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Research paper

Distribution Characterisation of Coil Spring Strain Histories Using Mixed Weibull Analysis

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Abstract

This paper investigates the scatter of strain histories obtained from coil spring of a vehicle suspension system. Statistical characterisation is essential in fatigue analysis due to the random nature of fatigue process. The element of uncertainty can be dealt with appropriately by applying probabilistic approach. In this study, four different road profiles were used to obtain strain signal data. Initial description of the strain histories was achieved by computing the global statistics values. The strain range data was calculated from the counted fatigue cycle and distribution fitting was performed using the Anderson Darling test. Four different types of distribution; the exponent, Gamma, 3P-Weibull, and 2P-Weibull; were compared to find the appropriate fit to model the strain range data. The results showed that strain range data are highly skewed with thick-tailed indicating a non-Gaussian distribution. All four tested distribution turned out to be insignificance, with the closest *p*-value of 0.01 produced by the 2P-Weibull distribution at significance level of 0.05. Due to the non-straight probability plot of data, the mixed Weibull distribution was chosen to model the data. The campus area and highway profiles follow this distribution with 2 sub-populations while rural and housing area resulted in 3 sub-populations. Finally, this distribution is found suitable for analysing the strain range data of vehicle coil spring and hence can be used in time-domain fatigue life evaluation.

Keywords: *Coil Spring; Distribution; Mixed Weibull; Strain Signal; Suspension.*

1. Introduction

Deterministic and probabilistic models have been widely proposed in the existing literature of fatigue analysis. The deterministic models are continuously developed in open literature [1, 2] despite numerous suggestions that experimental data involved scatter and uncertainty [3]. Most of the deterministic models proposed were supported by arbitrary selection of functional form based on convenience reasons such as simplicity [4]. However, the model still do not account for the random nature of fatigue process [3]. The uncertainties associated with the behaviour of fatigue experimental data [5] have motivated considerable efforts in investigating and quantifying this element in research data. Thus, various probabilistic models are presented and applied when dealing with uncertainties [6].

A considerable number of probabilistic methods have been developed over the years to deal with fatigue data scatter and uncertainty [7, 8]. Klemenc and Fajdiga [9] suggested an approach for estimating strain−life curves and their scatter using evolutionary algorithms. Paolino et al. [10] proposed an equation and procedure that should be applied for evaluation of the parameter error due to different sources of uncertainty. Another extensive statistical works were performed by Fernández-Canteli et al. [11]. They stated that deterministic models are very limited and should be replaced by probabilistic models in fatigue analysis, based on physical and statistical knowledge.

In order to perform probabilistic analysis for appropriate fatigue investigations, the initial step is the characterisation of variation in fatigue properties [12]. Therefore, this paper aims to characterise and investigate the scatter and statistical properties of strain histories specifically for the vehicle coil springs using appropriate distribution analysis. The findings about distribution behaviour are useful as an input for further fatigue analysis on coil spring component. Discussion on the distribution behaviour of the strain range data obtained from cycle counting technique is also presented. The strain range magnitude is important because it is closely related to the occurrence of fatigue damage for component in service.

2. Theoretical Background

To enable a correct results interpretation of fatigue test data, statistical evaluation methods could be adopted. The initial statistical description of the strain histories was achieved using the global statistics values. The most common statistic to measure the central tendency is the arithmetic mean. For *n* number of data points, the mean can be computed as follows:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \quad (1)$$

For a random fatigue process in service, the mean value is often not constant due to non-stationary loadings [13]. The other statistics that is often used is the root-mean-squares (r.m.s). It represents the overall energy content of an oscillatory signal [14]. High energy content is frequently associated with high fatigue damage for components in service. The equation is written as follows

$$\text{r.m.s} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x^2{}_i} \tag{2}$$

