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Research paper

Brief Review and Mathematical Modelling of Air Cushioned Pressurized Paper Machine Headbox

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Abstract

Today, the requirement of high grade papers is increasing day by day. To cater the need of high quality papers, new high-speed paper machines are being developed constantly. The whole economy of a pulp and paper industry depends on the quality of paper produced, quality of paper depends on the efficient operation of the major part of paper industry – paper machine. Paper machine is the heart of paper industry and has many subsystems. One of the important subsystem of paper machine is Headbox. To have the desired quality papers, efficient working of headbox is highly required. To have desired operation from headbox, its precise control is necessary. The precise modelling of headbox, leads to design of efficient controllers for its proper control to get the desired response. This paper presents the mathematical modelling of headbox along with a brief review of various research performed on headbox.

Keywords: *Paper machine, Headbox, Pressure/Stock Level, Headbox Consistency, Mathematical Modeling.*

1. Introduction

The first ever paper was invented in china by Ts'ai Lun around 20 centuries ago [73]. Since then, paper has been used in variety of areas i.e. from kitchen to office. Due to the numerous applications of paper, its demand and production is increasing every year. According to a report, in India only the paper industry is booming, and it is expected that it will grow in coming years. Till 2011, china has produced around 25% of the total paper production (399 million tons) in the world, which is the highest paper production by any country. Asia accounts for around 45% of the total paper production which is nearly equal to 179 million tonnes. In 2012, the total production or paper around the world was 400 million tonnes. This shows that the demand of paper is increasing, and paper industry has become a billion-dollar industry around the globe [47]. The economy of paper industry majorly depends on the quality of paper, and operation of paper machine. Paper machine is known to be the heart of any paper industry. It is the combination of various subsystems synchronized to operate for desired output i.e. good quality paper. A glimpse of fourdrinier machine is shown in figure 1. It has four major sections viz. wet end, wet press, dryer and post drying operation. Among various subsystems of paper machine, the most important is, Headbox. Headbox belongs to the wet end section of paper machine and plays an important role in the economy of the paper industry.

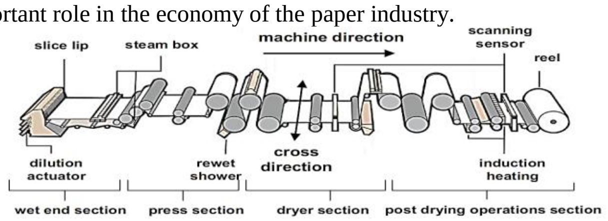

Fig. 1 Fourdrinier paper machine [42, 47].

2. Headbox

As mentioned, headbox is one of the most important subprocesses of paper machine. The function of headbox is to spread the stock uniformly on the wire. That means, headbox is designed to cause a steady state movement of the pulp to wire. Headbox is generally categorized as:

a) Open type headbox
b) Pressurized headbox

Further, pressurized headbox is categorized into, air cushioned type and hydraulic type. Most of the paper machines use pressurized headboxes in which the pressure is maintained through pumps (such as air pump). A schematic of pressurized headbox is shown in figure 2.

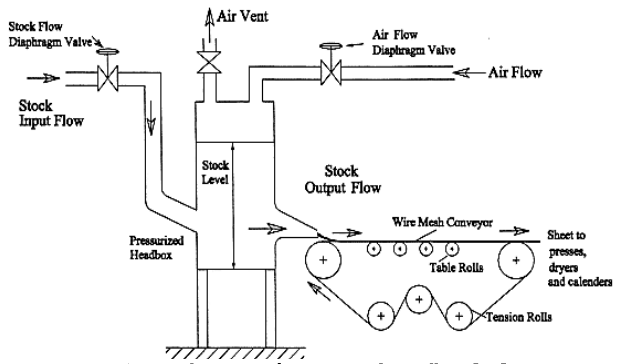

Fig. 2 Schematic of Pressurized Headbox [31]

