

# Outgoing longwave radiation prediction using dynamic mode decomposition

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## Abstract

Weather prediction is a very tedious process since lot of factors affect it and because of that it is very non-linear in nature. Many research works have shown that the Outgoing Longwave Radiation (OLR) has a very concrete linear relation with many weather parameters including rainfall and it is one of the key factor in determining the global energy budget. In this work we are predicting the global surface OLR by using past OLR data and loading it onto Dynamic Mode Decomposition (DMD) algorithm. The DMD is a technique which uses data driven dimensionality reduction approach for extracting dynamically relevant features which uses time-resolved numerical data for prediction and analysis.

**Keywords:** Arcgis; Dynamic Mode Decomposition (DMD); Outgoing Longwave Radiation (OLR); Prediction; Spatio-Temporal

## 1. Introduction

Outgoing Longwave Radiation is the energy radiating from the Earth as infrared radiation in the range of  $4\ \mu\text{m}$  and  $100\ \mu\text{m}$  to Space. The amount of OLR going out from the earth surface and the incoming shortwave radiation absorbed by earth from Sun will give us the energy budget or global heating and cooling of the Earth. OLR helps in the prediction of global warming since the greenhouse gases absorb OLR waves going out and thus increasing the atmospheric temperature and some of the infrared waves are reflected back to the Earth surface increasing the surface temperature. Since IR doesn't penetrate much through the clouds the position of the clouds could be understood by the OLR measurements. DMD is a recent dimensionality reduction algorithm formulated in the year 2008 by Peter J. Schmid. DMD is very efficient because it exploits the low dimensional structure of the data it is computationally very fast and effective. Because of this property, DMD is useful in many applications in various fields.

## 2. Related works

Since the clouds are opaque to the IR radiation the temperature at the top of the cloud will be sensed by the OLR sensors. The relation between summer monsoon rainfall and the OLR was studied by [3] and they found that rainfall is having a good relation with Sea Surface Temperature but it has a much better relation with OLR. The negative relationship between the OLR and rainfall was shown by [4] by taking into account the region of the South East Asia. They have also found out the correlation coefficients of each month. [5] has revealed a linear equation connecting the OLR and rainfall in which the amount of rainfall can be calculated by taking the number of days in which the OLR value is less than  $240\ \text{Wm}^{-2}$  in a month. The relation between earthquake and OLR was studied by [7] and they showed that a short-lived anomaly in OLR occurs before an

earthquake. The work done by [9] showed that neural network based model named as NeuroFlux can be used to successfully derive OLR. They showed that it is computationally quicker than other old models like WBM and Automated Atmospheric Absorption Atlas line-by-line model (4A). [11] showed the advantage of using neural network over radiative transfer model for radiative transfer calculation in the terrestrial infrared. The work done by [6] revealed the use of artificial neural networks and satellite data of previous years for the prediction of land surface temperature. The use of passive microwave measurements and neural network approach to get information about longwave flux was done by [10] using the satellite data over oceanic areas and they got an accuracy of  $[5]\ m^{-2}$ .

DMD decomposes a dynamical system into modes and these modes can be used to identify the evolutionary patterns of the system [1]. There are many other decomposition methods including Proper Orthogonal Decomposition (POD). POD modes are orthogonal in space with multi-frequency time signals, while DMD modes are non-orthogonal in space with single frequency time signal is the only difference between the two. DMD was applied on a univariate time series data to decompose the data into different components, such underlying data trend and the noise by [8]. They showed that the sampling window size should be between 8 to 45 percent to capture the underlying trend and for preventing under fitting and over fitting of the data. [2] used DMD to a sequence of a slow helium jet flow images entering quiescent fluid. They showed that the dynamically relevant coherent structures that play vital role in understanding the fluid flow was detected. [6] used DMD for stock market prediction. They used minute wise data for prediction and they also showed that DMD is more powerful than ARIMA for stock price prediction.

## 3. Methodology

### 3.1. Dataset

The dataset used for this work was obtained from National Climatic Data Centre (<http://www.ncdc.noaa.gov/>). It is a global gridded dataset with each grid of size 2.5x2.5 degree and in each grid the monthly averages of the OLR covering the region is represented. 39 years OLR data starting from 1979/01 till 2018/01 was used. We have used only the tropical regions OLR data in the range from 45° N to 20° S, since tropical regions have the maximum anomalies in OLR values when compared with temperate and frigid zones. The data of each month was vectorised and made into column vectors corresponding to each snapshot  $X = [X_1, X_2, \dots, X_p]$ , thus each row represents each locations and each column represents each snapshots i.e. each month.