To assess the distribution of data in terms of whether it is symmetric or skewed, the third statistical moment can be used [15]. Non-zero skewness value designates a non-Gaussian load distribution. The mathematical expression is given by

$$S = \frac{1}{n(\text{r.m.s})^3}\sum_{i=1}^{n}(x_i - \bar{x})^3 \tag{3}$$

The kurtosis measures the peaked-ness of data distribution and the thickness of distribution tails. It is highly sensitive to outliers and can be used to quantify the deviation of load from Gaussianity and load stationarity. The formula is given by

$$K = \frac{1}{n(\text{r.m.s})^4}\sum_{i=1}^{n}(x_i - \bar{x})^4 \tag{4}$$

The commonly used distribution to model life data analysis is the 2P-Weibull. This type of distribution is often used in fatigue reliability [16] and durability analysis because of the versatility of its shape parameter. The cumulative distribution function of a 2P-Weibull can be expressed as follows [17]:

$$F(N_f) = 1 - \exp - \left(\frac{N_f}{\eta}\right)^{\beta} \tag{5}$$

where $F(N_f)$ is the fraction failed in time or cycles N_f, N_{f0} is the minimum time or cycles to failure, η is the characteristic life, and β is the Weibull slope or shape parameter. For data with probability plots which do not fall on a straight line, the mixed Weibull distribution is more appropriate to be used. This type of distribution has been recognized as a suitable model for the lives of mechanical component or systems which caused by more than one failure modes [18]. The probability density function for a 3-subpopulation distribution is written as

$$f_{1,2,3}(t) = \sum_{i=1}^{3} \frac{N_i\beta_i}{N\eta_i}\left(\frac{t}{\eta_i}\right)^{\beta_i - 1} e^{-\left(\frac{t}{\eta_i}\right)^{\beta_i}} \tag{6}$$

where N is the number of sample data; β and η are distribution parameters.

3. Methodology

All strain signals data series can be observed in Fig. 1. It is apparent that each road profile produces different trend of data series. The trends were according to the specific type of manoeuvre of the vehicle demonstrated by the maximum and minimum of the oscillatory signals [19].

Similar vertical axes were used in the data series so that all data are directly comparable.

The strain histories used in this study were obtained by performing real experiments [20]. The process flow of methodology is detailed in Fig. 2. A strain gauge was attached to the coil spring surface and it was connected to data acquisition device and computer. The vehicle was driven on the preselected road according to the location profile as seen in Table 1.

Table 1: The location of each signal obtained

Signal	Type of location
S1	Campus (UKM)
S2	Highway
S3	Rural
S4	Housing area

There are four types of distinct locations, i.e. UKM campus (S1), highway (S2), rural (S3), and housing area (S4). The material used in the coil spring is the SAE 5160. All the signals were recorded using sampling frequency of 500 Hz for 132 seconds. Recording load histories at a frequency of 500 Hz is sufficient to detect and capture all damaging load cycles [21], and the selection of 500 Hz was adequate for the on-site strain signal collection. This is also to ensure that the essential components of the signal are not lost during measurement [22].

The irregular strain histories recorded in the experiment were analysed using the Rainflow cycle counting for individual fatigue cycle determination. Then, descriptive statistics analysis was performed to identify the load data behaviour. There are four types of commonly used global statistics for signals characterisation considered here; the mean, root-mean-square (r.m.s), skewness, and kurtosis. The Anderson Darling (AD) test for normality was implemented to confirm the load cycle distribution.

The Rainflow cycle counting provided the maximum and minimum magnitudes of each counted cycle. The difference of these two values gave the strain range, which is also corresponding to the size of that particular fatigue cycle. The strain range data for each signal was determined and denoted by a different notation, R*i* to differentiate with the original load amplitude. The global statistics were calculated for statistical description of the strain range data. Then, four types of distribution; the exponent, Gamma, 3P-Weibull, and 2P-Weibull; were selected as the candidates for distribution fitting of the data. These candidate distributions were chosen based on the obtained characteristics from global statistics analysis. Due to the suggestions from previous research, a mixed Weibull analysis was also performed on the strain range data.

4. Results and Discussion

The global statistics values are depicted in Table 2 for each signal. The mean values are obtained in the range of −100.84 $\mu\varepsilon$ (for S3) to 17.95 $\mu\varepsilon$ (for S4) with three out of four values are negatives. The negative mean value indicates that most of the strain signals are in compression state rather than tension. This outcome is consistent with the type of strain signals collected from the coil spring of suspension system [20].