3. Literature Review

The complexity and interactions in the paper making process and difficulty in measuring the major quality variables online, it is required to design such an advanced and robust control system that can ensure the stability of the process in presence of various disturbances and uncertainties. There has been a substantial study done on paper machine headbox control. Kothari et. al. [1] describes the various effects of headbox variables on the formulation of paper. An adaptive control algorithm has been developed for the control of moisture and basis weight in paper [2]. The algorithm proved to be good for reduction in the losses and any variance in the moisture and basis weights of the paper [2]. A multivariable controller for headbox has been proposed and designed using computer-based designs such as Rosenbrock's Inverse Array Technique for controlling the pressure head and stock level of headbox [3], Extended Kalman filter [4], [5] and two stage estimation algorithm techniques have been used in the control of a non-interacting paper machine headbox and the responses have been compared with different conventional loops. Also, the controller is designed using quadratic optimization principle. Hulster et. al. evaluated the performance of various advanced control methodologies developed for paper machine headbox [6]. A new adaptive controller for paper machine has also been proposed in [7] and the performance is compared with conventional PI controller. Self-tuning technique for moisture control in [9], [10] discusses the use of Kalman filters on pressurized headbox, bilinear control strategy which includes bilinear decoupling control, observer and disturbance compensator in [11], A systematic decoupling control of pressurized headbox was introduced in [12], A Models and on-line identification techniques was presented in [13], Robust GMV control for the cross direction control of headbox [14], Nonlinear predictive control in [15], An Internal model control with reference model was proposed in [16], [24]. The 2-D coupling of Cross Direction and Machine Direction and its control has been discussed in [18]. Robust control through loop-shaping design in [19], Generalized Predictive Control with Modified Smith predictor and Internal Model Control [20], Robust Control for CD response in [21], a multivariable PI controller for headbox has been proposed in [22]. A model based predictive control method using multiple PIDs has been applied and tested on simulated headbox in [25]. An object-oriented control using modelling language OMOLA in [26], [27] describes a tuning technique which is centered on the identification of the process and disturbance models, fuzzy-PID method in [28]. A multivariable predictive control method has been designed and applied on simulated headbox in [29]. MIMO digital-linear-quadratic-regulator in [30], The fragility issues related to the controllers and the aspect of robustness that has been neglected in analytical treatments of control system design of paper machine headbox is discussed in [31], Shape optimization and optimal control techniques were developed for numerical control of paper machine headbox flows in [32]. A decoupling application has been proposed in [34] and a nonlinear geometric control has been implemented on headbox. The CD control limitations due to model uncertainty have been examined and discussed in [35], A technique based on discrete wavelet transforms for CD control has been described in [36], Effects of disturbances on first – pass retention and basis weight discussed in [37], A dynamic simulation of a paper machine wet end was presented.in [38]. [39] explains, how the model uncertainties can affect the design of CD control systems in headbox. Design of CD control of paper machine through multivariable problem, Optimal minimum control effort for a Fourdrinier machine headbox in [40]. An adaptive model predictive control of consistency control [41], Spatially-distributed feedback control technique for CD control of paper machines in [42], A non-smooth bi-objective optimization technique for the design of the shape of a slice channel [43], Interactive multi-objective optimization method NIMBUS [44]. [46] demonstrates a technique of PI controller tuning using single way lead and/or lag decoupling tuning for headbox. The tuning is based on finite frequency response data. A simplified dynamic model for the wet-end in a paper mill was proposed in [48], Artificial Neural Networks based moisture and basis weight control in [49], Advanced control methods and decoupling algorithms [52], Decoupling and Time-delay Control Approach [54], A neural network (NN) based decoupling control technique has been developed in [56], Artificial neural network (ANN) based retention control [58], Adaptive fuzzy controller [59], Various controllers and tuning methods have been studied on paper machine headbox in [60]. Neural network decoupler control system [61], Model Predictive Control for consistency and liquid level control [62], GA based Neural PID decoupling control [63], Advanced prediction based control approach [64], Various PID controller algorithms for consistency control [65], A decoupling control system design [67], Non-fragile bilinear state feedback control [68]. A robust PID control technique have been designed and applied on paper machine headbox for the control of stock flow, air pressure & stock level [69], A robustness analysis of different controllers based on PID tuning methods have been discussed in [70] for the control of consistency, Optimal Linear Quadratic Gaussian control scheme used for controlling the pulp total head & pulp stock level of a paper making headbox system in [71] & [74]. MIMO control design for headbox using fuzzy logic control strategy in [72], fuzzy tuned PID control for nonlinear control problem in air cushion headbox in [75] and for the total pressure and stock level control of paper machine headbox [76], A non-fragile controller has been proposed and designed for headbox in [77], Kalman – Filter based model predictive controller has been designed for headbox in [78] while [79] proposes an internal model controller for headbox.

Table 1: Nomenclature

Parameter Notation	Description
m_{fb}	- Amount of stock present in the flow box
m_{in}	- Amount of Incoming stock
m_{out}	- Amount of outgoing stock
m_{ol}	- Stock flow through the overflow line
t	- time
A_{fb}	- Average cross-section area of flow box
h_1	- Hydrostatic press
ρ_s	- Density of suspension
ρ_a	- Density of air
v_1	- Relative deviation of level
P	- Air Press (Air cushion)
H	- Height of stock to the axis of value
μ_1	- Relative change in the opening of inlet valve
A_c	- Cross section of valve opening
Cd_1	- Discharge coefficient k1
C_1	- Inlet Valve
P_{11}	- Press before entering valve C_1
P_{12}	- Press after valve C_1
P'	- Relative change in pressure of air
C_2	- Slice opening
A_{lip}	- Cross -sectional area of lip opening
Cd_2	- Discharge coefficient k2
Cd_3	- Discharge coefficient k3
b	- Width of air flow
h_3	- Height of air flow
T_{fbl}	- Time constant for flow-box for level of stock in the box

w_1, w_2, w_3	- Load factors
K_{1v_1}, K_{2v_1} $K_{1\rho}, K_{2\rho}$	- Constant factors depending on the speed of the machine
m_{ch}	- Amount of the stock present in the channel
m_2	- flow of stock
A_{ch}	- Average area of cross-section of the channel
h_{ch}	- Height of stock level in channel
V_{ch}	- Volume of channel
A_{op}	- Cross-sectional area of outlet pipe
C_{d4}	- Discharge coefficient k4
T_{fblo}	- Time constant for flow box stock level in the overflow pipe
w_{21}	- Factor depending on v1
$K_1 v_2$, $K_3 \rho$	- Constant factor depending on the paper machine speed
m_{air}	- amount of air present above the stock level in the flow box
$m_{\sup}$, m_{rem}	- Amount of air supplied and removed from the flow-box
V_3	- Volume of air above stock level in the flow box
A_{c_5}, A_{c_6}	- Cross – sectional area of overflow valves C5 and C6
k_5, k_6	- Loss factor
P_{51}, P_{52}	- Inlet and outlet press of valve C5
P_{61}, P_{62}	- Inlet and outlet press of valve C6
T_{air}	- time constant of flow box for air cushion
w_4	- Load factor
K_4, K_5	- Constant factors depending on the machine speed
T_{tv_1}, T_{tv_2}	- Time constant for turbulence in the channel depending on v1 & v2
Index ∞	- Indicates the fixed value of given quantity

