot of the new Hamiltonian chaotic system (10) in the (u,x) plane for $(a,b) = (1,1)$ and $X(0) = (0.2,0,-0.2,0)$

4. Dynamical Analysis of the new Hamiltonian chaotic system

It is well-known that a conservative chaotic system has no strange attractor [53]. In fact, a chaotic attractor is named *strange* due to its fractional dimension. The fractional dimension of a conservative chaotic system is an integer and hence it does not exhibit any strange attractor. Some authors call the phase plots of a conservative chaotic system as *chaotic sea* [53]. Here, we use *chaotic orbits* to call the phase plots of a conservative chaotic system. In addition, as can be seen from the diagrams of bifurcation and the Lyapunov exponents, there is no classical route to chaos such as forward period-doubling route or reverse period-doubling route. From the coexisting bifurcation model, we can see that the new Hamiltonian chaotic system (10) is in quasi-periodic state at first and then it goes into chaos.

First, we describe the bifurcation diagrams and Lyapunov exponents. Fix $b = 1$, initial condition $X(0) = (0.2,0,-0.2,0)$ and vary a in the region of $[0,2]$. The corresponding bifurcation diagram and Lyapunov exponents for the new Hamiltonian chaotic system (10) is shown in Figure 9.

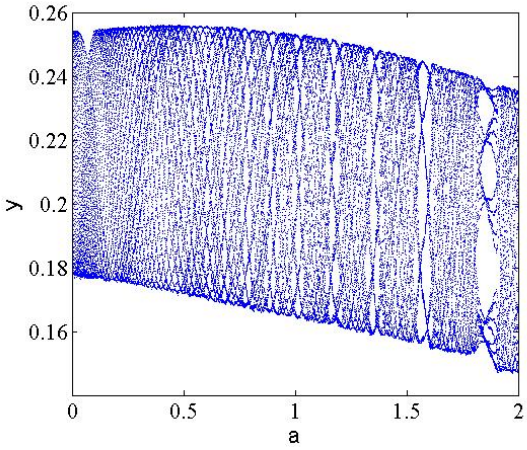
Fix $a = 1$, initial condition $X(0) = (0.2,0,-0.2,0)$ and vary b in the region of $[0,2]$. The corresponding bifurcation diagram and Lyapunov exponents for the new Hamiltonian chaotic system (10) is shown in Figure 10.

Figure 11 shows the coexisting bifurcation diagram where we fix $b = 1$ and vary a in the region of $[0,2]$. We note that the blue color shows the orbit of the new system (10) starting from the initial condition $X(0) = (0.2,0.2,0.2,0)$ and the red color shows the orbit of the new system (10) starting from the initial condition $Y(0) = (0.2,-0.2,0.2,0)$.

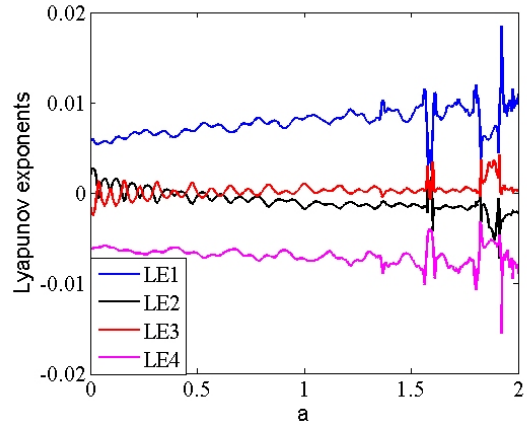
Figures 12-14 depict the coexisting orbits for the new Hamiltonian system (10) for various values of a and b. Here, we note that blue color shows the orbit of (10) starting from the initial condition $X(0) = (0.2,0.2,0.2,0)$ and the red color shows the orbit of (10) starting from the initial condition $Y(0) = (0.2,-0.2,0.2,0)$.

5. Conclusion

Hamiltonian chaotic systems form an important class of mechanical conservative chaotic systems. A famous Hamiltonian chaotic system is the Hénon-Heiles system (1964), which was modeled by Hénon and Heiles, describing the nonlinear motion of a star around a galactic centre with the motion restricted to a plane. By modifying the dynamics of the Hénon-Heiles system (1964), we have derived a new Hamiltonian chaotic system. We have described the chaotic



(a)



(b)

Figure 9: (a) Bifurcation diagram of the new system (10) versus the parameter a for $b = 1$ and initial condition $X(0) = (0.2,0,-0.2,0)$ and (b) Lyapunov exponent spectrum of the new system (10) when varying the parameter a for $b = 1$ and the initial condition $X(0) = (0.2,0,-0.2,0)$

orbits of the new Hamiltonian chaotic system and found its Lyapunov chaos exponents. We analyzed the dynamical properties of the new Hamiltonian chaotic system with bifurcation diagrams, Lyapunov exponents and coexisting chaotic orbits.