Table 2: Global statistics for all strain signals

Signal	Mean ($\mu\varepsilon$)	r.m.s ($\mu\varepsilon$)	Skewness	Kurtosis
S1	−78.85	126.43	0.36	2.82
S2	−15.14	44.60	0.42	4.45
S3	−100.84	218.79	−0.33	2.04
S4	17.95	110.62	1.11	5.56

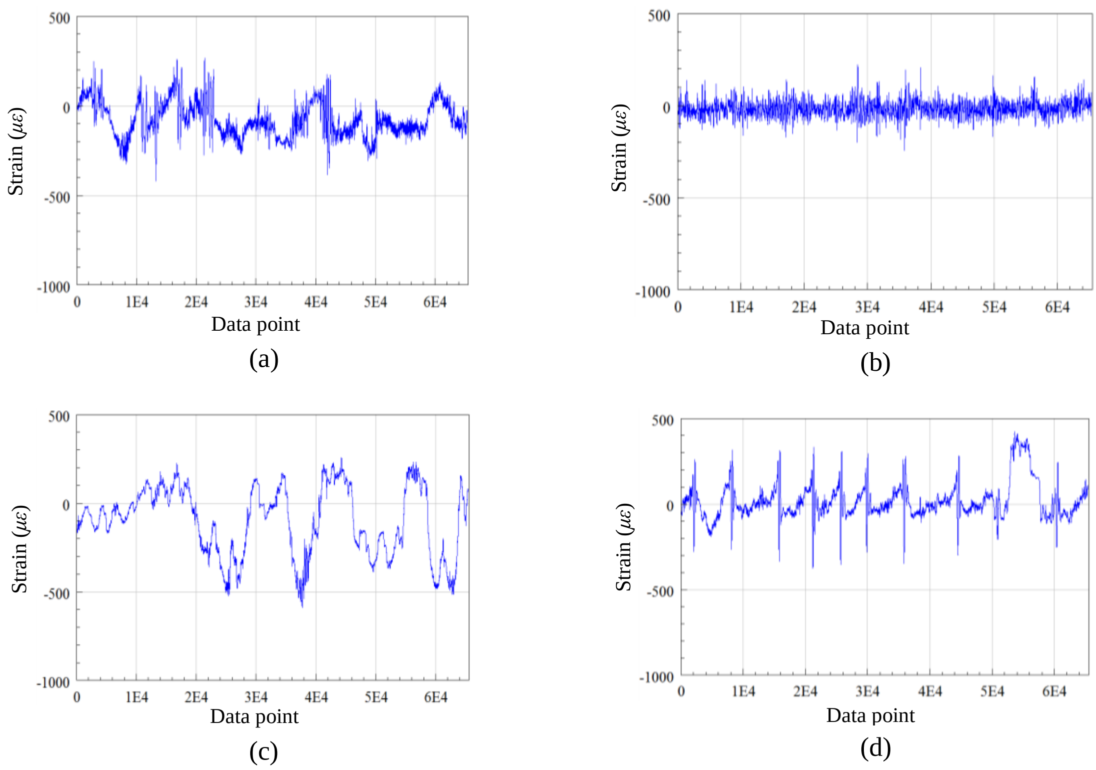

Fig. 1: Data series of strain signals for: (a) S1, (b) S2, (c) S3, and (d) S4

The positive mean of S4 is because of the existence of higher strain signal values in tension state. The obvious repetitive trend seen in measured data for S4 was due to the condition of road surfaces found in housing area which have multiple humps for safety purposes. The skewness values for all signals data are positive (except for S3) due to the existence of numerous small amplitude signals. Generally, all data are distributed asymmetrically with S1, S2, and S4 data that skewed to the right while S3 is skewed to the left. This outcome indicates that the strain signal data produced by coil spring component in service does not follow a Gaussian distribution.

The kurtosis values obtained ranging from 2.04 to 5.56 but none of the signals has kurtosis value approximately 3. Therefore, it is apparent that none of the signals follows Gaussian distribution. For data with kurtosis value greater than 3 such as S2 (4.45) and S4 (5.56), this indicates the presence of extreme values more than could be found in Gaussian distribution [14]. This means that the tail of distribution is also thicker than the Gaussian distribution tail. These extreme values most possibly contribute significantly to fatigue damage because they are most likely related to the higher magnitude of fatigue cycles.

In the context of oscillatory energy content, S3 provides the highest r.m.s value of 218 $\mu\varepsilon$ while S2 has the lowest r.m.s of 44.60 $\mu\varepsilon$. These results can be explained by the location background of the data captured.S3 was recorded in the rural area where the road surfaces are rough and mostly braking and cornering manoeuvre involved. This characteristic is demonstrated by the non-stationary time series of strain signal (as observed previously in Fig. 1 (c)) and the high energy content shown by the r.m.s value. On the other hand, S2 data which was recorded from vehicle moving on the highway exhibits a different characteristic. Due to the smoother highway road surfaces, the energy content shown by the strain signal is also consistently the lowest among all signals considered.

