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Research paper

A New Chaotic Jerk System with Three Nonlinearities and Synchronization via Adaptive Backstepping Control

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Abstract

Jerk systems are popular in mechanical engineering and chaotic jerk systems are used in many applications as they have simple structure and complex dynamic properties. In this work, we report a new chaotic jerk system with three nonlinear terms. Dynamical properties of the chaotic jerk system are analyzed through equilibrium analysis, dissipativity, phase portraits and Lyapunov chaos exponents. We show that the new chaotic jerk system has a unique saddle-focus equilibrium at the origin. Thus, the new chaotic jerk system has a self-excited strange attractor. Next, global chaos synchronization of a pair of new chaotic jerk systems is successfully achieved via adaptive backstepping control.

Keywords: *Chaos, Chaotic systems, jerk systems, backstepping control, synchronization.*

1. Introduction

Chaos theory deals with nonlinear dynamical systems that are very sensitive to even small changes in the initial conditions, known as *chaotic systems* ([1]-[2]). Chaos theory has applications in several areas of science and engineering such as plasma systems [3], weather systems ([4]-[5]), chemical reactions ([6]-[8]), ecology ([9]-[10]), biology ([11]-[14]), encryption ([15]-[18]), robotics [19], neural networks ([20]-[21]), oscillations ([22]-[29]), circuits ([30]-[41]), etc.
Gottlieb [42] established that 3-D chaotic systems can be expressed in the form of single ordinary differential equations, which are also termed as *jerk* differential equations. The jerk systems have many applications in science and engineering such as circuits [43], thermal arc plasma [44], biological reactions [45], mechanical oscillations [46], etc.
In physics, a jerk differential equation can be represented as the third order dynamics

$$\dddot{x} = f(x, \dot{x}, \ddot{x}), \tag{1}$$

where $x(t)$ represents the *displacement*, $\dot{x}(t)$ the *velocity*, $\ddot{x}(t)$ the *acceleration* and $\dddot{x}(t)$ the *jerk*.
It is convenient to express (1) in system notation by defining two additional phase variables as

$$y = \dot{x} \quad \text{and} \quad z = \dot{y} = \ddot{x} \tag{2}$$

Thus, the state variables x, y, z have the mechanical interpretation of *displacement*, *velocity* and *acceleration*, respectively.

Thus, the differential equation (1) can be expressed in system notation as

$$\begin{cases} \dot{x} &= y \\ \dot{y} &= z \\ \dot{z} &= f(x,y,z) \end{cases} \tag{3}$$

There are many jerk systems reported in the chaos literature such as Sprott system [47], Coullet system [48], Munmuangsaen system [49], Kengne system [50], Elsonbaty system [51], Vaidyanathan systems ([52]-[57]), etc.
In this paper, we modify the Sprott jerk dynamics [47] so as to obtain a new chaotic jerk dynamics. We point out that our new jerk system has three nonlinearities, *viz.* two cubic nonlinearities and a sinusoidal nonlinearity. With the help of a rigorous bifurcation analysis, we show that our new chaotic jerk system displays the *bistability phenomenon* for a certain range of the parameters, and Symmetry Breaking (SB) bifurcation followed by Symmetry Restoring (SR) bifurcation. We illustrate bistable chaotic attractors with detailed numerical simulations in our work.
In this paper, we also derive new results for the global chaos synchronization of a pair of new jerk systems considered as the *master* and *slave* systems. The synchronization of chaotic systems deals with finding suitable feedback control laws so that the trajectories of the slave system follows the corresponding trajectories of the master system asymptotically with time. Many techniques like active control, adaptive control and sliding mode control have been used to study the synchronization of chaotic systems in the literature ([58]-[63]).

Backstepping control is a recursive design procedure which is very useful for establishing stability results in Lyapunov stability theory [64]. Backstepping control has been effectively applied in the control literature to tackle various problems of stabilizing dynamical systems ([65]-[68]).
In this work, we use backstepping control for the global chaos synchronization of the new jerk systems with unknown parameters.
In Section 2, we describe the dynamics and qualitative properties of the new chaotic jerk system. As an engineering application, we derive global hyperchaos synchronization results for the new chaotic jerk system with unknown parameters using adaptive backstepping control in Section 3. Finally, the conclusions are drawn in Section 4.