### 3.2. DMD algorithm

DMD is based on the Koopman operator which tries to map a non linear problem into higher dimensions to make it a linear problem. The data collected from a dynamic system can be defined as

$$\frac{dz}{dt} = f(z, t; \nu) \tag{1}$$

Where  $z(t) \in R^p$  is a vector symbolising the status of dynamical system at time  $t$ ,  $\nu$  contains system parameters and  $f(\cdot)$  represents the dynamics. A discrete time representation may evolve due to the continuous time from (1). This discrete time representation of the system with sampling interval of  $\Delta t$  is obtained by a discrete time flow map as

$$z_{k+1} = G(z_k) \tag{2}$$

The measurement function of the system can be denoted as

$$y_k = F(z_k) \tag{3}$$

Where  $k = \text{one}, 2, \dots, p$ , where  $p$  is the total measurement. The initial condition is given as  $z(0) = z_0$

We will have no idea regarding the governing non-linear equations and the function  $G$  which defines these equations. Only thing known to us is the initial condition and the measurements taken i.e. the OLR values at each location. The DMD establishes an approximate locally linear dynamical system

$$\frac{dz}{dt} = Bz \tag{4}$$

With a well-known equation and initial condition  $z(0)$

$$z(t) = \sum_{k=1}^n \phi_k \exp(\omega_k t) b_k \tag{5}$$

Here  $\omega_k$  and  $\phi_k$  are eigenvalues and the eigenvectors of the matrix  $B$  and  $b_k$  contains the initial condition  $z(0)$  coordinates. The discrete time sampled system of (5) can be denoted as

$$z_k = \sum_{j=1}^r \phi_j \lambda_j^k b_j \tag{6}$$

A low rank eigendecomposition is produced by DMD algorithm to optimally fit the measured observations  $z_k$  in a least square manner such that

$$\|z_{k+1} - Bz_k\|_2 \tag{7}$$

Is minimised throughout the instances for  $k=1, 2, 3, \dots, p-1$ . By looking at the real part of the eigenvalue we can say whether the OLR values is increasing or decreasing. If the value is positive it is increasing and if it is negative it is decreasing. The month-wise arranged data is given as input to DMD algorithm. At any particular instance the month-wise OLR values of all the grids are taken as DMD snapshot. DMD decomposes these snapshots to different modes which spans the spatially and temporal frequencies linked with them. To minimize the approximation error along all the points

given by (7) and to find out the inner dynamics, two large data matrices are formed by arranging the  $p$  snapshots.

$$Z_1 = [z_1, z_2, \dots, z_{p-1}] \tag{8}$$

$$Z_2 = [z_2, z_3, \dots, z_p] \tag{9}$$

By using a linear operator  $z_i$  and  $z_{i+1}$  is mapped and  $Z_2$  can be represented as

$$Z_2 = [Bz_1, Bz_2, \dots, Bz_{p-1}] \tag{10}$$

$$Z_2 \approx BZ_1 \tag{11}$$

With the help of SVD, we will get a rank reduced form of  $Z_1$

$$Z_1 = U\Sigma V^* \tag{12}$$

Where  $U \in C^{n \times r}$ ,  $\Sigma \in C^{r \times r}$  and  $V \in C^{p \times r}$ . Where  $V$  and  $U$  are unitary matrix and  $\Sigma$  is a diagonal matrix. The psuedoinverse of  $Z_1$  obtained via SVD will give matrix  $B$ .

$$B = Z_2 V \Sigma^{-1} U^* \tag{13}$$

Direct computation of  $B$  will be a tedious process as the dimension of one snapshot is very big. So DMD does the eigendecomposition of  $B$  by making use of a rank-reduced representation of  $B$  with the help of POD projected matrix  $Q$ .

$$Q = U^* B U = U^* Z_2 V \Sigma^{-1} \tag{14}$$

Eigenvectors and eigenvalues of  $Q$  can be found out using

$$QW = W\Lambda \tag{15}$$

where eigenvectors are columns of  $W$  and eigenvalues are diagonals of  $\Lambda$ . Finally, the eigendecomposition of  $B$  can be reconstructed from  $\Lambda$  and  $W$  where eigenvalues of  $B$  are given by  $\Lambda$  and eigenvectors or DMD modes are given by columns of  $\Phi$

$$\Phi = Z_2 V \Sigma^{-1} W \tag{16}$$

These DMD modes can be converted to Fourier modes by  $\omega_k = \ln(\lambda_k) / \Delta t$  and reconstructed system can be defined as

$$Z_{DMD}(t) \approx \sum_{k=1}^r b_k \phi_k \exp(\omega_k t) \tag{17}$$

Where  $Z_{DMD}$  represents the OLR at time  $t$ . The sampling window size taken was 44.