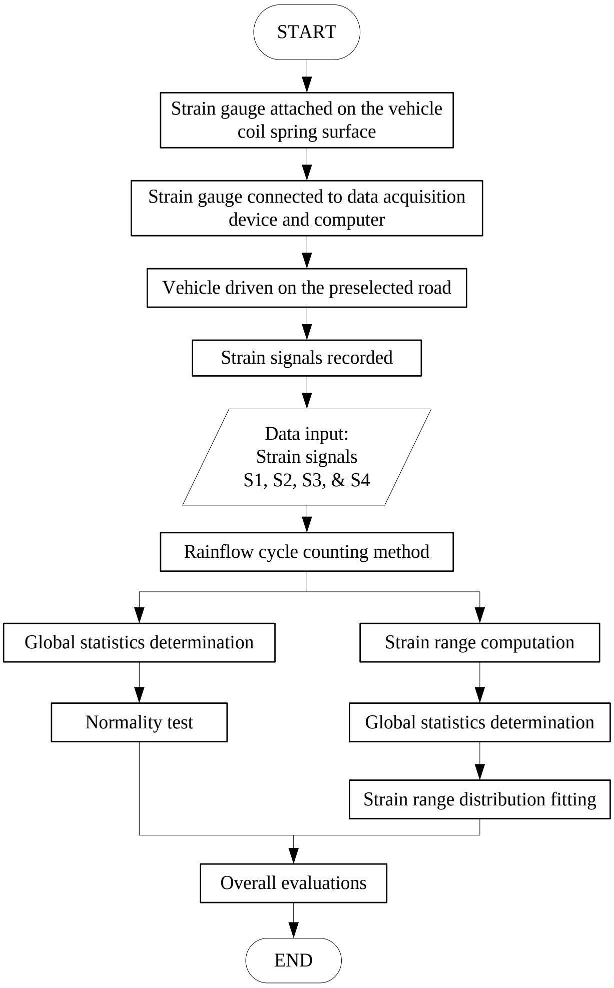

Fig. 2: Process flow of the methodology

The findings based on global statistics indicate the non-Gaussian distribution of strain data. To investigate further, the probability density function (PDF) plots of all signals are shown in Fig. 3. The normal curves were superimposed in the PDF plots so that a direct comparison can be achieved. Overall observations on the PDF plots further support that the strain data follows non-Gaussian distribution. However, the PDF of S2 appears to resemble the normal curve. For confirmation, a normality test was performed using the AD test and the results are detailed in Table 3.

Table 3: Normality test results for each signal

Signal	AD	*p*-value	Normality?
S1	439.8	< 0.005	No
S2	111.8	< 0.005	No
S3	746.2	< 0.005	No
S4	2264.2	< 0.005	No

The hypotheses used were H_0 : the data follows a normal distribution; versus H_1 : the data does not follow a normal distribution. Using a 5% significance level, all *p*-values obtained from AD test were less than 0.005 which is also less than the significance level. Consequently, the null hypothesis cannot be accepted and it is concluded that all strain data are non-Gaussian including S2 that has PDF plot almost resembles the normal curve. The AD value shows indication regarding the goodness of fit of data distribution to the normal distribution. The higher the AD value the less likely that the data is Gaussian [12]. In Table 3, S2 has the smallest AD value (111.8) while S4 produces the highest one (2264.2) signifying S4 has the farthest departure from normality. Load non-normality has a great effect on fatigue damage and under certain circumstances may be responsible for the increase in the rate of damage accumulation [15]. Therefore, it is important to identify the load distribution behaviour.

The relation between the size of cycles and fatigue damage are closely related [23]. The size of fatigue cycles can be represented by the magnitude of fatigue strain range ($\Delta\varepsilon$). Thus, it is necessary to know the distribution behaviour of strain range data especially for the analysis of fatigue life prediction. The strain range value was obtained by subtracting the minimum strain from the maximum.

Overall statistical description of strain range data is given in Table 4. The symbol R1, R2, R3, and R4 represent the strain range data to differentiate with original strain signals S1, S2, S3, and S4 in previous analysis.

