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Technical paper

Estimating Non-Conformance Using the Modified Tolerance Region Method and the Target Distance Method

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Abstract

In many cases, the quality of a manufactured product is determined by more than one characteristic and often, these quality characteristics are correlated. A number of methods for dealing with quality evaluation of multivariate processes have been proposed in the literature. However, some of these studies do not consider correlation among quality characteristics. In this paper, two new approaches for estimating the proportion of non-conformance for correlated multivariate quality characteristics with nominal specifications are proposed: (i) the modified tolerance region approach and (ii) the target distance approach. In the first approach, the p number of correlated variables are analysed based on the projected shadow of the p-dimensional hyper ellipsoid so that the ability to visualise the tolerance region and the process region for $p > 2$ is preserved. In the second approach, the correlated variables are combined and a new variable called the target distance is introduced. The proportion of non-conformance results estimated using both methods were used to compute the multivariate capability index and the total expected quality cost. This study also suggest modification to the NMC_p index as proposed in Pan and Lee (2010) such that the process capability for $p > 2$ can be measured correctly. The application of both approaches is demonstrated using two examples and it is shown that both methods i.e. the modified tolerance region and the target distance methods are capable of estimating the capability of multivariate processes.

Keywords: *Multivariate quality control; Correlated characteristics; Tolerance region; Proportion of non-conformance; Mahalanobis Distance*

1. Introduction

The common goals for any profit-based manufacturing companies are to have financial controls and to gain substantial share in the global market. One of the important fiscal components in many manufacturing companies is the quality costs. These costs can be categorised into four elements i.e. prevention costs, appraisal costs, internal failure costs and external failure costs [1]. Prevention costs are costs associated to activities required in avoiding occurrence of poor quality at design and production stage. Appraisal costs are costs due to monitoring and inspection of incoming materials, products, consumables and equipment. When non-conformance of products is detected within the vicinity of the manufacturing site, activities such as rework, scrap, retest and machine unscheduled downtime may be required and these incur internal failure costs. External failure costs are costs associated to the negative financial impact that occurs after the end product is procured by the customer such as warranty claims, product recall and loss of market share. External failure costs are difficult to quantify, however the opportunity to identify and mitigate these external costs can provide significant benefit to the manufacturer.

[2] defined quality from the customers' perspective and highlighted that loss incurs on the society when a quality characteristic deviates away from its target value. This definition of quality relates the quality loss to the internal and external failure costs, thus advocates that these costs can be estimated using the quality loss function (LF). When a product quality is defined by more than one key quality characteristics, applying the univariate LF for individual quality characteristic to estimate total quality loss may results in erroneous conclusion, especially when correlation exists. Motivated by this need, this paper intends to answer the following research question: "How can we measure the probability of non-conformance for products with correlated multivariate quality characteristics so that the costs of quality failure can be estimated effectively?"

Therefore, the objectives of this paper are: (i) to review and analyse related models as proposed by past researches, (ii) to present two new models for estimating the proportion of non-conformance (P_{NC}) for multivariate data, as developed in this study, and (iii) to estimate the internal and external failure costs based on the estimated P_{NC}. The following parts of this paper are organised as follows: Section 2 reviews some of the multivariate LF and multivariate process capability index (mPCI) published in the literature related to this study. Section 3 presents both the modified tolerance region (MTR) and the target distance (TD) methods developed in this study, Section 4 discusses the application and results obtained using both methods, and Section 5 concludes this paper.

2. Literature Review

2.1. Multivariate Loss Functions

Earlier studies on multivariate LF neglects the factor of correlation among the quality characteristics and mostly suggest that the total quality loss is equal to the sum of losses for each quality characteristics [3-5]. [6] included the covariance structure of the quality characteristics in the multi-response LF, as shown in Eq. (1):

$$E(Q) = (\boldsymbol{\mu} - \boldsymbol{T})'\boldsymbol{C}(\boldsymbol{\mu} - \boldsymbol{T}) + \; trace\;(\boldsymbol{C\Sigma}) \quad (1)$$

where $\boldsymbol{\mu}$ is the process mean vector, $\boldsymbol{T}$ is the target mean vector, $\boldsymbol{\Sigma}$ is the process variance-covariance matrix and $\boldsymbol{C}$ is the cost matrix. However, the multivariate LF in Eq. (1) does not identify any specification region, as recommended by [7]. [8] proposed a LF for multivariate normal quality characteristics with nominal specifications and identified specification region based on [7] i.e. the region of joint intersection by the specification limits for each quality characteristic.