### 3.3. Analysis and reconstruction

Mean Absolute Percentage Error (MAPE) analysis is done to find out the error in the predicted data. MAPE is calculated using the following equation,

$$MAPE = \frac{1}{m} \left( \sum_{i=1}^m \frac{A_i - P_i}{A_i} \right) * 100$$

where  $m$  is the total number of grids considered,  $A_i$  is the actual  $i$ th value and  $P_i$  is the predicted  $i$ th value. After the prediction using DMD algorithm the predicted data was reconstructed by making each column vector in the predicted data back to its original dimension. It is then georeferenced and mapped using ArcGIS software. The full flow chart of the methodology followed is shown in Figure 1.

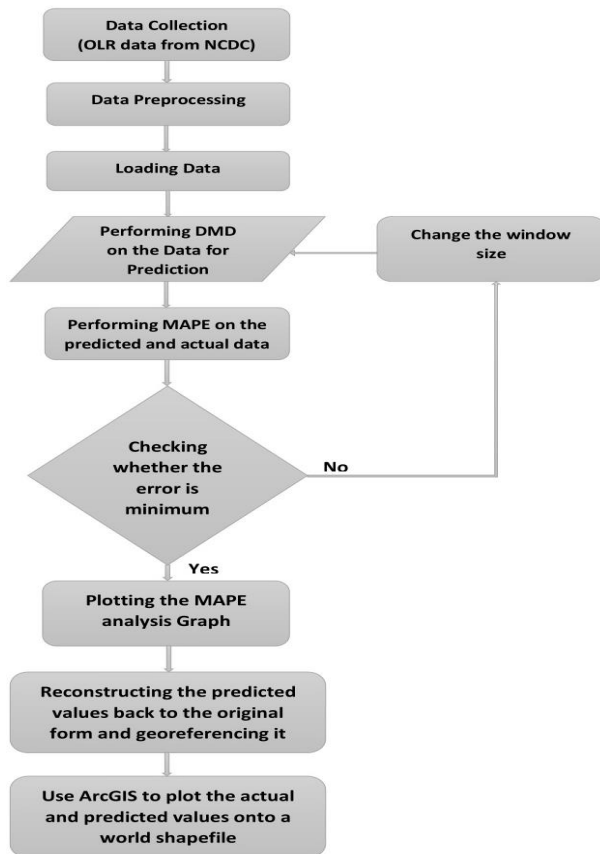


Fig. 1: Flow Chart of Methodology.

### 4. Results

In our work the sampling window and prediction window was kept 44 throughout. A total of 3745 grids were considered for the analysis, and a total of 469 months from 39 years were considered. The sampling window size was determined by trial and error method, and the window size which gave minimum MAPE was chosen. The MAPE was calculated in two ways. One is considering m as the number of grids i.e. spatial analysis and other considering m as the number of snapshots i.e. temporal analysis.

Figure 2 shows the comparison between the actual and DMD predicted OLR values calculated considering m as number of snapshots and the spatial region considered is a random grid in the Southwest region of India.

Figure 3 shows the comparison of actual and DMD predicted OLR values computed along all the grids in one random month.

Figure 4 shows the MAPE values of one random location.

The low values of OLR in the figure shows the presence of clouds in that region and the possibility of rainfall. Fig. 5 shows the reconstructed OLR grids from the predicted data at the top and the actual OLR grids at the bottom. The predicted grids were georeferenced and was plotted and overlaid on a world shape file using ArcGIS software.

The overall time taken for DMD computation was less than five seconds using MATLAB.

An overall MAPE of 5.3 was obtained between the actual data and predicted data after applying DMD.