2. A new chaotic jerk system with three nonlinear terms

Dissipative chaotic jerk systems have received significant attention in the chaos literature ([1]-[2]). A classical jerk system with two cubic nonlinear terms was discovered by Sprott in 1997 [47]. The Sprott jerk system [47] is described by the third-order ordinary differential equation

$$\frac{d^3x}{dt^3} = -a\frac{d^2x}{dt^2} + x\left(\frac{dx}{dt}\right)^2 - x^3 \tag{4}$$

where a is a constant parameter.
In the system form, it is possible to express the ODE (4) as

$$\begin{cases} \dot{x} = y \\ \dot{y} = z \\ \dot{z} = -az + xy^2 - x^3 \end{cases} \tag{5}$$

To simplify the notations, we set $X = (x,y,z)$.
For the initial state $X(0) = (0.2, 0.2, 0.2)$ and $a = 3.6$, the Lyapunov exponents of the Sprott jerk system (5) can be computed using Wolf's algorithm [69] for $T = 1E5$ seconds as follows:

$$L_1 = 0.1357, \;\; L_2 = 0, \;\; L_3 = -3.7357 \tag{6}$$

Thus, the Sprott jerk system (5) is chaotic and dissipative. Moreover, the Kaplan-Yorke dimension of the Sprott jerk system (5) is found as

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.0363 \tag{7}$$

Figures 1-3 show the 2-D plots of the Sprott jerk system (5) for $X(0) = (0.2, 0.2, 0.2)$ and $a = 3.6$.

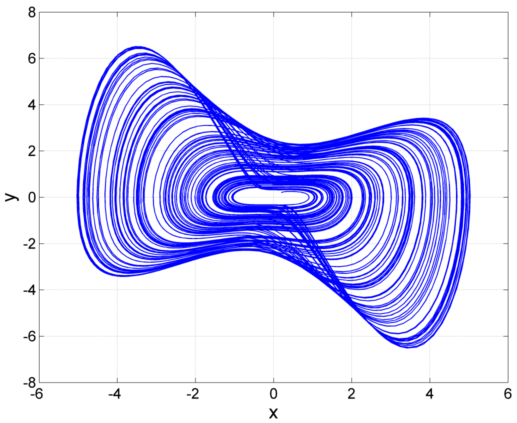

Figure 1: 2-D phase plot of the Sprott jerk system (5) in the (x,y) plane for $a = 3.6$ and $X(0) = (0.2, 0.2, 0.2)$

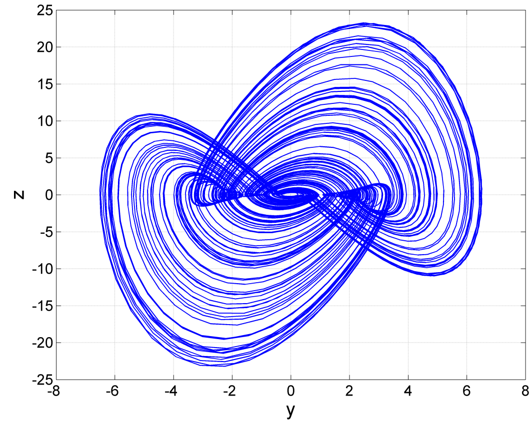

Figure 2: 2-D phase plot of the Sprott jerk system (5) in the (y,z) plane for $a = 3.6$ and $X(0) = (0.2, 0.2, 0.2)$

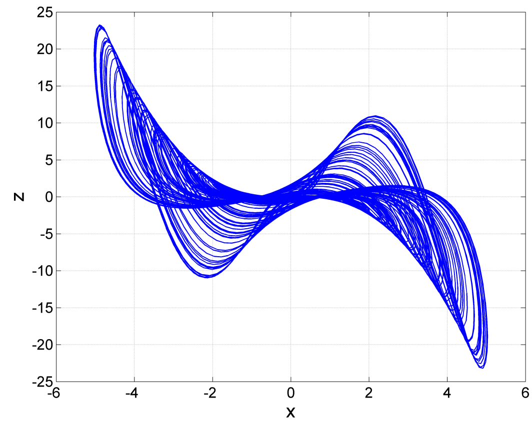

Figure 3: 2-D phase plot of the Sprott jerk system (5) in the (x,z) plane for $a = 3.6$ and $X(0) = (0.2, 0.2, 0.2)$