4. Headbox Control Techniques

Based on the literature review discussed in section 3, the identified control techniques for paper machine headbox are listed below:

a) Adaptive Control [2], [41], [59]
b) Predictive Control [36], [47], [62], [78]
c) Robust Control [14],[19], [21], [27], [69], [70], [71], [74]
d) Optimal Control [28], [30], [33], [40], [44]
e) Multivariable Non-linear Control [15], [35], [72], [77]
f) Bilinear Control [11], [68]
g) Intelligent Control [49], [58], [75], [76]
h) Decoupling Control [12], [52], [54], [56], [63], [67]
i) Digital Control [2], [4], [30]
j) Internal Model Control [20], [24], [79]

As listed above, there have been many control strategies developed for different types of paper machine headbox. However, the efficient control can be ensured only by perfect modeling of a dynamical system. Efficient modeling of system is possible only if complete information of the system is available. Since, a headbox is a multivariable dynamical system which is difficult to control. The prime objective of headbox is to spread the stock on the wire uniformly to obtain desired paper quality. However, headbox is under the effect of various disturbances which affect the quality of paper produced. Hence, the design a suitable controller for headbox, it is necessary to have prefect mathematical modeling of it. The section 5 discusses the mathematical modeling of air cushioned headbox.

5. Headbox Mathematical Modelling

Headbox is a nonlinear 2-inputs and 2-outputs MIMO process with significant loop interaction. Because of its non-linearity, it has been a center of attraction for researchers. Many control methodologies have been developed for headbox control in past few decades. The major parameters required to be controlled in headbox are: Total pressure head, Stock level, Consistency, Jet velocity, and rush/drag [3], [10], [11], [15], [16], [19], [20], [27], [28], [30], [31], [52], [56], [61], [63], [64], [68], [69], [71], [74]. To have the desired quality of paper, a precise control of headbox is highly required. The control and stability analysis of headbox is done through it mathematical model. Since, it is difficult to develop the perfect mathematical model of a system, so develop efficient control algorithms are also difficult. A Stochastic state-variable model of headbox was developed by Sinha et. al. in [3]. This section gives a complete mathematical model of air pad pressure, level and consistency for an air cushioned pressurized headbox.

5.1 Modelling of Air Cushioned Head Box

In paper industry, the high-speed paper machine is equipped with pressurized flow box with air cushion. Pressurized headbox can be used for complete automation, so it is necessary to know dynamic processes which are associated with the incoming and outgoing stock on the paper machine wire. One type of pressurized headbox is shown in figure 3.

The pressurized headbox may be divided into three zones in which dynamics may be conceived independently. Kikiewich et al (8) developed partly the dynamics of the same but could not analyze. This problem has been solved in this present investigation. The paper machine headbox may be divided into three zones in which dynamic may be conceived independently. These three zones are:

- *Portion of box filled with stock*
- *Space above stock surface filled with air cushion*
- *Overflow system*

5.1 (a) Headbox Filled with Stock

$$\frac{dm_{fb}}{dt} = m_{in} - m_{out} - m_{ol} \quad (1)$$

where,
m_{fb} = Amount of stock present in the flow box
m_{in} and m_{out} = Incoming and Outgoing stock through flow box
m_{ol}= Stock flowing through overflow line.
Eqn. (1) may be represented by the following relationship

$$\frac{d\Delta m_{fb}}{dt} = \Delta m_{in} - \Delta m_{out} - \Delta m_{ol} \quad (2)$$

where

$$\Delta m_{fb} = m_{fb} - m_{fb\infty}$$
$$\Delta m_{in} = m_{in} - m_{in\infty}$$
$$\Delta m_{out} = m_{out} - m_{out\infty}$$
$$\Delta m_{ol} = m_{ol} - m_{ol\infty}$$
$$m_{fb} = V_1 \rho_s$$