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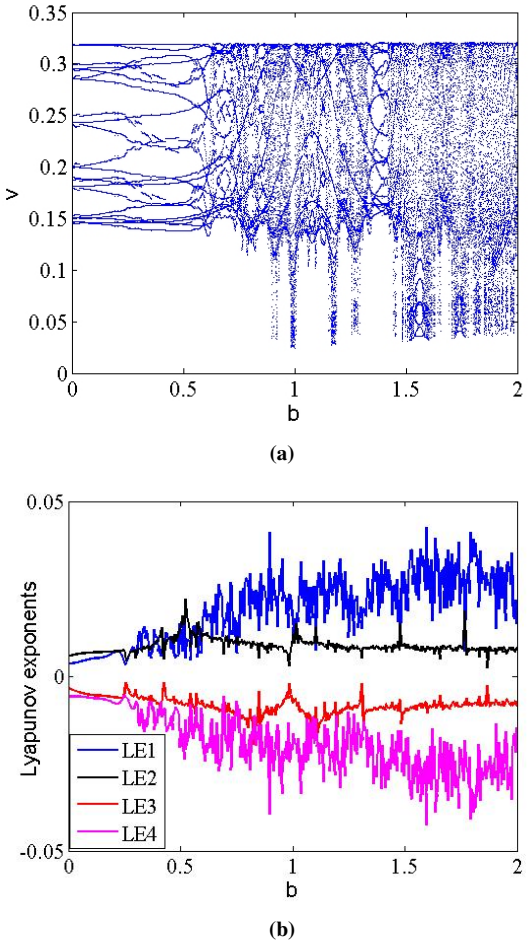
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(a)

(b)

Figure 10: (a) Bifurcation diagram of the new system (10) versus the parameter b for $a = 1$ and initial condition $X(0) = (0.2, 0, -0.2, 0)$ and (b) Lyapunov exponent spectrum of the new system (10) when varying the parameter b for $a = 1$ and the initial condition $X(0) = (0.2, 0, -0.2, 0)$

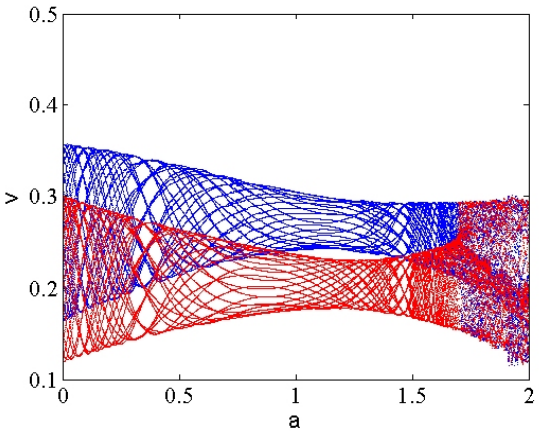
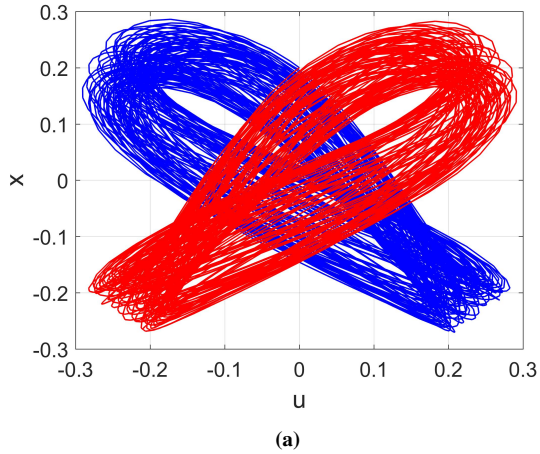
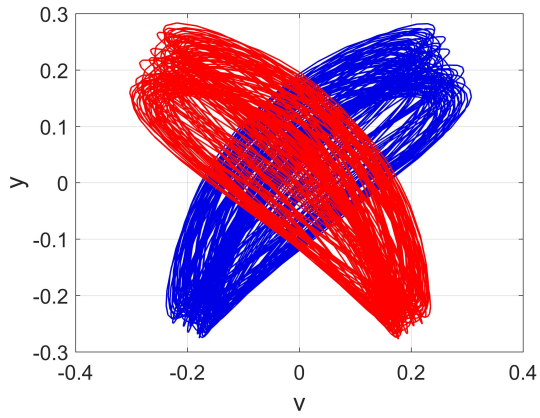


Figure 11: Coexisting bifurcation diagram of the new Hamiltonian chaotic system (10) where we fix $b = 1$, vary a in the region of $[0, 2]$. (Blue color shows the orbit of (10) with $X(0) = (0.2, 0.2, 0.2, 0)$ and the red color shows the orbit of (10) with $Y(0) = (0.2, -0.2, 0.2, 0)$.)