In this work, we announce a new jerk system by adding a linear term $-bx$ and a sinusoidal nonlinearity $c\sin(\dot{x})$ in the Sprott jerk function given in the ODE (4), where b and c are constant parameters. These terms are added to modify the Sprott jerk system (4) so as to derive a new chaotic system with better chaotic properties.
Our new jerk system is given by the third-order ODE:

$$\frac{d^3x}{dt^3} = -a\frac{d^2x}{dt^2} + x\left(\frac{dx}{dt}\right)^2 - x^3 - bx + c\sin(\dot{x}) \tag{8}$$

where a, b, c are positive parameters.
Thus, the new jerk ODE (8) has three nonlinearities, *viz.* two cubic nonlinearities and a sinusoidal nonlinearity. In system form, it is possible to express the ODE (8) as

$$\begin{cases} \dot{x} = y \\ \dot{y} = z \\ \dot{z} = -az + xy^2 - x^3 - bx + c\sin y \end{cases} \tag{9}$$

The new jerk system (9) is dissipative because

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -a < 0 \tag{10}$$

The equilibrium points of the new jerk system (9) are got by solving the system

$$y = 0 \tag{11a}$$

$$z = 0 \tag{11b}$$

$$-az + xy^2 - x^3 - bx + c\sin y = 0 \tag{11c}$$

From Eq. (11a), we get $y = 0$.
From Eq. (11b), we get $z = 0$.
Substituting $y = z = 0$ in Eq. (11c), we get

$$-x^3 - bx = 0 \text{ or } x(b + x^2) = 0 \tag{12}$$

Since $b > 0$, we must have $x = 0$.
This shows that $O = (0,0,0)$ is the only real equilibrium of the new jerk system (9).
We note that the new jerk system (9) is invariant under the change of coordinates

$$(x,y,z) \mapsto (-x,-y,-z) \tag{13}$$

which holds for all values of the parameters a, b and c¿
This shows that the new jerk system (9) has point reflection symmetry about the equilibrium at the origin $O = (0,0,0)$.
The Jacobian matrix of the new jerk system (9) at the equilibrium point $O = (0,0,0)$ is given by

$$J = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -b & c & -a \end{bmatrix} \tag{14}$$

The Jacobian matrix J has the characteristic equation

$$\lambda^3 + a\lambda^2 - c\lambda + b = 0 \tag{15}$$

Since a, b, c are positive parameters, it is immediate that the equilibrium point $O = (0,0,0)$ is unstable according to the Routh-Hurwitz criterion.
Dynamics of the proposed jerk oscillator (9) is investigated by considering the effect of the parameters a, b, c on the system's behaviour. Our simulations show that the proposed jerk oscillator displays rich dynamics for the parameter c. We study the dynamics of the proposed jerk oscillator by fixing the parameters a and b as $a = 4.0$, $b = 1.0$ and by varying the value of the parameter c. Bifurcation diagrams and the largest Lyapunov exponents of the proposed jerk oscillator (9) versus the parameter c are presented in Figure 4. Bifurcation diagrams are obtained by scanning the parameter c upwards (black) and downwards (gray). The acronym SB means *symmetry-breaking* while the acronym SR corresponds to *symmetry-restoring*.

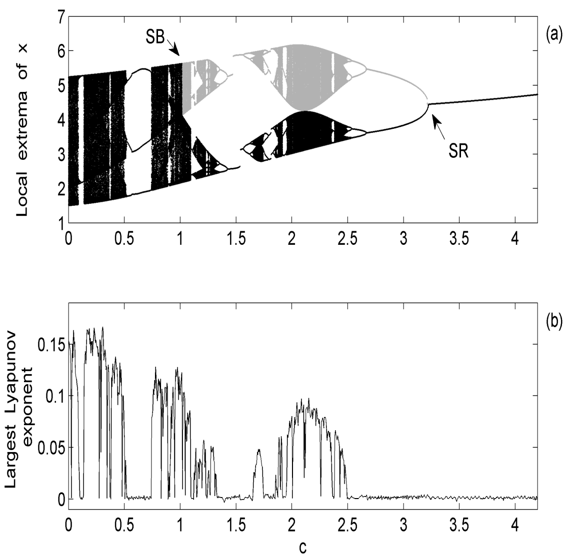

Figure 4: The bifurcation diagrams for the new jerk system (9) for the parameter c, where $a = 4.0, b = 1.0$. (a) Local maxima of $x(t)$ and (b) Largest Lyapunov exponent of the new jerk system (9).