Based on this definition, the specification region for a bivariate case for example, is represented by a rectangular shape defined by the original specification limits of both quality characteristics. When the quality characteristics falls within this specification region, no observable loss incurs on the manufacturer but the deviation from target value may incur loss on the customer, hence results in the external failure costs. When the quality characteristics falls outside of the specification region, rework or scrap costs will be directly incurred by the manufacturer, thus resulting in internal failure costs. For p-variate of quality characteristics y with lower specification limit (LSL) and upper specification limit (USL), estimated the total quality cost per unit for multivariate normal quality characteristics with nominal specifications by defining the rejection region as the region outside of the rectangular tolerance region. [9] showed that the multivariate LF based on rectangular region, as described in Fig. 1 underestimates the rejection cost for correlated quality characteristics, thus improved the model in [8] by assuming a modified tolerance region defined by the correlation structure between the bivariate normal quality characteristics. For example, the modified tolerance region for positively correlated bivariate normal quality characteristics is represented by an elliptical shape as shown in the following Fig. 1:

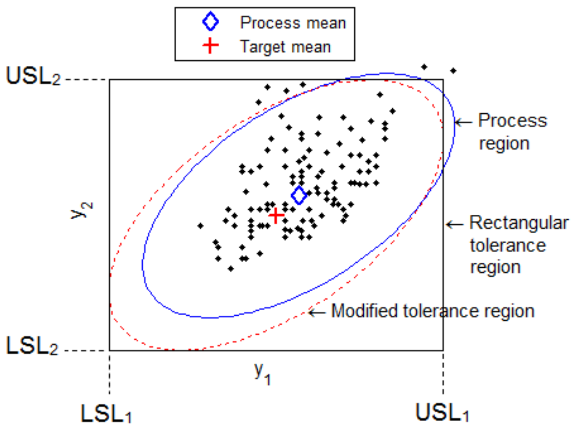

Fig. 1: Tolerance regions and process region

As shown in Fig. 1, for a bivariate normal distribution, the modified tolerance region forms a slanted elliptical shape with the orientation of its major axis described by the correlation coefficient. With reference to the modified tolerance region, a total of 6 points are detected as rejects or non-conformance. However, when judging by the original tolerance region only 2 points are detected as rejects. For products with bivariate quality characteristics, discriminating the quality characteristics against the modified tolerance region is necessary because it assist in the estimation of the total rejection cost, Q_{RC} and the total quality cost, Q_{QC}.

2.2. Multivariate Process Capability Index

For products with single key quality characteristic, the capability of the in-control process to provide output within the specified LSL and USL and also around the target value is commonly measured using the indices C_p and C_{pk}. The intricacy in dealing with the multivariate quality characteristics increases when the number of quality characteristics involved is large, thus often demands for the application of multivariate statistical techniques. Furthermore, a robust method for assessing the quality level of a product or process must be able to address the correlation between multiple quality characteristics. This subsection includes a brief review of two common techniques used in the literature for assessing the capability of processes with multivariate quality characteristics i.e. (i) finding the proportion of non-conformance, and (ii) finding the ratio of the tolerance region against the process region.

2.2.1 Finding the Proportion of Non-Conformance

[10] suggested that the multivariate capability index MC_{pk} is calculated based on the P_{NC} value obtained from the probability density of the multivariate normal distribution:

$$MC_{pk} = -\tfrac{1}{3}\Phi^{-1}(\theta) \quad (2)$$

where $\Phi^{-1}(\theta)$ is the inverse cumulative distribution of the standard normal random variable and θ is the multivariate normal probability of multivariate data that falls within a multivariate specification limits. [11] used a nonparametric approach of multivariate kernel-density estimation to find P_{NC} and computed capability index using Eq. (1). [12] estimated the theoretical P_{NC} for bivariate normal distribution with rectangular tolerance region using the assumption of convex polygons. [13] developed a multivariate capability index MPCNCV based on the conformance and non-conformance volumes for unilateral and bilateral specification limits, however the modified tolerance region defined does not considers correlation.

Similar approaches of using distances and dimension reduction have been proposed in [14-16]. [14] recommended to combine correlated quality characteristics of any distribution using the generalised distance, GD variable, as follows:

$$GD = \sqrt{(\boldsymbol{Y} - \boldsymbol{T})'(\boldsymbol{Y} - \boldsymbol{T})} \quad (3)$$

where $\boldsymbol{Y}$ is the measurement vector. Based on this, [15] suggested that the P_{NC} value for each new GD variable can be estimated as,

$$P_{NC} = 1 - \int_0^{MRD} f(x)\, dx \quad (4)$$

where MRD is the maximum radial distance calculated as $MRD = \sqrt{Tol_i{}^2 + \cdots + Tol_p{}^2}$, $Tol_i = USL_i - LSL_i$, and $f(x)$ is the probability density function of each GD variable. The Luceno capability index, C_{pc} is then used to measure the capability of each GD variable:

$$C_{pc} = \frac{MRD - 0}{3\sqrt{\pi/2}\; \times \; Average\; (\boldsymbol{Y}{-}\boldsymbol{T})} \quad (5)$$

[16] employed the dimension reduction technique for the correlated multivariate data and included the process variance-covariance matrix $\boldsymbol{\Sigma}$ for computing the covariance distance, CD, as follows:

$$CD = \sqrt{(\boldsymbol{Y} - \boldsymbol{T})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{Y} - \boldsymbol{T})} \quad (6)$$

In Eq. (6), the distance for each measurement is weighted by $\boldsymbol{\Sigma}$ however, in estimating P_{NC} the non-conformance value is underestimated as the MRD value adopted is the same as in Eq. (4).