$m_{fb} = A_{fb} h_1 \rho_s$

A_{fb} = Average cross-sectional area of flow box

h_1 = Hydrostatic press

ρ_s = Density of suspension

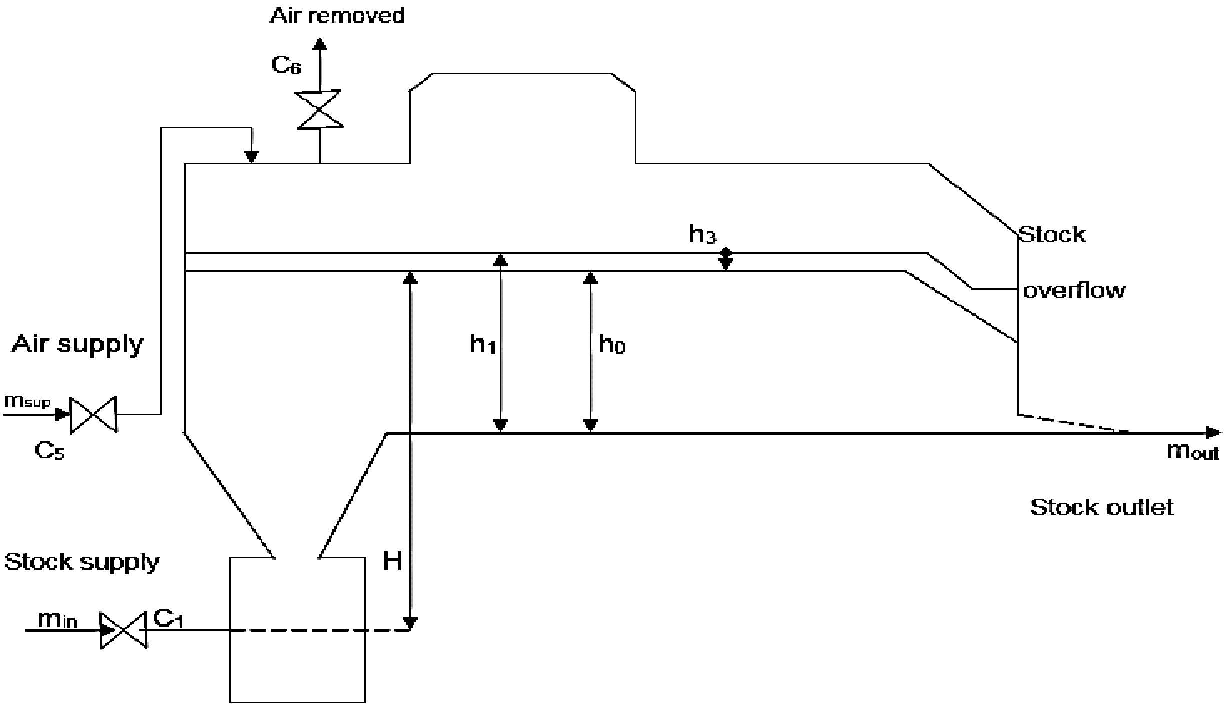

Fig. 3 Air Cushioned Pressurized Headbox

Rise in Δm_{fb} amounts to $A_{fb}\Delta h_1 \rho_s$ or $\Delta m_{fb} = A_{fb} h_{1\infty} \upsilon_1 \rho_s$ or $\Delta m_f = m_{fb\infty}\upsilon_1$

Amount of inlet stock m_{in} depends on opening of inlet valve C_1, the term m_{in} can be written as

$$m_{in} = m_{in}\left(C_1,\ P_{11,} P_{12}\right)$$

$$\begin{aligned} \Delta m_{in} = & \left(\partial m_{in} / \partial C_1\right)_\infty \Delta C_1 \ldots \\ & +\left(\partial m_{in} / \partial H\right)_\infty \Delta P_{11} \ldots \\ & +\left(\partial m_{in} / \partial P_1\right)_\infty \Delta P_{12} \end{aligned} \quad (3)$$

C1 as under

$$m_{in} = A_c . C_{d1}\sqrt{2\rho_s\left(P_{11} - P_{12}\right)}$$

$$\left(\frac{\partial m_{in}}{\partial H}\right)_\infty \Delta P_{11} = \left(\frac{-1}{2}\right) m_{in\infty}\rho_s g \left(\frac{h_{1\infty}}{P_{11} - P_{12}}\right) \upsilon_1 \quad (5)$$

$$\left(\frac{\partial m_{in}}{\partial P}\right)_\infty \Delta P_{12} = \left(\frac{-1}{2}\right) m_{in\infty}\left(\frac{P_\infty}{P_{11} - P_{12}}\right) \mathrm{P}' \quad (6)$$

Flow of stock from slice lip

$$m_{out} = m_{out}\left(C_2,\ h_{1,} P\right)$$

$$\Delta m_{out} = \begin{bmatrix} \left(\frac{\partial m_{out}}{\partial C_2}\right)_\infty \Delta C_2 \ldots. \\ +\left(\frac{\partial m_{out}}{\partial h_1}\right)_\infty \Delta h_1 \ldots. \\ +\left(\frac{\partial m_{out}}{\partial P}\right)_\infty \Delta P \end{bmatrix} \quad (7)$$

$$\left(\frac{\partial m_{out}}{\partial C_2}\right)_\infty = \frac{m_{outmax}}{C_{2max}}$$

$$\left(\frac{\partial m_{out}}{\partial C_2}\right)_\infty \Delta C_2 = \frac{m_{outmax}}{C_{2max}}\Delta C_2$$

Assume that the characteristic of inlet valve of stock is linear, then

$$\left(\frac{\partial m_{in}}{\partial C_1}\right)_\infty = \frac{m_{inmax}}{C_{1max}} \text{ or } \left(\frac{\partial m_{in}}{\partial C_1}\right)_\infty \Delta C_1 = \left(\frac{m_{inmax}}{C_{1max}}\right)\Delta C_1$$

$$\left(\frac{\partial m_{in}}{\partial C_1}\right)_\infty \Delta C_1 = m_{inmax}\mu_1 \quad (4)$$