(a)



(b)

Figure 12: Coexisting quasi-periodic orbits for $a = 1$ and $b = 1$. Blue color shows the orbit of (10) with $X(0) = (0.2, 0.2, 0.2, 0)$ and the red color shows the orbit of (10) with $Y(0) = (0.2, -0.2, 0.2, 0)$.

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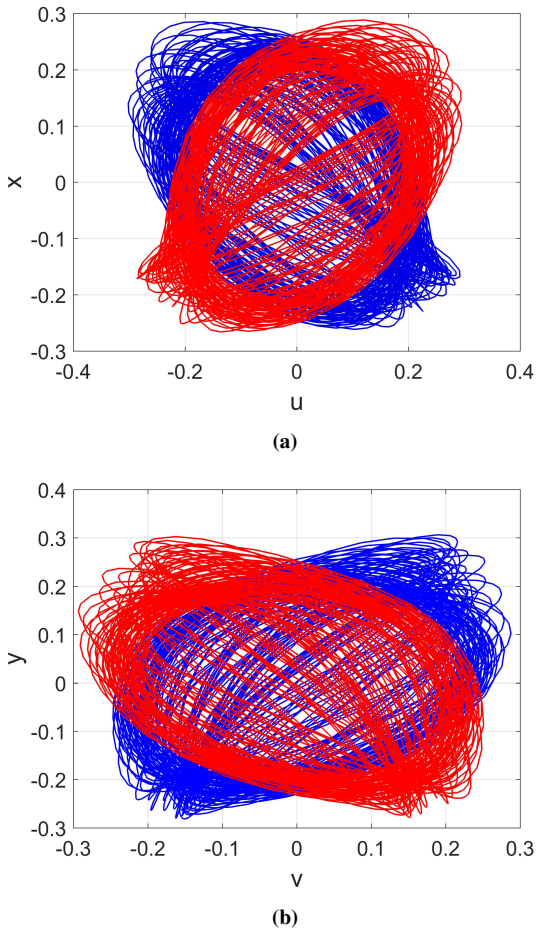


Figure 13: Coexisting chaotic orbits for $a = 1.69$ and $b = 1$. Blue color shows the orbit of (10) with $X(0) = (0.2, 0.2, 0.2, 0)$ and the red color shows the orbit of (10) with $Y(0) = (0.2, -0.2, 0.2, 0)$.

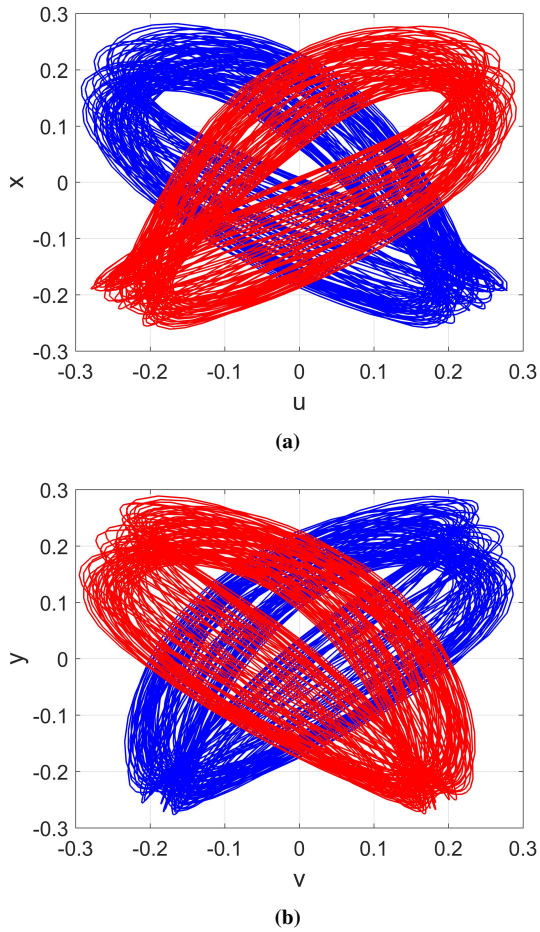


Figure 14: Coexisting chaotic orbits for $a = 1.5$ and $b = 1.5$. Blue color shows the orbit of (10) with $X(0) = (0.2, 0.2, 0.2, 0)$ and the red color shows the orbit of (10) with $Y(0) = (0.2, -0.2, 0.2, 0)$.

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