When the parameter c increases from 0.0 to 4.2 [see black dot in Figure 4 (a)], the bifurcation diagram of the output $x(t)$ shows a reversed period-doubling bifurcation to chaos interspersed with periodic windows. When performing the same analysis by ramping the parameter c [see gray dot in Figure 4 (a)], the output $x(t)$ displays the same dynamical behaviours as in Figure 4 (a) (see black dot) but its amplitude is different to the one in Figure 4 (a) (see black dot) in the range $1.0 \leq c \leq 3.2$. Comparison of the two sets of data [for increasing (black) and decreasing (gray)] used to plot Figure 4 (a) shows that the new jerk oscillator (9) displays the bistability phenomenon in the range $1.0 \leq c \leq 3.2$ and symmetry breaking bifurcation followed by symmetry restoring bifurcation. The chaotic behaviour is confirmed by the largest Lyapunov exponents shown in Figure 4 (b).
For $a = 4.0, b = 1.0$ and $c = 0.95$, the proposed jerk oscillator (9) exhibits chaos as illustrated in the Figures 5-7.

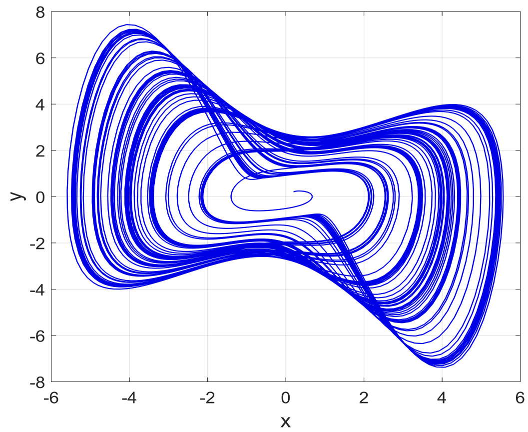

Figure 5: 2-D phase plot of the new chaotic jerk system (9) in the (x,y) plane for $a = 4.0, b = 1.0, c = 0.95$ and $X(0) = (0.2, 0.2, 0.2)$

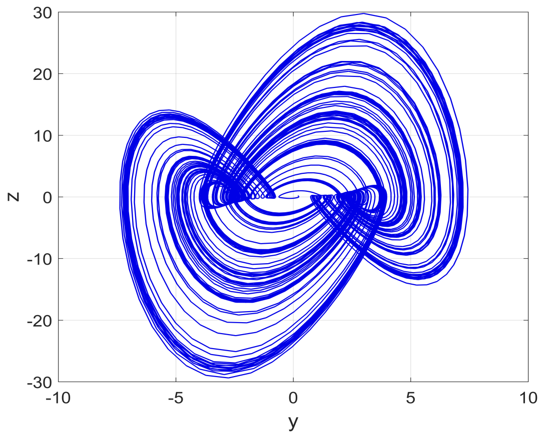

Figure 6: 2-D phase plot of the new chaotic jerk system (9) in the (y,z) plane for $a = 4.0, b = 1.0, c = 0.95$ and $X(0) = (0.2, 0.2, 0.2)$

From Figures 5-7, we notice that the shape of the chaotic attractor is different to the one of Sprott chaotic attractor (5) shown in the Figures 1-3. In fact, Figures 5-7 describe the signature of a double-scroll chaotic attrator.
Figures 8-10 describe the bistability phenomenon of the new chaotic jerk system for the parameter values $a = 4.0, b = 1.0, c = 1.2$ and for two different initial conditions given by $X(0) = (0.2, 0.2, 0.2)$ and $Y(0) = (-0.2, -0.2, -0.2)$. The chaotic phase portrait of the new chaotic jerk system (9) for $X(0) = (0.2, 0.2, 0.2)$ is shown in blue colour and the chaotic phase portrait of the new chaotic jerk system

