International Journal of Engineering &Technology, 7 (3.12) (2018) 407-410

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

Probabilistic Fracture Energy Assessment of Natural Fibre Reinforced Concrete by Two Parameter Weibull Distribution

Neha P Asrani[1], Murali G[2], Arthika J[3], Karthikeyan. K[4], Haridharan. M.K[5]

[1,2,3] School of Civil Engineering, SASTRA Deemed University, Thanjavur, India
[4]SMBS, VIT Deemed to Be University, Chennai, Tamil Nadu, India.
[5]Amrita School of Engineering, Amrita Vishwa Vidyapeetham Deemed To Be University, Coimbatore, India.
**Corresponding Author: murali_220984@yahoo.co.in[2]*

Abstract

Fracture energy is the post-crack energy absorption ability of the material that represents the energy absorbed by the structure at the time of failure. Its analysis has gained importance and hence requires a powerfulmethod for its development. A two parameter Weibull distribution proves to be an efficient tool in analysing the scattered experimental test results. In this paper, the specific fracture energy of plain concrete and concrete reinforced with natural fibres of hemp, wheat straw and elephant grass are statistically analysed by two parameter Weibull distribution by using graphical method. For determining Weibull parameters, 21 equations have been used and the best equation is taken for the reliability analysis. A Weibull reliability curve is plotted, which shows the specific fracture energy at each reliability level. This curve enables an engineer to choose the fracture energy of a particular mix based on its reliability requirement and safety limit. Therefore, reliability curves are a pioneer in statistical analysis as they eliminate the time-consuming and costly experimental process. This method can be applied in areas with similar uncertainties.

Keywords: *Fracture energy, Weibull Distribution, Reliability, Shape parameter, Scale parameter, Natural fibre.*

1. Introduction

Particle fragmentation is highly profuse and is seen in many engineering fields. In consequence, it has attracted undivided interest in research over the past few years [1].Past research has explained several strategies for enhancing the fracture energy of concrete using various types of manmade fibres [2]. Due to its limitations in terms of sustainability, the use of alternate natural fibres has gained importance. Due to their excellent mechanical properties, they are considered as the most promising structural materials in sustainable engineering technologies [37]. In the current day scenario, both plant based (jute, hemp, sisal kenaf, coir, flax, bamboo, banana) and animal based (Cocoon silk, chicken feather, wool, spider silks) fibres are used as they are economical, sustainable and exhibit good strength characteristics.

Merta and Tschegg [4] studied the specific fracture energy of plain concrete and concrete reinforced with wheat straw, hemp and elephant grass fibres. Wedge splitting test was carried out on specimens of concrete containing 40 mm long fibres, which were 0.19% by weight. It was found that asmore fibres were added, the fracture toughness of concrete increased. It was examined that all its mixes showed a large scatter in results.This requires an enhancement in mixing technique so as to obtain a uniform matrix.

Currently, the focus of research is on predicting the failure life of concrete, for which many methods are available. Weibull distribution is one such method. It was initially established byWallodi Weibull [5]. Weibull parameters can be estimated by Maximum Likelihood method, Graphical Method, Empirical Method of Justus, Empirical Method of Lysen, Energy Pattern Factor Method and Maximum Likelihood method [6].

In the present study, Graphical method has been used by applying 21 probability estimators [7-31] to the experimental data of specific fracture energy conducted by Merta and Tschegg [4]. Results are then presented, not as the average of the data but in terms of various reliability levels. This provides more safety and accuracy.

2. Experimental Procedure

2.1 Weibull Distribution Theories

The two parameter Weibull distribution function is a broadly utilized method in determining fatigue life, wind distribution and impact strength investigations [32]. The Weibull statistical analysis of mathematical probability distribution function is given by [32]:

$$f(n) = \frac{\gamma}{\alpha}\left(\frac{n}{\alpha}\right)^{\gamma-1} exp\left[-\left(\frac{n}{\alpha}\right)\right] \quad (1)$$

The cumulative function can be obtained by integrating Eq. (1):

$$F(n) = 1 - exp\left[-\left(\frac{n}{\alpha}\right)^{\gamma}\right] \quad (2)$$

Where, n = random variable N; γ = shape parameter and α = scale parameter.