Flow of stock through the regulating valve

$$\left(\frac{\partial m_{out}}{\partial C_2}\right)_\infty \Delta C_2 = m_{outmax}\mu_2 \quad (8)$$

Flow of stock through the slice

$$m_{out} = A_{lip}.C_{d2}\sqrt{2\rho_s\left(h_1 g\ \rho_s + P\right)}$$

$$\left(\frac{\partial m_{out}}{\partial h_1}\right)_\infty \Delta h_1 = \left(\frac{1}{2}\right) m_{out\infty}\rho g \left(\frac{h_{1\infty}}{h_1 g\ \rho_s + P}\right) \upsilon_1 \quad (9)$$

$$\left(\frac{\partial m_{out}}{\partial P}\right)_\infty \Delta P = \left(\frac{1}{2}\right) m_{out\infty}\left(\frac{P_\infty}{h_1 g\ \rho_s + P}\right) P' \quad (10)$$

Flow of stock through overflow

$$m_{ol} = C_{d3} b \rho_s \sqrt{2} g (h_3)^{1.5}$$

$$\Delta m_{ol} = \frac{3}{2} C_{d3} b \rho_s h_{3\infty}\sqrt{2} g h_3 \frac{\Delta h_3}{h_{3\infty}}$$

$$\Delta m_{ol} = \frac{3}{2} m_{ol\infty} u_3 \quad (11)$$

Substituting above all eqns. in eqn. (2), one can write

$$d\left(\frac{V_1\rho_s\upsilon_1}{dt}\right)=\begin{bmatrix} m_{in\max}\mu_1-\frac{1}{2}m_{in\infty}\rho_s g\left(\frac{h_{1\infty}}{P_{11}-P_{12}}\right)\upsilon_1.... \\ -\frac{1}{2}m_{in\infty}\left(\frac{P_\infty}{P_{11}-P_{12}}\right)P'.... \\ ...-m_{out\max}\mu_2-\frac{1}{2}m_{out\infty}\rho_s g\left(\frac{h_{1\infty}}{h_1 g\rho_s+p}\right)\upsilon_1.... \\ -\frac{1}{2}m_{out\infty}\left(\frac{P_\infty}{h_1 g\rho_s+p}\right)P'-\frac{3}{2}m_{ol\infty}\upsilon_3 \end{bmatrix} \quad (12)$$

If C2 is constant value therefore μ2=0 or $M_{outmax}\mu_2 = 0$, hence equ.[12] can be written as

$$\left(V_1\rho_s\frac{d\upsilon_1}{dt}\right)=\begin{bmatrix} m_{in\max}\mu_1-\frac{1}{2}m_{in\infty}\rho_s g\left(\frac{h_{1\infty}}{P_{11}-P_{12}}\right)\upsilon_1... \\ -\frac{1}{2}m_{in\infty}\left(\frac{P_\infty}{P_{11}-P_{12}}\right)P'... \\ ...-\frac{1}{2}m_{out\infty}\rho_s g\left(\frac{h_{1\infty}}{h_1 g\rho_s+P}\right)\upsilon_1... \\ -\frac{1}{2}m_{out\infty}\left(\frac{P_\infty}{h_1 g\rho_s+P}\right)P'-\frac{3}{2}m_{ol\infty}\upsilon_3 \end{bmatrix}$$

$$\left(\frac{V_1\rho_s}{m_{in\max}}\right)*\left(\frac{d\upsilon_1}{dt}\right)=\begin{bmatrix} \mu_1-\left[\left(\frac{m_{in\infty}}{m_{in\max}}\right)\frac{1}{2}\rho_s g\left(\frac{h_{1\infty}}{P_{11}-P_{12}}\right)\right]\upsilon_1.. \\ ..-\left[\left(\frac{m_{in\infty}}{m_{in\max}}\right)\frac{1}{2}\left(\frac{P_\infty}{P_{11}-P_{12}}\right)\right]P'... \\ ...-\left[\frac{1}{2}\rho_s g\left(\frac{h_{1\infty}}{h_1 g\rho_s+p}\right)\left(\frac{m_{out\infty}}{m_{in\max}}\right)\right]\upsilon_1.. \\ ..-\left[\left(\frac{m_{out\infty}}{m_{in\max}}\frac{1}{2}\left(\frac{P_\infty}{h_1 g\rho_s+p}\right)\right)\right]P'.. \\ ..-\frac{3}{2}\frac{m_{out\infty}}{m_{in\max}}\upsilon_3 \end{bmatrix}$$

$$T_{fbl}\frac{d\upsilon_1}{dt}=\begin{bmatrix} \mu_1-\left(w_1k_{1v1}+w_2k_{2v1}\right)\upsilon_1.. \\ ..-\left(w_1k_1\rho+w_2k_2\rho\right)P'... \\ -\frac{3}{2}w_{31}\upsilon_3 \end{bmatrix} \quad (13)$$

Considering that

$h_1 = h_0 + h_3$ or $\Delta h_1 = \Delta h_3 = 1$,

so $\upsilon_3 = \left(\frac{h_{1\infty}}{h_{3\infty}}\right)\upsilon_1$ or $a_{13}\upsilon_1$