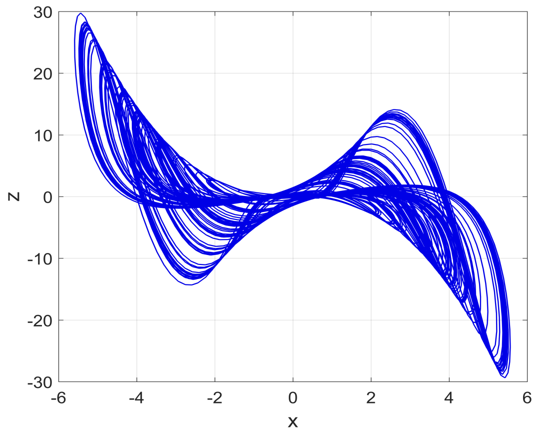

Figure 7: 2-D phase plot of the new chaotic jerk system (9) in the (x,z) plane for $a = 4.0, b = 1.0, c = 0.95$ and $X(0) = (0.2, 0.2, 0.2)$

(9) for $Y(0) = (-0.2, -0.2, -0.2)$ is shown in red colour. From Figures 8-10, we notice that the new chaotic jerk system (9) displays bistable chaotic attractors with the same shape and parameter values but different initial conditions.

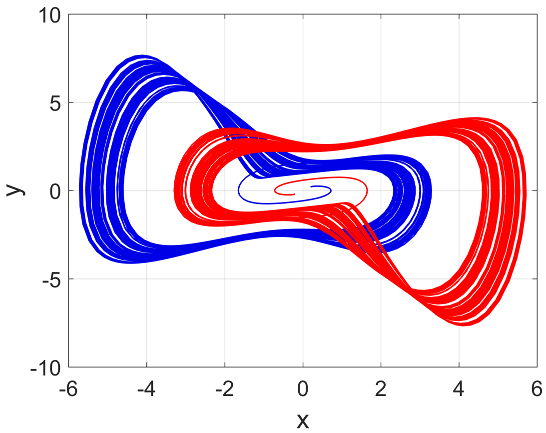

Figure 8: The 2-D phase plot of the bistable chaotic attractors for the new jerk system (9) in the in the (x,y) plane for $a = 4.0, b = 1.0, c = 1.2$ and specific initial conditions: $X(0) = (0.2, 0.2, 0.2)$ (blue colour) and $Y(0) = (-0.2, -0.2, -0.2)$ (red colour)

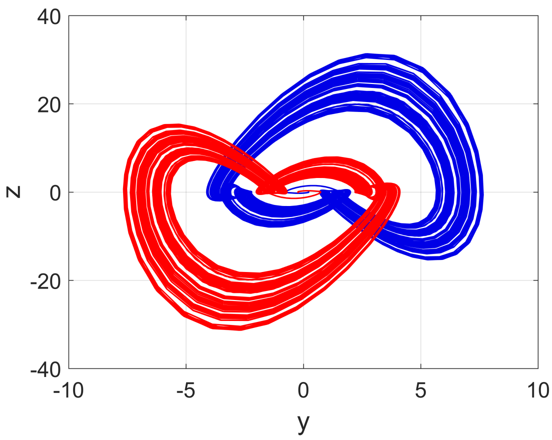

Figure 9: The 2-D phase plot of the bistable chaotic attractors for the new jerk system (9) in the in the (y,z) plane for $a = 4.0, b = 1.0, c = 1.2$ and specific initial conditions: $X(0) = (0.2, 0.2, 0.2)$ (blue colour) and $Y(0) = (-0.2, -0.2, -0.2)$ (red colour)

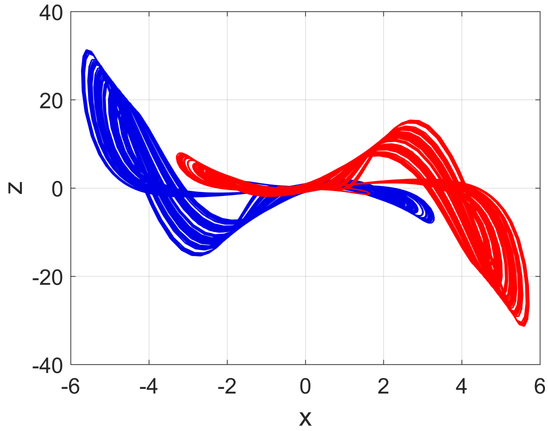

Figure 10: The 2-D phase plot of the bistable chaotic attractors for the new jerk system (9) in the in the (x,z) plane for $a = 4.0, b = 1.0, c = 1.2$ and specific initial conditions: $X(0) = (0.2, 0.2, 0.2)$ (blue colour) and $Y(0) = (-0.2, -0.2, -0.2)$ (red colour)