Table 4: Global statistics values for each strain range data

Data	Mean ($\mu\varepsilon$)	Skewness	Kurtosis
R1	17.86	9.82	155.60
R2	15.93	6.46	62.88
R3	5.54	27.07	995.19
R4	7.25	19.91	483.75

In Table 4, all mean values are in the range of 5.54 $\mu\varepsilon$ to 17.86 $\mu\varepsilon$ implying that the average loads endured by the component in service are the low amplitudes. In terms of shape of distribution, all signals are found to be asymmetric based on the non-zero skewness level. R3 produces the highest skewness value of 27.07 while R2 has the lowest value of 6.46. Observation on the kurtosis of all signals shows extremely high magnitudes up to 995.19 given by R3. This indicates the existence of numerous extreme values and very thick distribution tails.

Overview of the strain range distribution is depicted in Fig. 4 using boxplots. The asterisks in each boxplot are the outlier data points or extreme values. The outliers clearly signify that the coil spring component endured multiple, very high amplitude fatigue cycles which most likely contributing significantly to fatigue damage. These outcomes confirmed that the strain range distribution is highly skewed due to existence of numerous extreme values indicating non-Gaussian distribution. This characteristic is highlighted in the probability density plots (PDF) as shown in Fig. 5. There is a clear evidence of departure from normality in the PDF plots based on the asymmetric distribution shapes. Also, it is apparent that the strain range data has frequent small cycles as highlighted at the lower magnitudes.

Thus, consideration regarding the appropriate type of distribution for modelling of strain range data has to be based on the following characteristics: skewed and thick-tailed distribution. Therefore, several types of distribution which are often used in the modelling of engineering data are considered in this analysis. The distributions are the exponent, Gamma, 2P-Weibull, and 3P-Weibull.

Distribution fitting evaluation was performed using the AD test. The corresponding results are detailed in Table 5. Based on the four types of candidate distributions, the 2P-Weibull provides the highest *p*-value of 0.01. Even though this value is slightly less than 0.05, consideration has been made to accept this type of distribution for further analysis. This is due to the fact that it has been used as linear combinations to model strain range data in previous research [24].

Previously, Tovo [25] and Klemenc and Fajdiga [24] have applied the linear combinations of 2P-Weibull distribution to model the strain range data in fatigue life analysis. Thus, this distribution type was used as a candidate in modelling the strain range of coil spring data in this study. The strain range data are multimodal with multiple peaks and can be modelled by a distribution with a non-straight probability plot such as the mixed Weibull distribution. Accordingly, the strain range data for all signals were fitted to the probability plot of (i) 2P-Weibull and (ii) mixed Weibull distribution side by side as seen in Fig. 6.

In fatigue damage analysis, the distribution of high level of strain range data is emphasized more compared to the low level data as highlighted in Fig. 6 (a,i) and (a,ii). The fitting at lower magnitude of strain range appears inaccurate due to various small cycles. Because fatigue damage is caused mostly by the high level of strain range data [23], therefore the fitting at lower amplitude is not very significant to be considered. Apparently, the mixed Weibull distribution fitting appeared to be more accurate than the 2P-Weibull for the strain range data. The number of peaks for each data is provided by the number of sub−population as given in Table 6. It is observed that R1 and R2 have two sub−population, while R3 and R4 have three sub−population depending on the number peaks or modals possessed by the data. The portions for each data should sum up to unity for a complete mixed Weibull distribution.