2.2.2 Finding the Ratio of Tolerance Region to Process Region

The univariate approach of comparing tolerance length, *Tol* against the actual process length, 6σ can be extended to multivariate cases as tolerance volume against actual process volume. This approach translates to the commonly known univariate indices such as C_p and C_{pk} thus is prevailing among researchers and practitioners. However, the complexity of using this approach in a multivariate setting arises when large number of variables is involved as estimating hyper volumes can be difficult especially for non-central distributed process data.

In the quality control literature, the development of mPCI using this technique observed a number of different interpretations of the tolerance volume. [17] defined the volumes based on the shape of the process region. For example, for a bivariate normal distribution with both centred target means, the process region R_2 is an ellipse scaled at 99.73% and the tolerance region R_1 is also an ellipse completely enclosed within the original or rectangular tolerance region. The major and minor axes of the elliptical tolerance are parallel to the edges of the rectangular tolerance. Using this assumption, the capability index is calculated as

$$MC_{pm} = \frac{vol.(R_1)}{vol.(R_2)} \cdot \frac{1}{D} \quad (7)$$

where $D = \sqrt{(1 + (\boldsymbol{\mu} - \boldsymbol{T})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu} - \boldsymbol{T})}$. In [18], the process volumes is modified to the shape of the original tolerance volumes so that the capability index is estimated as:

$$C_{pm} = \left[\frac{vol.(tolerance\ box)}{vol.(process\ box)}\right]^{\frac{1}{p}} \quad (8)$$

The multivariate capability indices in Eq. (7) and Eq. (8) fail to include the correlation structure for the multivariate quality characteristics, which may result in the overestimation of product quality level. To rectify this shortcoming, [19] redefined the modified tolerance region in [17] as the largest ellipsoid centered at the target mean with its axes parallel to the direction of the eigenvectors of the process covariance matrix. The modified tolerance region in [20] is similar to [19] however the elliptical tolerance region is tangent to all sides of the original tolerance region. Based on the proposed multivariate indices by [20] as shown in Eq. (9) and Eq. (10), [20] developed a model for estimating the expected quality loss, as in Eq. (11):

$$NMC_p = \left(\frac{|S|}{|\Sigma|}\right)^{\frac{1}{2}} \quad (9)$$

$$NMC_{pm} = \frac{NMC_p}{D} \quad (10)$$

$$E(Q) = k_N\left(\left(\frac{NMC_p}{NMC_{pm}}\right)^2 - 1\right) + \frac{1}{NMC_{pm}{}^2}|\boldsymbol{C}||\boldsymbol{S}|\sum_{j=1}^{p}\frac{1}{|\Lambda_j|} \quad (11)$$

where k_N is the ratio of $(\boldsymbol{\mu} - \boldsymbol{T})'\boldsymbol{C}(\boldsymbol{\mu} - \boldsymbol{T})$ and $(\boldsymbol{\mu} - \boldsymbol{T})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu} - \boldsymbol{T})$, $\boldsymbol{S}$ is the target covariance matrix, $\boldsymbol{C}$ is the cost matrix and $|\boldsymbol{\Lambda_j}| = \prod_{k=1,k\neq j}^{p}\lambda_k, \lambda_j$ is the jth eigenvalue of $\boldsymbol{C\Sigma}$ matrix.

3. Methodology

This section describes the development of the MTR and the TD methods based on the review done on past researches and its proposed models. The MTR and the TD methods presented in this study were developed by improving the models as presented in earlier studies [16],[21]. Both models were constructed and evaluated using the MATLAB R2014a software package. Verification and validation of the models were done by comparing the results to prior models, based on the literature data related to the manufacturing industry. The methodology for employing the MTR and the TD methods are discussed in subsections 3.1 and 3.2, respectively.

3.1. The MTR Method

In [9], the definition of modified tolerance volume based on [21] is adopted as it addresses the correlation within the multivariate normal data and it represents the original tolerances of each quality characteristic involved.

The construction of the elliptical process region and the elliptical tolerance region is done based on the Σ and S matrices, respectively. For number of variables denoted as $i, j = 1, 2, \ldots, p$, the elements of $\boldsymbol{S}$ is calculated as:

$$\rho_{ij}\left(\frac{USL_i - LSL_i}{2d}\right)\left(\frac{USL_j - LSL_j}{2d}\right) \quad (12)$$

where $\boldsymbol{\rho_{ij}}$ is the correlation coefficient matrix and $\boldsymbol{d^2 \sim \chi_p{}^2(\alpha)}$. In working with bivariate data, Tanjong et al. (2014), estimated the proportion falling inside and outside of the MTR as the regions of intersection and subtraction, respectively which is determined using the MATLAB's function *polybool*.