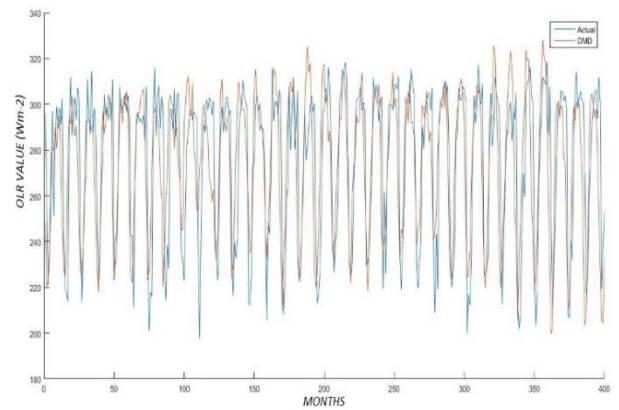


Fig. 2: Comparison Actual and Predicted Values of One Location.

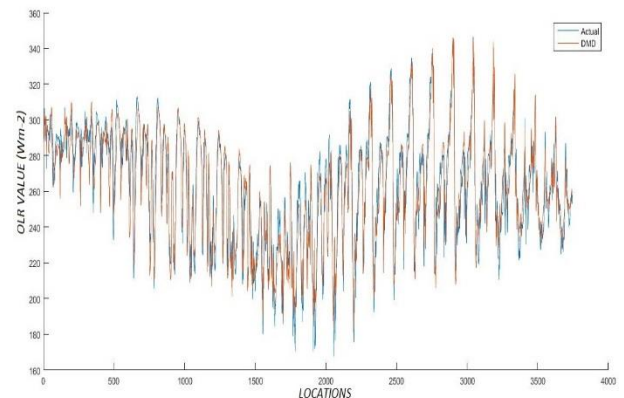


Fig. 3: Comparison of Actual and Predicted Values of One Month.

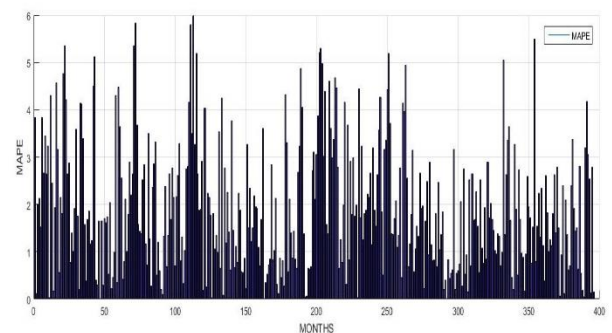


Fig. 4: MAPE of One Location.

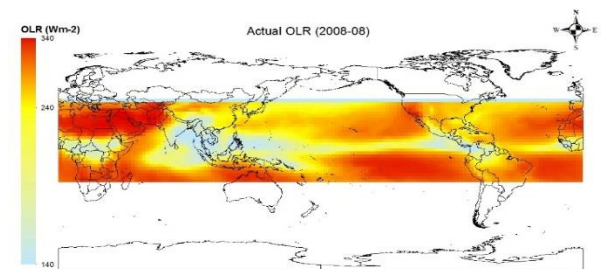
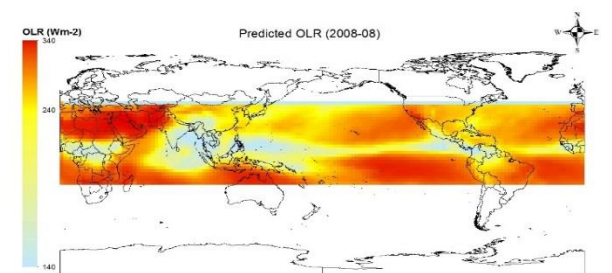


Fig. 5: Predicted OLR vs. Actual OLR.

## 5. Conclusion and future work

Sensing of Environment, 80(1), 157164.  
[https://doi.org/10.1016/S0034-4257\(01\)00297-8](https://doi.org/10.1016/S0034-4257(01)00297-8).

DMD is computationally very efficient than the neural networks and other algorithms as it take very less time to compute and it predicts the future states of a system without any prior knowledge or the understanding of the underlying equations. Since it is a completely data driven approach it depends only on the past data and so when there is an abrupt change in the dynamics DMD fails to register it and thus our prediction fails. From the results it was seen that DMD was correctly predicting the trends. As DMD is computationally fast and efficient we can input a large amount of data and get predictions quickly. So as a future work we can do it for day-wise data. We can also predict the rainfall data from this predicted data by utilizing the results obtained from the work done by other authors as a future work. Since OLR has an impact on many weather and environmental parameters the prediction of OLR will help in the understanding and forecasting of those parameters.

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