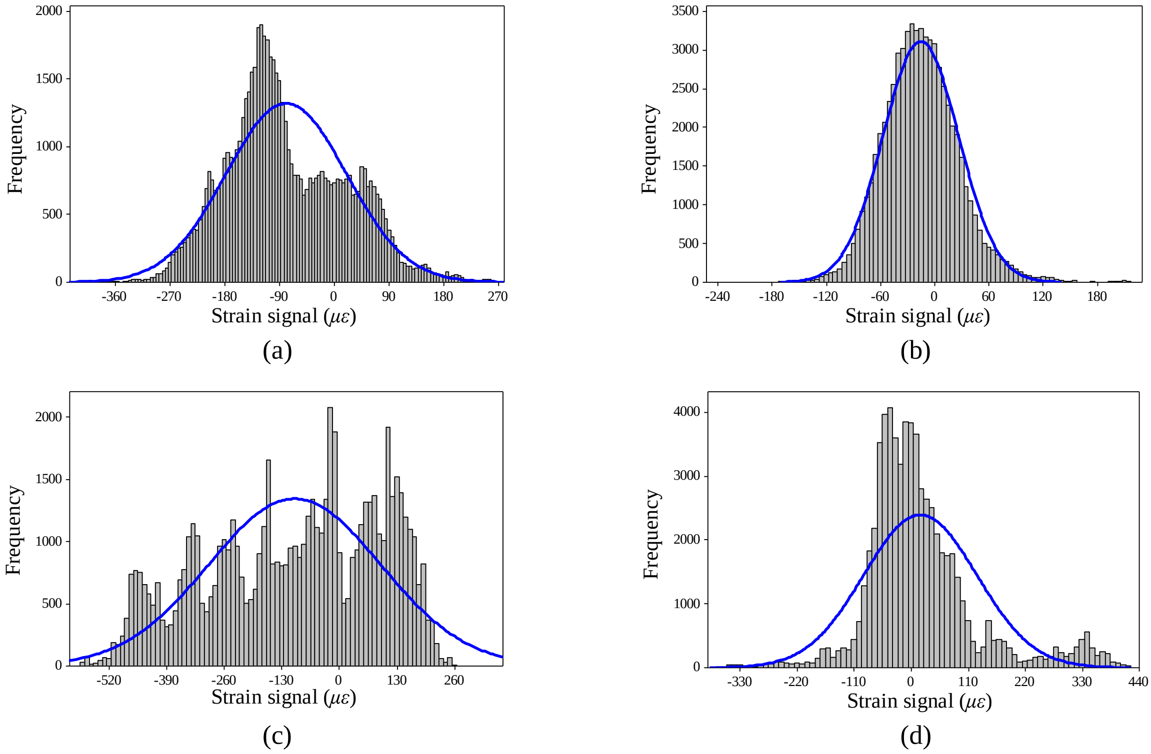

Fig. 3: Histogram of data distribution for all strain histories: (a) S1, (b) S2, (c) S3, and (d) S4

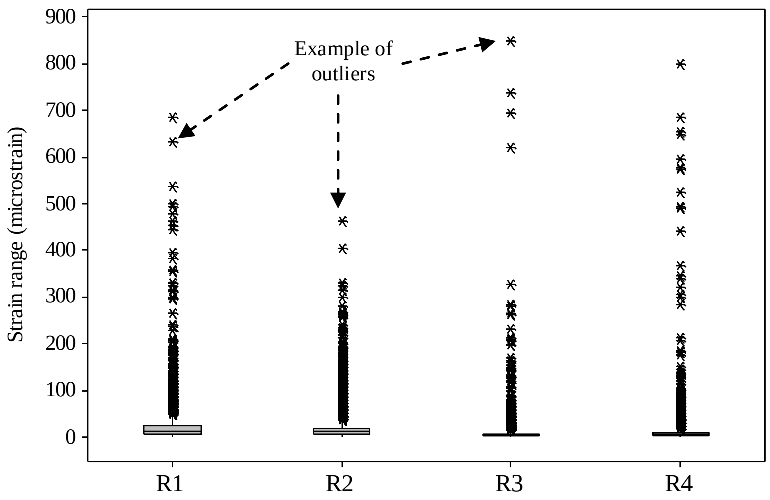

Fig. 4: Boxplots of each signal strain range distribution

Table 5: Distribution fitting results for each strain range data

Data	Exponent		Gamma		3P-Weibull		2P-Weibull	
	AD	***p*-value**	**AD**	***p*-value**	**AD**	***p*-value**	**AD**	***p*-value**
R1	24.9	<0.003	27.9	<0.005	41.5	<0.005	38.4	<0.01
R2	142.3	<0.003	129.6	<0.005	164.0	<0.005	159.8	<0.01
R3	261.5	<0.003	143.5	<0.005	98.4	<0.005	105.2	<0.01
R4	212.2	<0.003	167.5	<0.005	149.2	<0.005	152.1	<0.01