In this study, similar technique to [9] was employed but with an improvement on the method of estimating the probability density by using multiple integrations on the joint probability density function $f(y_1, \ldots, y_p)$ bounded by the MTR. For example, Fig. 2 illustrates $n = 20$ number of rectangular strips bounded by the elliptical tolerance region so that the probability of the process region falling inside of the tolerance region can be estimated using multiple n-integral with its limits defined by the rectangular strips. The accuracy of the probability distribution density estimation can be improved by selecting an optimum value of n.

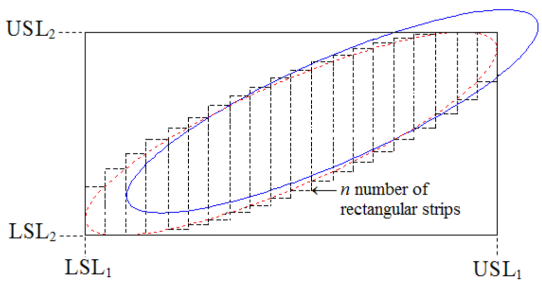

Fig. 2: Rectangular strips of multiple integrations bounded by MTR

For the bivariate quality characteristics in Fig. 2, the proportion of the process region that falls outside of the modified tolerance region can be estimated using Eq. 13:

$$P_{NC,y_1y_2} = 1 - \left(\int_{y_{2,1}}\int_{y_{1,1}} f(y_1, y_2)dy_1dy_2 + \int_{y_{2,2}}\int_{y_{1,2}} f(y_1, y_2)dy_1dy_2 + \cdots + \int_{y_{2,n}}\int_{y_{1,n}} f(y_1, y_2, \ldots, y_v)dy_1dy_2\right) \quad (13)$$

The ability to visualise process region against the specified tolerance region assists in providing insight to the actual process performance in graphical form. To allow the visualisation of the process region against the MTR for more than 2 correlated normal variables, the $\boldsymbol{\Sigma}$ and $\boldsymbol{S}$ ellipses are plotted as the projected shadow of the p-dimensional hyperellipsoid, thus each pair of correlated quality characteristics is defined by $\{y: (\boldsymbol{Y} - \boldsymbol{T})'\boldsymbol{S}^{-1}(\boldsymbol{Y} - \boldsymbol{T}) \leq \chi_2{}^2(\alpha)\}$ where $\chi_2{}^2(\alpha)$ denotes the upper $(100\alpha)th$ percentile of the chi-square distribution with 2 degrees of freedom. Table 1 summarises the proposed MTR method.

Table 1: Summary of the MTR method

1. Collect sample data and perform basic statistics calculation and normality test.
2. Find correlation coefficient matrix $\boldsymbol{\rho}$ and process covariance $\boldsymbol{\Sigma}$.

For each pair of correlated quality characteristics:

3. Calculate the corresponding bivariate process covariance matrix $\boldsymbol{\Sigma_{ij}}$ and target covariance matrix $\mathbf{S_{ij}}$ using the selected $\chi_2{}^2(\alpha)$.
4. Plot the ellipses $\boldsymbol{\Sigma_{ij}}$ and $\mathbf{S_{ij}}$.
5. Define the number of n for multiple integral.
6. Calculate $P_{NC,\, y1y2}$ using Eq. (13).

The total P_{NC} is estimated as,

$$P_{NC} = \max\{P_{NC}, y_1y_2\,,\ P_{NC}, y_1y_3\,, \dots, P_{NC}, y_iy_j\,\} \quad \text{for } i \neq j \qquad (14)$$

3.2. Target Distance Method

To reduce the dimension of the correlated quality characteristics, [16] proposed the computation of CD variable as in Eq. (6). The technique of dimension reduction has the advantage of reducing the dimensionality of the correlated multivariate data into a single variable however it requires data fitting and the definition of upper specification limit for the new variable. In this paper, we further develop this method by specifying the Mahalanobis distance of the measurement point to target mean with respect to the target covariance matrix, $\boldsymbol{S}$ instead of $\boldsymbol{\Sigma}$, and define this new variable as target distance, TD:

$$TD = \sqrt{(\boldsymbol{Y}-\boldsymbol{T})'\boldsymbol{S}^{-1}(\boldsymbol{Y}-\boldsymbol{T})} \qquad (15)$$

To estimate the P_{NC} for a multivariate normal data with nominal specifications, we defined the maximum target distance or the upper specification limit for the new variable TD as d where $d^2 \sim \chi_p{}^2(\alpha)$, so that:

$$P_{NC,TD} = 1 - \int_0^d f(x)\,dx \qquad (16)$$

where $f\,(x)$ is the probability density function of the TD variable. Table 2 summarises the proposed TD method.

Table 2: Summary of the TD method

1. Collect sample data and perform basic statistics calculation and normality test.
2. Determine the correlated and uncorrelated group of variables.

For each group of correlated and uncorrelated quality characteristics:

3. Find correlation coefficient matrix $\boldsymbol{\rho}$.
4. Calculate the target covariance matrix $\boldsymbol{S}$ using Eq. (12).
5. Calculate TD using Eq. (15).
6. Find best-fitting distribution for each variable TD.
7. Calculate $P_{NC,TD}$ for each variable using Eq. (16).