Eqn. (13) can be rewritten as

$$T_{fbl}\frac{d\upsilon_1}{dt}=\begin{bmatrix} \mu_1-\left(w_1k_{1v1}+w_2k_{2v1}\right)\upsilon_1... \\ -\left(w_1k_1\rho+w_2k_2\rho\right)P'... \\ -\frac{3}{2}w_{31}a_{13}\upsilon_1 \end{bmatrix}$$

$$T_{fbl}\frac{d\upsilon_1}{dt}+A\upsilon_1=\mu_1-BP' \quad (14)$$

Where

$$A = w_1k_{1v1}+w_2k_{2v1}+\frac{3}{2}w_{31}\upsilon_3$$

$$B = w_1k_1\rho+w_2k_2\rho$$

$$a_{13}=\frac{\upsilon_3}{\upsilon_1}$$

5.1 (B) Material Balance for Overflow System

Equation for material balance for overflow can be written as

$$\frac{dm_{ch}}{dt}=m_{ol1}-m_{ol2} \quad (15)$$

or $m_{ch} = A_{ch}h_{ch}\rho_s = V_{ch}\rho_s$

Rise in m_{ch}

$$\left(\frac{d\Delta m_{ch}}{dt}\right)=\Delta m_{ol1}-\Delta m_{ol2} \quad (16)$$

$$\Delta m_{ch}=A_{ch}h_{ch\infty}\rho_s\upsilon_2 \quad (17)$$

Flow of stock can be written as

$$m_{ol2}=A_{op}C_{d4}\sqrt{2\rho s\left(h_{ch}\rho_s g+P\right)}$$

$$\Delta m_{ol2}=\begin{bmatrix} \frac{1}{2}m_{ol2\infty}\rho_s g\left(\frac{h_{ch\infty}}{h_{ch}g\rho_s+P}\right)\upsilon_2.... \\ +\frac{1}{2}m_{ol2\infty}\left(\frac{P_\infty}{h_{ch}g\rho_s+P}\right)p' \end{bmatrix} \quad (18)$$

Substituting the values of Δm_{ol1}, Δm_{ol2} & Δm_{ch} into eqn. (16), one can get

$$\left(\frac{d(V_2\rho_s\upsilon_2)}{dt}\right)=\begin{bmatrix} \frac{3}{2}m_{ol\infty}\upsilon_3-\frac{1}{2}m_{ol2\infty}\left(\frac{P_\infty}{h_{ch}g\rho_s+P}\right)P'.. \\ ..-\frac{1}{2}m_{ol2\infty}\rho_s g\left(\frac{h_{ch\infty}}{h_{ch}g\rho_s+P}\right)\upsilon_2 \end{bmatrix}$$

$$V_2\rho_s\left(\frac{d\upsilon_2}{dt}\right)=\begin{bmatrix} \frac{3}{2}m_{ol\infty}a_{13}\upsilon_1-\frac{1}{2}m_{ol_2\infty}\left(\frac{P_\infty}{h_{ch}g\rho_s+P}\right)P'.. \\ ..-\frac{1}{2}m_{ol_2\infty}\rho_s g\left(\frac{h_{ch\infty}}{h_{ch}g\rho_s+P}\right)\upsilon_2 \end{bmatrix}$$

$$\left(\frac{V_2\rho_s}{m_{ol2\infty}}\right)\left(\frac{d\upsilon_2}{dt}\right)=\left[\frac{\frac{3}{2}m_{ol\infty}a_{13}\upsilon_1-\frac{1}{2}m_{ol2\infty}\left(\frac{P_\infty}{h_{ch}g\rho_s+P}\right)P'..\quad ....-\frac{1}{2}m_{ol2\infty}\rho_s g\left(\frac{h_{ch\infty}}{h_{ch}g\rho_s+P}\right)\upsilon_2}{m_{ol2\infty}}\right]$$

$$T_{fblo}\frac{d\upsilon_2}{dt}=w_{21}\upsilon_1-K_3\rho P'-K_{1v2}\upsilon_2$$

$$T_{fblo}\frac{d\upsilon_2}{dt}+K_{1v2}\upsilon_2=w_{21}\upsilon_1-K_3\rho P' \quad (19)$$

5.1 (C) Material Balance for Air Cushion

Equation for material balance for air cushion can be written in the form as under

$$m_{air}=m_{\sup}-m_{rem}$$

$$\frac{dm_{air}}{dt}=\Delta m_{\sup}-\Delta m_{rem} \quad (20)$$

$$\Delta m_{air}=m_{air}-m_{air\infty} \quad ; \quad \Delta m_{\sup}=m_{\sup}-m_{\sup\infty} \quad ;$$

$$\Delta m_{rem}=m_{rem}-m_{rem\infty};$$

$$m_{air}=V_3\rho_a \quad (21)$$

$$V_3=V_0-V_1-V_2 \quad (22)$$

$$\Delta m_{air}=\Delta V_3\rho_{a\infty}+V_{3\infty}\Delta\rho_a \quad (23)$$

Putting the value of V_3 in eqn.[23], then

$$\Delta m_{air}=\rho_{a\infty}\left(\Delta V_1-\Delta V_2\right)+V_{3\infty}\rho_a\frac{\Delta P}{P_\infty} \text{ or}$$

$$\Delta m_{air}=-\rho_{a\infty}\Delta V_1-\Delta V_2\rho_{a\infty}+V_{3\infty}\rho_a P' \quad (24)$$