For the initial state $X(0) = (0.2, 0.2, 0.2)$ and $(a, b, c) = (4, 1, 1.2)$, the Lyapunov exponents of the new jerk system (9) are computed using Wolf's algorithm [69] for $T = 1E5$ seconds as follows:

$$L_1 = 0.0359, \quad L_2 = 0, \quad L_3 = -4.0359 \tag{16}$$

Thus, the new jerk system (9) is chaotic and dissipative. Moreover, the Kaplan-Yorke dimension of the new chaotic jerk system (9) is found as

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.0089 \tag{17}$$

Figure 11 shows the calculation of the Lyapunov exponents of the new chaotic jerk system (9).

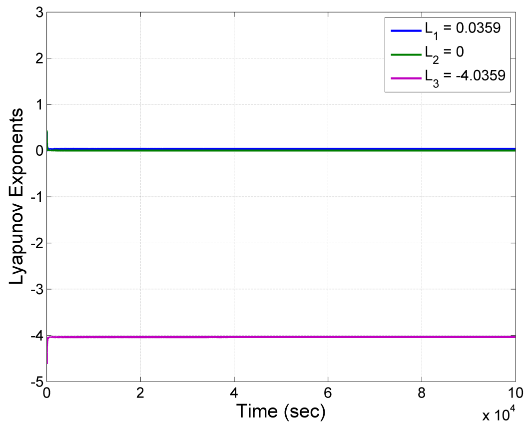

Figure 11: The Lyapunov exponents of the new jerk system (9) for $(a, b, c) = (4, 1, 1.2)$ and the initial state $X(0) = (0.2, 0.2, 0.2)$

3. Adaptive chaos synchronization of the new chaotic jerk systems

In this section, we study an engineering application of the new chaotic jerk system, *viz.* adaptive chaos synchronization of a pair of new chaotic jerk systems with unknown parameters considered as master and slave systems via adaptive backstepping control method. As the master system, we consider the new chaotic jerk system

$$\begin{cases} \dot{x}_1 &= y_1 \\ \dot{y}_1 &= z_1 \\ \dot{z}_1 &= -az_1 + x_1 y_1^2 - x_1^3 - bx_1 + c\sin y_1 \end{cases} \tag{18}$$

In Eq. (18), x_1, y_1, z_1 are the states and a, b, c are unknown parameters of the system.
As the slave system, we consider the new chaotic jerk system

$$\left\{\begin{array}{lll}\dot{x}_2 & = & y_2 \\ \dot{y}_2 & = & z_2 \\ \dot{z}_2 & = & -az_2 + x_2 y_2^2 - x_2^3 - bx_2 + c\sin y_2 + u\end{array}\right. \tag{19}$$

In Eq. (19), x_2, y_2, z_2 are the states and u is the backstepping control to be found.
The synchronization error between the new chaotic jerk systems (18) and (19) is defined by

$$\left\{\begin{array}{lll}e_x & = & x_2 - x_1 \\ e_y & = & y_2 - y_1 \\ e_z & = & z_2 - z_1\end{array}\right. \tag{20}$$

The error dynamics is calculated as follows.

$$\left\{\begin{array}{lll}\dot{e}_x & = & e_y \\ \dot{e}_y & = & e_z \\ \dot{e}_z & = & -ae_z - be_x + c(\sin y_2 - \sin y_1) + x_2 y_2^2 - x_1 y_1^2 \\ & & -x_2^3 + x_1^3 + u\end{array}\right. \tag{21}$$

Next, we define the estimation errors for the unknown parameters as

$$\left\{\begin{array}{lll}e_a(t) & = & a - A(t) \\ e_b(t) & = & b - B(t) \\ e_c(t) & = & c - C(t)\end{array}\right. \tag{22}$$

where $A(t), B(t), C(t)$ are estimates for a, b, c, respectively.
Differentiating (22), we get

$$\left\{\begin{array}{lll}\dot{e}_a(t) & = & -\dot{A}(t) \\ \dot{e}_b(t) & = & -\dot{B}(t) \\ \dot{e}_c(t) & = & -\dot{C}(t)\end{array}\right. \tag{23}$$

Using adaptive backstepping control method, we establish the key result of this section.