The total P_{NC} is estimated as,

$$P_{NC} = 1 - \prod_{i=1}^{k} \int_0^{d_i} f_i(x)\,dx \qquad (17)$$

where $\boldsymbol{f_i(x)}$ is the probability density function of the ith TD variable, and the number of the correlated and uncorrelated groups of variables is given as $i, j, \dots k$.

3.3. Modification to NMC_p

In section 4 of this paper, the multivariate capability indices obtained using the MTR and TD approaches are compared to the indices as proposed by [21] i.e. Eq. (9) and (10). To improve the efficiency of the multivariate capability index NMC_p for $p > 2$, Eq. (10) was modified as follows:

$$NMC_p = \left(\frac{|\boldsymbol{S}|}{|\boldsymbol{\Sigma}|}\right)^{\frac{1}{p}} \qquad (18)$$

The revised NMC_p model incorporates the number of variables in the power instead of using the constant 2. Thus, to calculate the expected quality loss for number of variables, $p > 2$ we modifed Eq. (11) as follows:

$$E(Q) = k_N\left(\left(\frac{NMC_p}{NMC_{pm}}\right)^2 - 1\right) + \frac{1}{NMC_{pm}{}^p}|\boldsymbol{C}||\boldsymbol{S}|\sum_{j=1}^{p}\frac{1}{|\Lambda_j|} \qquad (19)$$

We verified that the E(Q) value computed using Eq. (19) is equal to the results obtained using Eq. (1).

4. Application

This section demonstrates the application of both the MTR and TD methods using data acquired from the literature. Subsection 4.1 presents the P_{NC} results for bivariate and trivariate cases using the data available in [22-23], respectively. Using the estimated P_{NC} values, we calculated the capability indices MC_{pk} in Eq. (2) and these indices were compared to the values obtained using NMC_p and NMC_{pm}, respectively. Subsection 4.2 exemplifies how the quality costs can be estimated using the P_{NC} values. In this study, the calculations and analysis for both the MTR and TD approaches is achieved using the combination of the statistical package Minitab 16 and the computing software MATLAB R2014a.

4.1 Target Distance Method

[22] presented an example of a bivariate manufacturing process where the quality of bobbin is characterised by the its height (BH) and weight (BW). A sample of size $N = 100$ is provided and the data is assumed to follow a bivariate normal distribution. The specification limits of the process are given as $[(BH, BW): 40 \leq BH \leq 42, 44 \leq BW \leq 46.5]$. The non-conformance percentage and capability index as calculated in Pal (1999) is 0.00106 and 0.976, respectively.

Using the MTR method, the elliptical regions for the bivariate normal quality characteristics are constructed based on $\chi^2_{2,0.9973}$ and a nominal set of specifications is assumed. The matrices $\boldsymbol{\Sigma}$ and $\boldsymbol{S}$ are calculated as

$$\boldsymbol{\Sigma} = \begin{bmatrix} 0.0735 & 0.0505 \\ 0.0505 & 0.1076 \end{bmatrix} \text{ and } \boldsymbol{S} = \begin{bmatrix} 0.0845 & 0.0600 \\ 0.0600 & 0.1321 \end{bmatrix}$$

The process region and tolerance region are illustrated in Fig. 3. The actual proportion of non-conforming based on the observed data with respect to both original tolerance region and MTR are both zero. However, as seen in Fig. 3, the positive shift in the process mean results in a significant portion of the process region falls outside of the MTR, thus the P_{NC} based on the MTR is estimated as 0.002272.

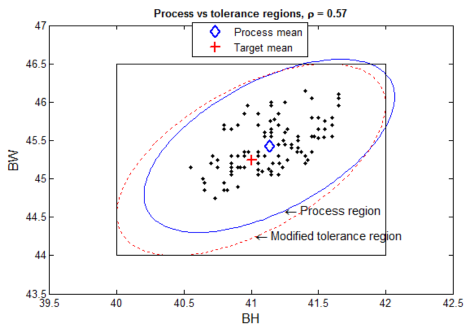

Fig. 3: Process region and tolerance region for the correlated BH and BW ($\rho = 0.57$)

The combined variable TD for the correlated variables BH and BW is calculated and fitted to the best-fitting distribution i.e. the Weibull distribution (scale parameter, $\alpha = 1.4277$ and shape parameter, $\beta = 2.2308$) with Anderson-Darling test statistic (AD) value 0.529 and P-value 0.19, as shown in Fig. 4. The upper specification limit for TD is given by, $d = 3.4393$. Thus, the P_{NC} of the bivariate data using the TD approach is estimated as 0.0008175. Table 3 summarises the P_{NC} results obtained using both the MTR and the TD method.