Loss of air through valve C_5 & C_6

$$m_{\sup}=A_{c5}K_5\sqrt{2\rho_a\left(P_{51}-P_{52}\right)} \quad (25)$$

$$m_{rem}=A_{c6}K_6\sqrt{2\rho_a\left(P_{61}-P_{62}\right)} \quad (26)$$

$$\Delta m_{\sup}=m_{\sup\max}\left(\frac{\Delta C_5}{C_{5\max}}\right)-\frac{1}{2}m_{\sup\infty}\left(\frac{P_{52\infty}}{P_{51}-P_{52}}\right)P'$$

if valve C_5 is linear, the term $\Delta m_{\sup}$ can be written as

$$\Delta m_{\sup}=m_{\sup\max}\mu_5-\frac{1}{2}m_{\sup\infty}\left(\frac{P_{52\infty}}{P_{51}-P_{52}}\right)P'$$

$$\Delta m_{rem}=m_{rem\max}\left(\frac{\Delta C_6}{C_{6\max}}\right)-\frac{1}{2}m_{rem\infty}\left(\frac{P_{61\infty}}{P_{61}-P_{62}}\right)P'$$

$$\Delta m_{rem}=m_{rem\max}\mu_6-\frac{1}{2}m_{rem\infty}\left(\frac{P_{61\infty}}{P_{61}-P_{62}}\right)P' \quad (27)$$

C_6 is constant, so $\mu_6=0$;

$$\Delta m_{rem}=-\frac{1}{2}m_{rem\infty}\left(\frac{P_{61\infty}}{P_{61}-P_{62}}\right)P' \quad (28)$$

Putting the values of Δm_{air}, $\Delta m_{\sup}$, Δm_{rem} into eqn. (20), one can get

$$\frac{d(\rho_{a\infty}\Delta v_3+v_{3\infty}\Delta\rho_a)}{dt}=\left[\begin{matrix}m_{\sup\max}\mu_5-..\\ ..\left[\left(\frac{1}{2}m_{\sup\infty}\left(\frac{P_{52\infty}}{P_{51}-P_{52}}\right)\right)...\quad ..-\frac{1}{2}m_{rem\infty}\left(\frac{P_{61\infty}}{P_{61}-P_{62}}\right)\right]P'\end{matrix}\right]$$

$$\frac{d(-\Delta v_1\rho_{a\infty}-\Delta v_2\rho_{a\infty}+v_{3\infty}\rho_a P')}{dt}=$$

$$\left[\begin{matrix}m_{\sup\max}\mu_5..\\ ..-\left[\left(\frac{1}{2}m_{\sup\infty}\left(\frac{P_{52\infty}}{P_{51}-P_{52}}\right)\right)-\frac{1}{2}m_{rem\infty}\left(\frac{P_{61\infty}}{P_{61}-P_{62}}\right)\right]P'\end{matrix}\right]$$

$$\left[\begin{matrix}\left(\frac{v_3\rho_a}{m_{\sup\max}}\right)\left(\frac{dP'}{dt}\right)+..\\ ..P'\left[\left(\frac{1}{2}m_{\sup\infty}\left(\frac{P_{52\infty}}{P_{51}-P_{52}}\right)+\frac{1}{2}m_{rem\infty}\left(\frac{P_{61\infty}}{P_{61}-P_{62}}\right)\right)\right]\end{matrix}\right]=\mu_5+v_1\rho_a\frac{du_1}{dt}+v_2\rho_a\frac{du_2}{dt}$$

$$T_{air}\frac{dP'}{dt}=w_4(k_4+k_5)P'=\mu_5+T_{tv1}\frac{du_1}{dt}+T_{tv2}\frac{du_2}{dt} \quad (29)$$

Where

$$T_{air}=\left(\frac{v_3\rho_a}{m_{\sup\max}}\right)$$

$$w_4=\frac{m_{\sup\infty}}{m_{\sup\max}}$$

$$k_4=\frac{1}{2}\left(\frac{P_{52\infty}}{P_{51}-P_{52}}\right)$$

$$k_5=\frac{1}{2}\left(\frac{P_{61\infty}}{P_{61}-P_{62}}\right)$$

$$T_{tv1}=\frac{v_1\rho_a}{m_{\sup\max}}$$

$$T_{tv2}=\frac{v_2\rho_a}{m_{\sup\max}}$$

Dynamic process in the flow box may be properly understood with the help of eqns. (14, 19 and 29). Equations may be represented in the following form as under:

$$T_{fb1}\frac{du_1}{dt}+Au_1=\mu_1-BP' \quad (30)$$

$$T_{fb1o}\frac{du_2}{dt}+Cu_2=Eu_1-DP' \quad (31)$$

$$T_{air}\frac{dP'}{dt}+FP'=\mu_5+T_{tv1}\frac{du_1}{dt}+T_{tv2}\frac{du_2}{dt} \quad (32)$$