Copyright © 2017 Authors. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

2.2 Analysis by Graphical Method

The probability of survival or reliability function, $L_N(n)$ of the two-parameter Weibull distribution, may be defined as $L_N(n) = 1-F(n)$, and this equation can be converted to following form [5]:

$$L_N(n) = \exp\left[\frac{n}{\alpha}\right]^{\gamma} \quad (3)$$

Taking twice logarithm for Eq. (3):

$$\ln\left[\ln\left(\frac{1}{L_N}\right)\right] = \gamma \ln n - \gamma \ln \alpha \quad (4)$$

$$Y = \gamma X - \beta \quad (5)$$

Where, $Y = \ln[\ln(1/L_N)]$, $X = \ln(n)$, and $\beta = \gamma \ln \alpha$. (6)

Eq. (4), represents a linear relationship between X and Y. The slope of the line represents the shape parameter (γ). The second term of Eq. (5) gives the scale parameter (α).

Experimental data of specific fracture energy (N/mm^2) from the tests conducted by Mertaand Tschegg [4] is shown in Table 1.

Table 1: Experimental test results of Mertaand Tschegg [4]

S.No.	Plain Concrete	Hemp FRC	Straw FRC	Elephant grass FRC
1	121.46	185.16	126.22	119.09
2	114.07	169.05	115.03	131.98
3	136.55	256.53	133.33	136.01
4	130.56	251.79	126.86	148.58
5	95.37	159.32	110.83	153.23

This data is arranged in an ascending order by assigning a serial number for each value. Then, the reliability function (L_N) for each value of 'n' is calculated by applying 21 equations as shown in Table 2, after which a graph is obtained which gives the values of γ and α.

Table 2: Probability Estimators

Eq. No.	Author	Year	Probability Estimator
(7)	[7]	(1814)	$F_1 = \frac{i+1}{k+2}$
(8)	[8]	(1914)	$F_2 = \frac{i-0.5}{k}$
(9)	[9]	(1923)	$F_3 = \frac{i}{k}$
(10)	[10]	(1923)	$F_4 = \frac{i-1}{k}$
(11)	[11]	(1939)	$F_5 = \frac{i}{k+1}$
(12)	[12]	(1943)	$F_6 = \frac{i-0.33}{k}$
(13)	[13]	(1953)	$F_7 = \frac{i-0.3}{k+0.4}$
(14)	[14]	(1958)	$F_8 = \frac{i-0.375}{k+0.25}$
(15)	[15]	(1962)	$F_9 = \frac{i-0.333}{k+0.333}$
(16)	(16)	(1963)	$F_{10} = \frac{i-0.44}{k+0.12}$
(17)	(17)	(1975)	$F_{11} = \frac{i-0.3175}{k+0.365}$
(18)	(18)	(1977)	$F_{12} = \frac{i-0.31}{k+0.38}$
(19)	(19)	(1978)	$F_{13} = \frac{i-0.4}{k+0.2}$
(20)	(20)	(1981)	$F_{14} = \frac{i-0.24}{k+0.5}$
(21)	(21)	(1981)	$F_{15} = \frac{i+0.5}{k+1}$
(22)	(22)	(1991)	$F_{16} = \frac{i-0.5}{k+0.25}$
(23)	(23)	(1994)	$F_{17} = \frac{i-0.44}{k+0.25}$
(24)	(24)	(2000)	$F_{18} = \frac{i-0.999}{k+1000}$

(25)	(25)	(2014)	$F_{19}=\frac{i-0.25}{k+0.5}$
(26)	[26]	(2009)	$F_{20}=1-\frac{i-0.3}{k+0.4}$
(27)	[27-31]	(2007, 2008, 2009, 2012, 2017)	$F_{21}=1-\frac{i}{k+1}$

Here, i is the failure order number and k is the total number of fracture energy data points.

3. Results and Discussion

Table 3 displays the shape and scale parameters of PC, HFRC, SFRC and EFRC by using various probability estimators. Results showed that F_{21} had the highest value of R^2, which was 0.9727, 0.8601, 0.9454 and 0.9791 respectively. Since the R^2 values for the four mixtures are greater than 0.85, it substantiated the existence of a linear relationship between Y and X. This validates that the distribution of fracture energy for all specimens follows the two-parameter Weibull distribution.