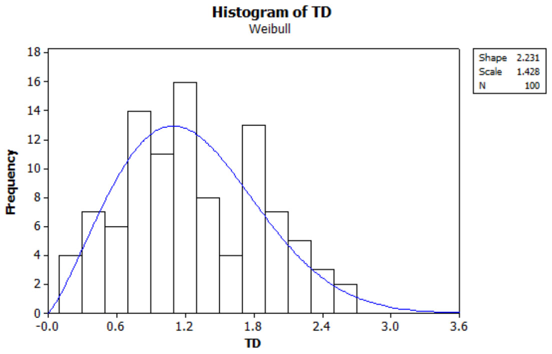

Fig. 4: Histogram of TD distribution for the bivariate data in Pal (1999), fitted to Weibull distribution.

[23] included a data set of a thermal spraying process with 3 variables i.e. light intensity (X_1), temperature (X_2) and velocity (X_3), which consists of 70 observations. The variables are assumed to have a multivariate normal distribution and correlations among the quality characteristics are evident. The target mean for all 3 variables are at the center of the bilateral specifications, as follows: $[(X_1, X_2, X_3): 394 \leq X_1 \leq 603, 2295 \leq X_2 \leq 2668, 98 \leq X_3 \leq 128]$. The matrices $\boldsymbol{\rho}$, $\boldsymbol{\Sigma}$ and $\boldsymbol{S}$ for the trivariate case are calculated as,

$$\boldsymbol{\rho} = \begin{bmatrix} 1 & 0.216 & 0.227 \\ 0.216 & 1 & 0.289 \\ 0.227 & 0.289 & 1 \end{bmatrix}, \boldsymbol{\Sigma} = \begin{bmatrix} 1166.5 & 395.6 & 25.9 \\ 395.6 & 2862.5 & 51.8 \\ 25.9 & 51.8 & 11.2 \end{bmatrix} \text{ and}$$

$$\boldsymbol{S} = \begin{bmatrix} 771.4 & 298 & 25.1 \\ 298 & 2457 & 57.2 \\ 25.1 & 57.2 & 15.9 \end{bmatrix}$$

The process regions and the tolerance regions for each pair of correlated variables are as described in Fig. 5:

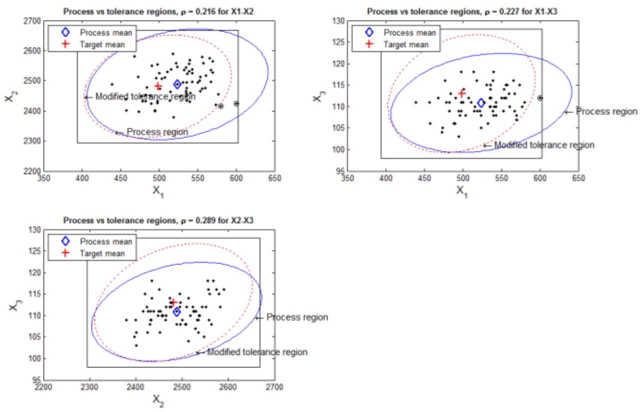

Fig. 5: Process region and tolerance region for each pair of correlated variables in the trivariate case.

Referring to Fig. 5, the actual proportion of non-conforming based on the observed data with respect to the original tolerance region is zero however 2 observations falls outside of the MTR. The total P_{NC} for the correlated variables is determined based on Eq. (14) where $P_{NC,x_1x_2} = 0.0342$, $P_{NC,x_1x_3} = 0.0369$ and $P_{NC,x_2x_3} = 0.0045$.

The combined variable TD for the correlated trivariate data is calculated and fitted to the best-fitting distribution i.e. the Normal distribution (scale parameter, $\alpha = 0.745$ and location parameter, $\mu = 2.059$) with AD value 0.326 and P-value 0.515, as shown in Fig. 6. The upper specification limit for the TD variable in the trivariate data is determined as $d = 3.7625$. The P_{NC} results of the correlated normal trivariate data based on the MTR and the TD approach is summarised as in Table 3.

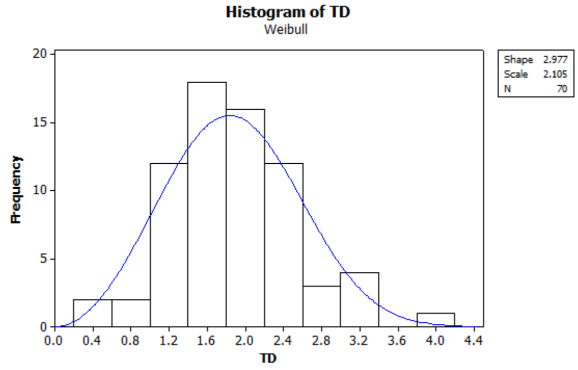

Fig. 6: Histogram of TD distribution for the trivariate case in Tano and Vännman (2012), fitted to Normal distribution.

Table 3: Summary of the P_{NC} results for the bivariate and the trivariate cases.