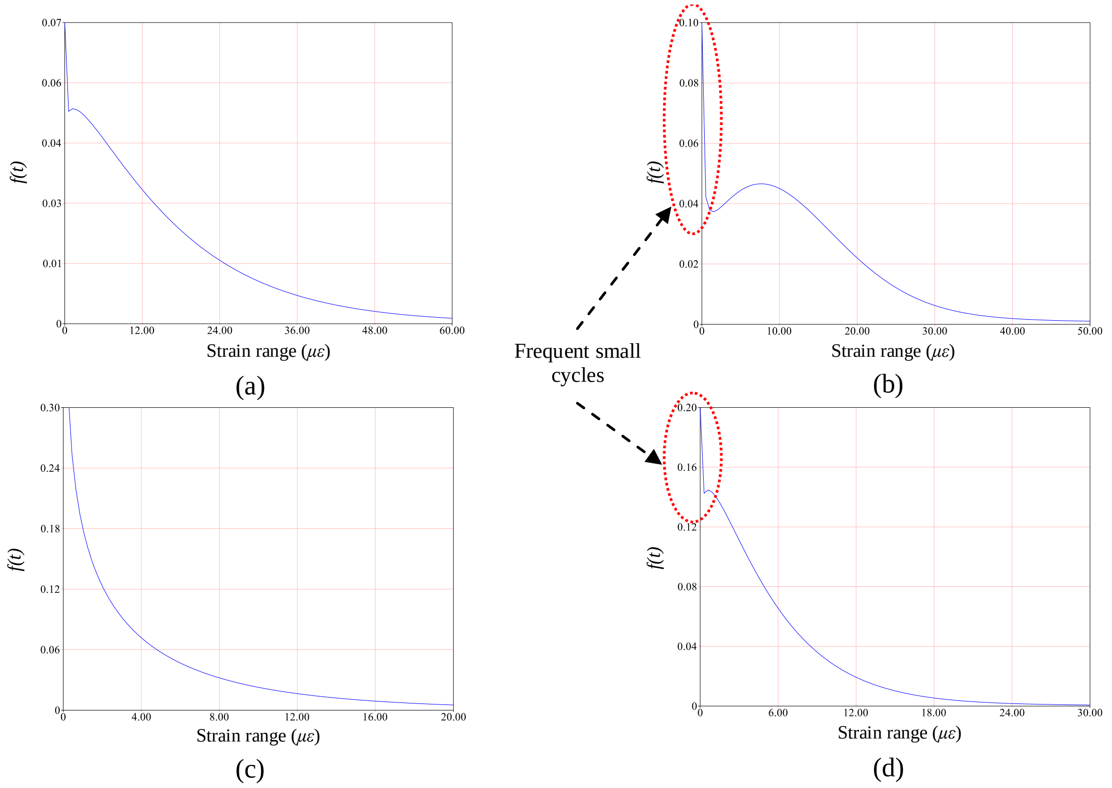

Fig. 5: Probability density function plot for each strain range data

Table 6: Mixed Weibull distribution parameter values for each strain range data

Data	Parameter	Sub-population I	Sub-population II	Sub-population III
R1	β	1.10	0.61	
	δ	16.14	37.41	–
	Portion	0.94	0.06	
R2	β	1.73	0.64	
	δ	14.40	16.61	–
	Portion	0.71	0.29	
R3	β	1.98	1.13	0.68
	δ	0.30	4.23	18.02
	Portion	0.05	0.87	0.08
R4	β	1.50	1.19	0.88
	δ	0.47	10.19	40.57
	Portion	0.05	0.92	0.03

5. Conclusion

This paper presented a characterisation of the scatter of strain histories obtained from the coil spring of vehicle suspension system driven on four different road profiles. Statistical distribution analysis was used to analyse the strain data. Initial description of the strain histories was determined by computing the global statistics (the mean, root-mean-square, skewness, and kurtosis) values. The findings showed that strain range data are highly skewed with thick-tailed indicating a non-Gaussian distribution. Based on these characteristics, distribution fitting process was performed in order to find the most suitable distribution type for modelling the strain range data. Four different types of distribution i.e. the exponent, Gamma, 3P-Weibull, and 2P-Weibull, were compared to find the best fit for the strain range data using the Anderson-Darling test. Even though all the tested distribution turned out to be insignificance, the closest significance value was produced by the 2P-Weibull distribution with p-value 0.01 at significance level 0.05. Further, due to the non-straight probability plot of data, the mixed Weibull distribution comprises of 2P-Weibull linear combinations was selected. Two of the data sets (campus area and highway) follow the mixed Weibull distribution with two sub-populations and another two data sets (rural and housing area) with three sub-populations. It can be concluded that the mixed Weibull distribution is suitable to model the strain range data of vehicle coil spring. These findings can be used in time-domain fatigue life evaluation.