Where: C= k1v2; E= w21; D=k3ρ ; F= w4(k4+k5)
To determine the transition process as originating in regulating channel of flow box, it is necessary to solve the set of eqns. (30 – 32) w.r.to P' and u. The eqn. (30) can be written as

$$(T_{fbl}s+A)u_1=\mu_1-BP'$$

$$u_1=\frac{\mu_1}{(T_{fbl}s+A)}-\frac{BP'}{(T_{fbl}s+A)}$$

If, $Q = \frac{1}{(T_{fbl}s + A)}$; $R = \frac{B'}{(T_{fbl}s + A)}$

Then, $u_1 = Q\mu_1 - RP'$ (33)

Eqn. (31) can be written as

$$(T_{fblo}s + C)u_2 = Eu_1 - DP'$$

Or

$$\left[(T_{fblo}s + C)(T_{tv2}s)\right]u_2 = (T_{tv2}sE)u_1 - (T_{tv2}sD)P' \quad (34)$$

Eqn. (32) can be written as

$$(T_{tv2}s)u_1 = (T_{tv1})u_1 - (T_{air}s + F)P' + \mu_5$$

$$\left[(T_{fblo}s + C)(T_{tv2}s)\right]u_2 = \begin{bmatrix} \left[(T_{fblo}s + C)(T_{tv1})\right]u_1 .. \\ ..-\left[(T_{air}s + F)(T_{fblo}s + C)\right]P' .. \\ ..+\mu_5(T_{fblo}s + C) \end{bmatrix} \quad (35)$$

Subtracting eqn. (35) from (34), one can find

$$P' = \begin{bmatrix} \frac{\left[T_{tv2}sE + (T_{fblo}s + C)(T_{tv1}s)\right]u_1}{\left[T_{air}T_{fblo}s^2 + (T_{air}C + T_{tv2}D + T_{fblo}F)s + FC\right]} + ... \\ .... \frac{u_5(T_{fblo}s + C)}{\left[T_{air}T_{fblo}s^2 + (T_{air}C + T_{tv2}D + T_{fblo}F)s + FC\right]} \end{bmatrix} \quad (36)$$

If $X = \left[\frac{T_{tv2}s(T_{fblo}s + C) + E(T_{tv1}s)}{\left[T_{air}T_{fblo}s^2 + (T_{air}C + T_{tv2}D + T_{fblo}F)s + FC\right]} \right]$

$$Y = \left(\frac{T_{fblo}s + C}{\left[T_{air}T_{fblo}s^2 + (T_{air}C + T_{tv2}D + T_{fblo}F)s + FC\right]} \right)$$

Then eqn. (36) can be written as

$$P' = Xu_1 + Y\mu_5 \quad (37)$$

From eqns. (33 – 37), both values u1 & P' are related with process conditions. Variable opening of valves µ1 or µ5 affect regulation in the value of these variables because they are related with the relationship. u1 and P' are replaced by L_{head}, and P_{head} respectively.

$$L_{head} = Q'(s)\mu_1 + R'(s)\mu_5 \quad (38)$$

$$P_{head} = X'(s)\mu_1 + Y'(s)\mu_5 \quad (39)$$

$$\begin{bmatrix} L_{head} \\ P_{head} \end{bmatrix} = \begin{bmatrix} X'(s) & Y'(s) \\ Q'(s) & R'(s) \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_5 \end{bmatrix} \quad (40)$$

Where

$$Q'(s) = \left[\frac{T_{air}T_{fblo}s^2 + \left(T_{air}C + T_{tv2}D + T_{fblo}F\right)s + FC}{a_1s^3 + a_2s^2 + a_3s + AFC} \right] \quad (41)$$

$$R'(s) = \left[\frac{BT_{fblo}s + BC}{a_1s^3 + a_2s^2 + a_3s + AFC} \right] \quad (42)$$

$$X'(s) = \left[\frac{T_{tv1}T_{fblo}s^2 + \left(ET_{tv2} + T_{tv1}C\right)s}{a_1s^3 + a_2s^2 + a_3s + AFC} \right] \quad (43)$$

$$Y' = \left[\frac{T_{fblo}T_{fbl}s^2 + \left(AT_{fblo} + CT_{fbl}\right)s + AC}{a_1s^3 + a_2s^2 + a_3s + AFC} \right] \quad (44)$$

Equation (38) to (41) can be used to find the MIMO transfer function of headbox. Further, the controller may be desired using various methods as discussed in section III. Since, headbox is a multivariable system with nonlinearity and loop interaction, so design of decoupler is necessary to minimize the loop interactions. The decoupler design is based on determining the relative gain array (RGA). RGA is calculated to know the suitable pairing among loops. The perfect mathematical model of headbox is key to successful design of controller. Since, there has been a significant research on headbox and many control strategies have been proposed and developed for it.

6. Conclusion

This paper presented the mathematical modeling of an air cushioned headbox. Headbox is the critical section of paper machine. The control of headbox is important in a way that it eventually affects the economy of pulp and paper industry which depends on the quality of the paper produced. So, to ensure the quality of the paper to be high, it is highly important that the paper machine headbox is controlled efficiently. There are many techniques available and applied on the headbox for its control but most of them fail in the sense that no effect of uncertainties and disturbances have been considered. However, headbox is a multi-variable system which is highly nonlinear, complex and has continuously changing parameters. So, the conventional techniques and other robust techniques developed do not give significant results. Hybrid intelligent robust control techniques can prove to be revolutionary step in control of paper machine headbox due to various advantages of robust and intelligent control.

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