Table 3: Shape and Scale Parameters obtained from different probability estimators

Probability Estimator	Parameters	PC	HFRC	SFRC	EFRC
F_1	γ	-5.32	-3.43	-10.12	-7.79
	α	107.57	163.54	114.04	125.50
F_2	γ	-7.77	-4.91	-14.62	-11.25
	α	110.93	179.93	117.86	130.96
F_3	γ	-3.29	-1.91	-5.14	-4.41
	α	106.49	166.21	113.98	126.56
F_4	γ	-3.75	-2.98	-8.08	-5.88
	α	107.91	177.70	116.89	129.16
F_5	γ	-5.90	-3.72	-11.06	-8.52
	α	109.81	177.07	117.23	130.04
F_6	γ	-8.32	-5.33	-15.81	-12.13
	α	109.03	175.41	116.82	129.47
F_7	γ	-6.86	-4.33	-12.88	-9.99
	α	110.50	178.82	117.61	126.22
F_8	γ	-7.16	-4.52	-13.47	-10.37
	α	110.65	179.25	117.69	130.72
F_9	γ	-8.13	-5.20	-15.42	-11.84
	α	109.22	175.85	116.93	129.59
F_{10}	γ	-7.46	-4.71	-14.03	-10.80
	α	110.80	179.61	117.77	130.88
F_{11}	γ	-6.92	-4.37	-13.01	-10.02
	α	110.52	178.90	117.61	130.64
F_{12}	γ	-6.90	-4.35	-12.96	-9.97
	α	110.52	178.86	117.62	130.62
F_{13}	γ	-7.27	-4.59	-13.68	-10.53
	α	110.72	179.37	117.72	130.77
F_{14}	γ	-6.68	-4.22	-12.56	-9.67
	α	110.27	178.24	117.49	130.42
F_{15}	γ	-6.87	-4.44	-13.09	-10.05
	α	105.32	166.46	114.74	126.47
F_{16}	γ	-7.00	-4.38	-13.08	-10.08
	α	112.13	182.85	118.50	131.89
F_{17}	γ	-7.06	-4.44	-13.24	-10.20
	α	111.40	181.07	118.12	131.31
F_{18}	γ	-2.56	-1.22	-4.17	-3.37
	α	255.95	1007.22	195.92	245.96
F_{19}	γ	-6.67	-4.21	-12.53	-9.65
	α	110.38	178.53	117.55	130.51
F_{20}	γ	7.32	4.25	13.20	10.24
	α	126.92	224.75	126.78	143.94
F_{21}	γ	6.23	3.66	11.30	8.75
	α	127.75	226.95	127.24	144.61

By using Weibull parameters obtained from F_{21}, the specific fracture energy in terms of reliability or probability of survival (N_R) can be computed by using Eq. (28) as given below [33-39]

$$N_R=\alpha(-\ln(R)^{\frac{1}{\gamma}}) \quad (28)$$

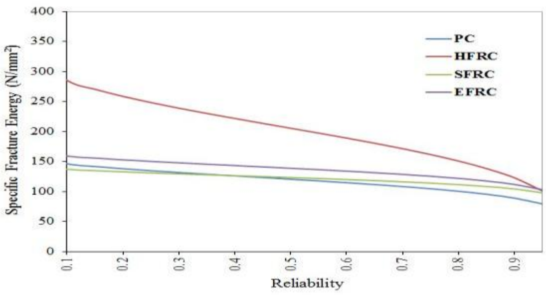

Fig. 1: Weibull Reliability Curve

Fig 1 shows the Specific fracture energy at various reliability levels for the four mixes. Forthe 0.90 reliability level, the specific fracture energy values are 89.03, 122.88, 104.27 and 111.82 for PC, HFRC, SFRC and EFRC respectively. Similarly, for 0.1 reliability level, the specific fracture energy obtained from the curve of PC, HFRC, SFRC and EFRC are 146.05, 284.89, 136.98 and 159.07 respectively. For 0.5% reliability level the specific fracture energy values are 120.46, 205.37, 123.18 and 138.68 for PC, HFRC, SFRC and EFRC respectively. These values are almost equal to the mean fracture energy values of experimental data. This shows that if the mean of the experimental data is taken, it gives 50% reliability according to probabilistic assessments.

4. Conclusions

In our study, the specific fracture energy of plain concrete and concrete reinforced with natural fibres of hemp, wheat straw and elephant grass are statistically analysed by two parameter Weibull distribution by using graphical method. For determining Weibull parameters using graphical method of analysis, 21 equations have been used and the best equation (F_{21}) is taken for reliability analysis. A Weibull reliability curve is plotted, which shows the specific fracture energy at each reliability level. This curve enables an engineer to choose the fracture energy of a particular mix based on its reliability requirement and safety limit. Therefore, reliability curves are a pioneer in statistical analysis as they eliminate the time- consuming and costly experimental process. This method can be applied in areas with similar uncertainties.