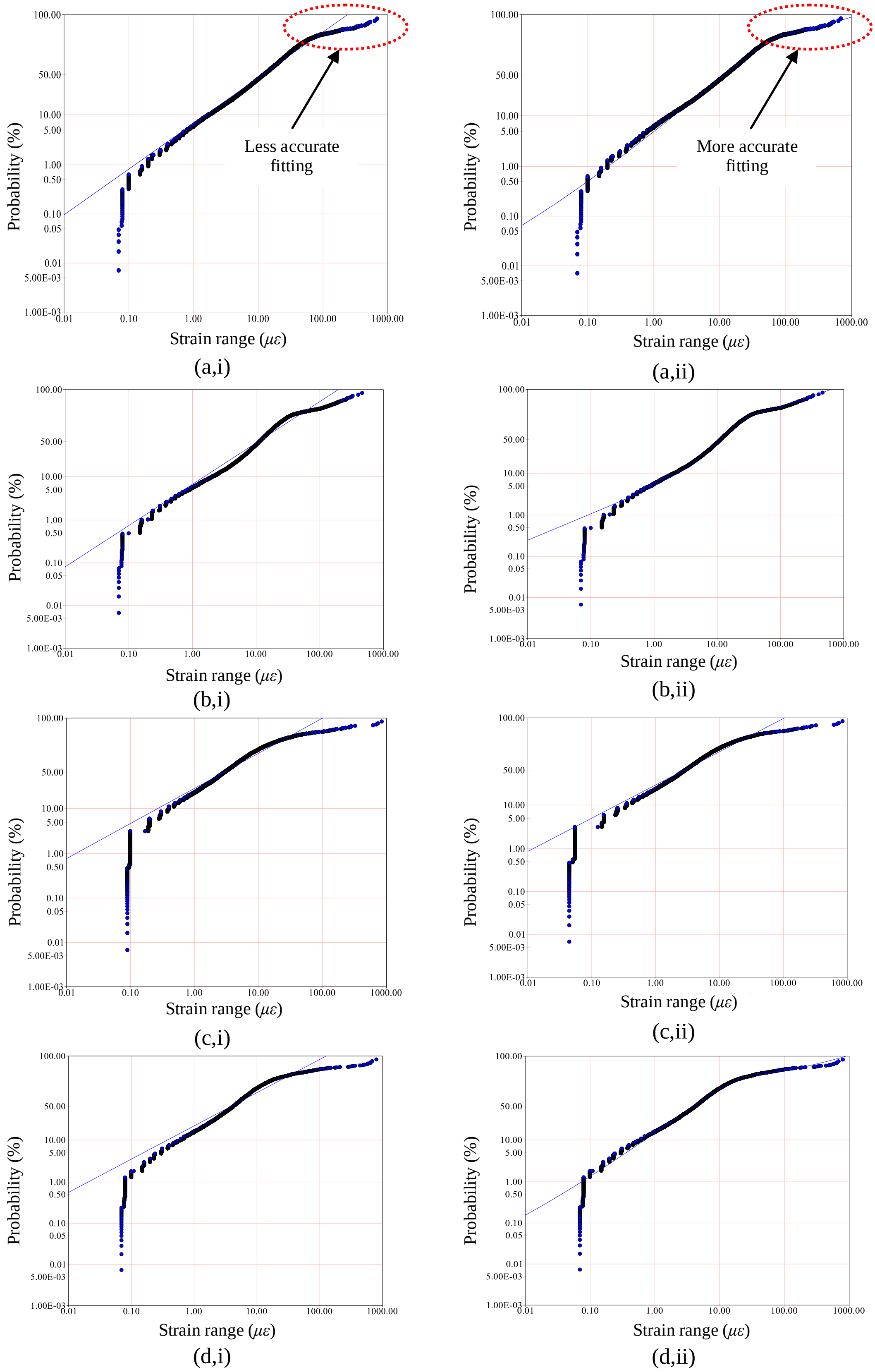

Fig. 6: Probability plots of each strain range data for: (i) 2P-Weibull and (ii) mixed Weibull distribution

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