Case	Actual P_{NC} based on observed data with respect to MTR	P_{NC} using MTR	P_{NC} using TD
Bivariate	0	0.00227	0.00082
Trivariate	0.02857	0.03695	0.01068

To analyse the effect of sample size in both the MTR and the TD approaches, we simulate multivariate random data for the bivariate and trivariate cases base on the original data. For the bivariate data we simulate N = 10^3, 10^4, 10^5 and similarly, for the trivariate data we simulate N = 70^2, 70^3, 70^4. The results are summarised in Fig. 7 and Fig. 8.

In Fig. 7, it is demonstrated that the P_{NC} results for both approaches are close for larger sample size. However, similar trend is not observable in Fig. 8, where the TD method estimates lower proportion of non-conformance as compared to the MTR method. Using Eq. (2) and the P_{NC} values obtained, we measure the multivariate capability index based on MTR and TD approaches. The results are then compared with indices obtained using Eq. (18) and Eq. (10). The results are summarised in Fig. 9 and Fig. 10.

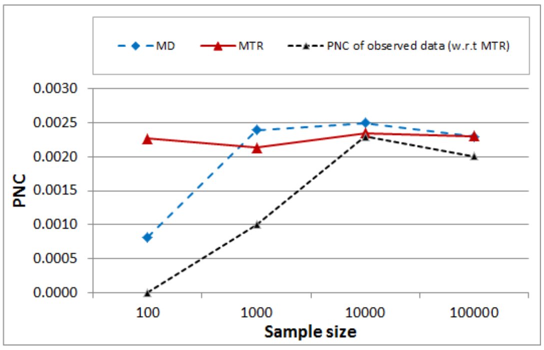

Fig. 7: P_{NC} with increasing sample size for the bivariate case

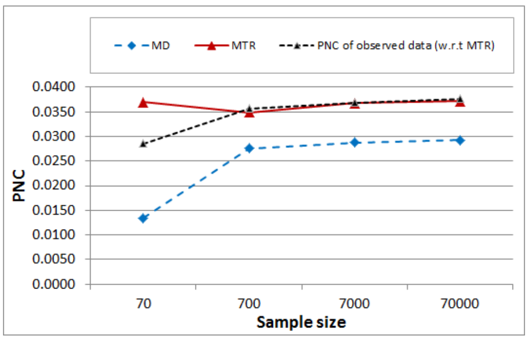

Fig. 8: P_{NC} with increasing sample size for the trivariate case

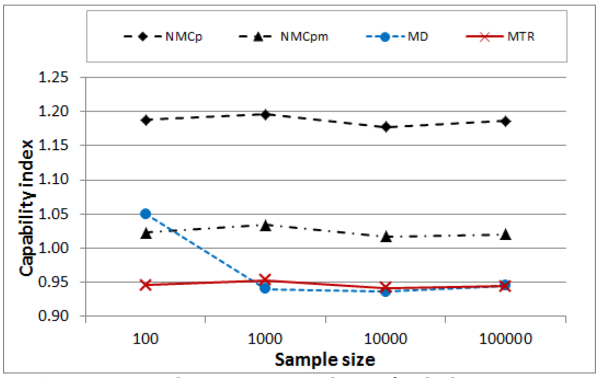

Fig. 9: mPCI with increasing sample size for the bivariate case

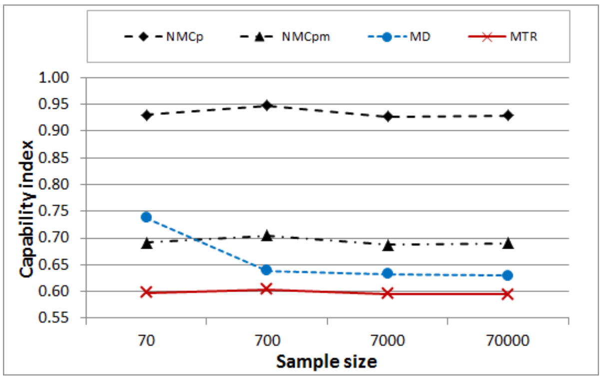

Fig. 10: mPCI with increasing sample size for the trivariate case

Referring to Fig. 9 and Fig. 10, both the MTR and TD methods are more stringent in assessing the capability of the process as compared to the NMC_{pm} index. Furthermore, for less capable process such as the trivariate case in Fig. 10, the MTR method penalise the process capability severely as the expected region of the process falling outside of the modified tolerance region is considerably large.

4.2 Estimating the Total Quality Costs

We estimate the total rejection costs and the total quality costs for the bivariate and trivariate cases discussed in the previous subsection based on the P_{NC} results obtained for the original data. In dealing with more than 2 correlated variables using the MTR method, we recommend using the projected shadow of the hyper ellipsoid to construct the elliptical regions thus the following modification to the loss function is proposed:

For number of variables denoted as $i, j = 1, 2, \ldots, p-1, p$,

$$L_{Q_{ij}} = \frac{k_{ii}s_{ii}}{p-1} + \frac{k_{ii}(\mu_i - T_i)^2}{p-1} + \frac{k_{jj}s_{jj}}{p-1} + \frac{k_{jj}(\mu_j - T_j)^2}{p-1} + k_{ij}s_{ij} + k_{ij}(\mu_i - T_i)(\mu_j - T_j) \ldots$$

$$L_{Q_{p-1p}} = \frac{k_{p-1p-1}s_{p-1p-1}}{p-1} + \frac{k_{p-1p-1}(\mu_{p-1} - T_{p-1})^2}{p-1} + \frac{k_{pp}s_{pp}}{p-1} + \frac{k_{pp}(\mu_p - T_p)^2}{p-1} + k_{p-1p}s_{p-1p} + k_{p-1p}(\mu_{p-1} - T_{p-1})(\mu_p - T_p) \quad (20)$$

Thus, the total rejection cost QRC and the total quality cost QQC for the multivariate case is estimated as:

$$Q_{RC} = L_Q P_{NC} \quad (21)$$

$$Q_{QC} = L_Q(1 - P_{NC}) \quad (22)$$

The ability to breakdown the quality losses with respect to process variance and deviation from target mean for each quality characteristics as in Eq. (20) offers the opportunity to identify and prioritise concerns for a correlated multivariate process.

The cost matrices for the bivariate and trivariate data is assumed as $C_2 = [2\ 2.5;\ 2.5\ 3.2]$ and $C_3 = [1\ 0.8\ 0.75;\ 0.8\ 0.75\ 0.72;\ 0.75\ 0.72\ 0.7]$, respectively. Using Eq. (19), the total quality loss for the bivariate and trivariate cases is estimated as \$1.0041 per unit and \$5136.3 per unit, respectively. The breakdown of the total quality costs calculated using Eq. (20) – (22) are summarised in the following Table 4 and Table 5:

Table 4: Q_{RC} and Q_{QC} for each quality characteristics for the bivariate data.

	Variable	Q_{RC}	Q_{QC}
Target loss	BH	0.0000	0.0362
	BW	0.0002	0.1029
	BH-BW	0.0003	0.1205
Variance loss	BH	0.0003	0.1467
	BW	0.0008	0.3435
	BH-BW	0.0006	0.2521
	Total	0.0022	1.0018
$EQ_{TC} = Q_{RC} + Q_{QC}$		1.0040	

Table 5: Q_{RC} and Q_{QC} for each quality characteristics for the trivariate data.

	Variable	Q_{RC}	Q_{QC}
Target loss	X_1	23.1430	627.1070
	X_2	0.6744	34.1515
	X_3	0.0657	3.1059
	X_1-X_2	9.5178	268.5050
	X_1-X_3	3.0082	78.4096
	X_2-X_3	0.0939	20.7929
Variance loss	X_1	41.5177	1125.0041
	X_2	41.5735	2105.2861
	X_3	0.1627	7.6879
	X_1-X_2	21.6684	611.2823
	X_1-X_3	1.4362	37.4334
	X_2-X_3	0.3356	74.3113
	Total	143.1972	4993.0769
$EQ_{TC} = Q_{RC} + Q_{QC}$		5136.2740	

Table 4 describes that a small percentage of 0.22% of the total expected quality cost EQ_{TC} is due to Q_{RC}, hence indicates a lower risk of internal failure cost occurrence. However, analysis on the Q_{QC} highlights that 26% of the potential occurrence of the external failure cost is due to the deviation of the process mean from target mean. Table 5 indicates that 3% of the total expected quality cost EQ_{TC} is due to Q_{RC} thus points out the opportunity for reducing the occurrence of internal failure cost. By referring to the breakdown of the quality costs in Table 5, the priority for corrective actions in reducing Q_{RC} is to shift the variable X_1 closer to its target mean and reduce the process variance with respect to X_1. Eq. (21) and Eq. (22) can be employed to estimate EQ_{TC} using the P_{NC} values obtained from the TD method, however the breakdown of losses with respect to each correlated variable cannot be determined.

5 Conclusion

Review on past studies related to the process capability analysis for multivariate data indicates that most models do not have direct interpretation of the capability index to P_{NC}. Some methods that estimate the multivariate capability index using P_{NC} fail to consider correlation among the variables when defining the rejection region. This paper presents two methods for estimating the P_{NC} for correlated multivariate normal quality characteristics i.e. the MTR and the TD methods. The MTR method is robust in estimating regions falling inside and outside of the specification volume irrespective of the number of samples. The strategy for analysing the multivariate data based on every correlated pairs provides the convenient means of visualising the process region against the tolerance region and allows the breakdown of quality costs for each quality characteristics. The TD method has the advantage of reducing the dimensionality of the multivariate data, which is a huge advantage for multivariate processes. Nevertheless, it requires large number of samples and can be less effective for analysing the effect of each quality characteristic to the correlated multivariate data. Both methods can appeal to the manufacturers in which the MTR method is helpful for identifying the mean and variation of the correlated quality characteristics against the combined specifications while the TD method is useful in reducing the complexity for working with higher dimension data. The work presented in this paper is limited to multivariate data with joint normal distributions. For future work, we will explore the potential of both methods for multivariate non-normal data with different types of specifications i.e. asymmetry and unilateral types of specifications.

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