References

[1] P.-S. Laplace, EssaiPhilosophique Sur Les Probabilités, Courcier, Paris, 1814.

[2] A. Hazen, Storage to be provided in impounding reservoirs for municipal water supply, Trans. Am. Soc. Civil Eng. 77 (1914) 1547–1550.

[3] California Department of Public Works, Flow in California streams, Calif., Dep. Public Works, Div. Eng. Irrig., Bull. 5 (1923).

[4] California Department of Public Works, Flow in California streams, Calif., Dep. Public Works, Div. Eng. Irrig., Bull. 5 (1923).

[5] W. Weibull, A statistical theory of strength of materials, Ing. Vet. Ak. Handl. (Stockholm) 151 (1939).

[6] L.R. Beard, Statistical analysis in hydrology, Trans. Am. Soc. Civil Eng. 108 (1943) 1110–1160.

[7] A. Benard, E.C. Bos-Levenbach, The plotting of observations on probability paper, Statistica 7 (1953) 163–173.

[8] G. Blom, Statistical Estimates and Transformed Beta-variables, J. Wiley and Sons, 1958.

[9] J.W. Tukey, The future of data analysis, Ann. Math. Stat. 33 (1) (1962) 21–24.

[10] I.I. Gringorten, A plotting rule for extreme probability paper, J. Geophys. Res. 68 (1963) 813–814.

[11] J. Filliben, The probability plot correlation coefficient test for normality, Technometrics 17 (1975) 111–117.

[12] A.F. Jenkinson, The Analysis of Meteorological and Other Geophysical Extremes, Meteorological Office, UK, 1977, Technical Note No. 58.

[13] C. Cunnane, Unbiased plotting positions—a review, J. Hydrol. 37 (1978) 205–222.

[14] K. Adamowski, Plotting formula for flood frequency, J. Am. Water Resour. Assoc. 17 (2) (1981) 197–202.

[15] R.M. Hirsch, Estimating probabilities of reservoir storage for the upper Delaware River basin, in: Open File Report 81–478, U.S. Geological Survey, Reston, VA, 1981.

[16] A. Khalili, K. Kromp, Statistical properties of Weibull estimators, J. Mater. Sci. 26 (24) (1991) 6741–6752.

[17] R. Ross, Graphical methods for plotting and evaluating Weibull distributed data, in: Proc. 4th Int. Conf. Properties Applicat. Dialectric Maters., June 3–8th 1994, Brisbane, Australia, 1994, pp. 250–253.

[18] J. Gong, A new probability index for estimating Weibull modulus for ceramics with the least-square method, J. Mater. Sci. Lett. 19 (2000) 827–829.

[19] S. Nohut, C. Lu, L. Gorjan, Optimal linear regression estimator in the fitting of Weibull strength distribution, J. Test. Eval. 42 (6) (2014) 1396–1407.

[20] R. Bedi, R. Chandra. Fatigue-life distributions and failure probability for glass-fiber reinforced polymeric composites. Composites Science and Technology 69 (2009) 1381–1387. doi:10.1016/j.compscitech.2008.09.016

[21] H. Li, M. Zhang, J. Ou, Flexural fatigue performance of concrete containing nano-particles for pavement, Int. J. Fatigue 29 (7) (2007) 1292–1301. http://dx.doi.org/10.1016/j.ijfatigue.2006.10.004.

[22] S. Raif, A. Irfan, Statistical analysis of bending fatigue life data using Weibull distribution in glass-fiber reinforced polyester composites, Mater. Des. 29 (6) (2008) 1170–1181. http://dx.doi.org/10.1016/j.matdes.2007.05.005.

[23] L. Wang, H. Wang, J. Jia, Impact resistance of steel-fibre-reinforced lightweight-aggregate concrete, Mag. Concr. Res. 67(2009) 539–547, http://dx.doi.org/10.1680/macr.2007.00128.

[24] S. Goel, S.P. Singh, P. Singh, Fatigue analysis of plain and fiber-reinforced self consolidating concrete, ACI Mater. J. 109 (2012) 573–582. http://dx.doi.org/10.14359/51684089.

[25] Y Ding, D Lia, Y Zhang, C Azevedo. Experimental investigation on the composite effect of steel rebars and macro fibers on the impact behavior of high performance self-compacting concrete. Construction and Building Materials 136 (2017) 495–505. http://dx.doi.org/10.1016/j.conbuildmat.2017.01.073

[26] Yeon, K.S., Choi, Y.S., Kim, K.K., and Yeon, J.H., "Flexural fatigue life analysis of unsaturated polyester-methyl methacrylate polymer concrete," Construction and Building Materials, Vol. 140, pp. 336–343 (2017). doi: 10.1016/j.conbuildmat.2017.02.116

[27] Merta, I., and Tschegg. E.K., "Fracture energy of natural fibre reinforced concrete," Construction and Building Materials, Vol. 40, pp. 991–997 (2013). doi: 10.1016/j.conbuildmat.2012.11.060

[28] Weibull, W., A statistical theory of the strength of materials, Series 151, General stabenslitografiskaanstaltsförlag, Stockholm, (1939).

[29] Ali, M.A.E.M., Soliman, A.M., and Nehdi, M.L., "Hybrid-fiber reinforced engineered cementitious composite under tensile and impact loading," Materials Design, Vol. 117, pp. 139–149 (2017). doi: 10.1016/j.matdes.2016.12.047

[30] Murali, G., Monika Vincy, T., Suraj, K., Ramkumar, V.R., and Karthikeyan, K., "Assessment of Impact Strength of Fibre Reinforced Concrete by Two Parameter Weibull Distribution," ARPN Journal of Engineering and Applied Sciences, Vol. 12(5), pp. 1440-1445 (2017).

[31] Murali, G., Muthulakshmi, T., NycilinKarunya, N., Iswarya, R., Hannah Jennifer, G. and Karthikeyan, K., "Impact Response and Strength Reliability of Green High Performance Fibre Reinforced Concrete Subjected to Freeze-Thaw Cycles in NaCl Solution," Materials Science Medziagotyra, Vol. 23(4), pp. 384-388 (2017).

[32] Murali, G., Santhi, A.S. and Mohan Ganesh, G., "Impact resistance and strength reliability of Fiber-reinforced concrete in bending under drop weight impact load," International Journal of Technology, Vol. 5(2), pp. 111-120 (2014).

[33] Murali, G., Santhi, A.S., and Mohan Ganesh, G., "Impact resistance and strength reliability of fiber reinforced concrete using two parameter Weibull distribution," ARPN Journal of Engineering and Applied Sciences, Vol. 9(4), pp. 554-559 (2014).

[34] Sakin, R., and Irfan, A.Y., "Statistical analysis of bending fatigue life data using Weibull distribution in glass-fiber reinforced polyester composites," Materials and Design, Vol. 29, pp. 1170–1181 (2008). doi: 10.1016/j.matdes.2007.05.005

[35] Sriram, V., Sirisha, K., Keerthi, P.R., Murali, G., Haridharan, M.K., Karthikeyan, K., "A New Method to Estimate Weibull Parameter for the Fatigue Life of Self Compacting Fibre Reinforced Concrete Beams," International Journal of Civil Engineering Technology, Vol. 8(7), pp. 326–331 (2017).

[36] G. Murali, T. Indhumathi, K. Karthikeyan and V.R. Ramkumar. Analysis of flexural fatigue failure of concrete made with 100% coarse recycled and natural aggregates. Computers and Concrete, Vol. 21, No. 3 (2018) 291-298.

[37] Jianqiang, Wei, and Christian, Meyer, " Degradation rate of natural fibre in cement composites exposed to various accelerated aging environmental conditions," Corrosion Science, 88, pp.118-132 (2014).

[38] Yongfu, Xu, "The fractal evolution of particle fragmentation under different fracture energy," Powder Technology, Vol. 323, pp. 337-345 (2018).

[39] Karahan, O., Ozbay, E., Atis, C.D., Lachemi, M., and Hossain, K.M.A., "Effects of Milled Cut Steel Fibers on the Properties of Concrete," KSCE. J. Civil. Eng, Vol. 20(7), pp. 2783-2789 (2016). doi: 10.1007/s12205-016-0577-3.

[40] Kin-tak, Lau, "Properties of natural fibre composites for structural engineering applications," Composites Part B: Engineering, Vol. 136, pp. 222-233 (2018).