Theorem 1. *The master and slave chaotic systems represented by the new jerk systems (18) and (19) with unknown parameters are globally and exponentially synchronized by means of the adaptive backstepping feedback control law given by*

$$\left\{\begin{array}{lll}u & = & -[3 - B(t)]e_x - 5e_y - [3 - A(t)]e_z \\ & & -C(t)(\sin y_2 - \sin y_1) - x_2 y_2^2 + x_1 y_1^2 \\ & & x_2^3 - x_1^3 - k\xi_3\end{array}\right. \tag{24}$$

where $k > 0$ is a gain constant,

$$\xi_3 = 2e_x + 2e_y + e_z \tag{25}$$

and the update law for the parameter estimates $A(t), B(t), C(t)$ is given by

$$\left\{\begin{array}{lll}\dot{A}(t) & = & -\xi_4 e_3 \\ \dot{B}(t) & = & -\xi_4(|y_2| - |x_2|) \\ \dot{C}(t) & = & -\xi_4(y_1^4 y_4 - x_1^4 x_4)\end{array}\right. \tag{26}$$

Proof. We establish this result via adaptive backstepping contorl method and Lyapunov stability theory [70].
We define the Lyapunov function

$$V_1(\xi_1) = \frac{1}{2}\,\xi_1^2 \tag{27}$$

where

$$\xi_1 = e_x \tag{28}$$

Differentiating V_1 along the error dynamics (21), we get

$$\dot{V}_1 = \xi_1\dot{\xi}_1 = e_x e_y = -\xi_1^2 + \xi_1(e_x + e_y) \tag{29}$$

We set

$$\xi_2 = e_x + e_y \tag{30}$$

Using (30), we can simplify Eq. (29) as

$$\dot{V}_1 = -\xi_1^2 + \xi_1\xi_2 \tag{31}$$

Next, we define the Lyapunov function

$$V_2(\xi_1, \xi_2) = V_1(\xi_1) + \frac{1}{2}\xi_2^2 = \frac{1}{2}\,(\xi_1^2 + \xi_2^2) \tag{32}$$

Differentiating V_2 along the error dynamics (21), we get

$$\dot{V}_2 = -\xi_1^2 - \xi_2^2 + \xi_2(2e_x + 2e_y + e_z) \tag{33}$$

We set

$$\xi_3 = 2e_x + 2e_y + e_z \tag{34}$$

Using (34), we can simplify Eq. (33) as

$$\dot{V}_2 = -\xi_1^2 - \xi_2^2 + \xi_2\xi_3 \tag{35}$$

To simplify the notation, we set $\xi = (\xi_1, \xi_2, \xi_3)$.
Finally, we define the quadratic Lyapunov function

$$V(\xi, e_a, e_b, e_c) = \frac{1}{2}(\xi_1^2 + \xi_2^2 + \xi_3^2) + \frac{1}{2}(e_a^2 + e_b^2 + e_c^2) \tag{36}$$

Clearly, V is a quadratic and positive definite function on $\mathbf{R}^6$.
Differentiating V along the error dynamics (21) and (26), we get

$$\dot{V} = -\sum_{i=1}^{3}\xi_i^2 + \xi_3(\xi_3 + \xi_2 + \dot{\xi}_3) - e_a\dot{A} - e_b\dot{B} - e_c\dot{C} \tag{37}$$

Eq. (37) can be written compactly as

$$\dot{V} = -\sum_{i=1}^{3}\xi_i^2 + \xi_3 S - e_a\dot{A} - e_b\dot{B} - e_c\dot{C} \tag{38}$$

where

$$S = \xi_3 + \xi_2 + \dot{\xi}_3 = \xi_3 + \xi_2 + (2\dot{e}_x + 2\dot{e}_y + \dot{e}_z) \tag{39}$$

A simple computation gives the result

$$\left\{\begin{array}{lll}S & = & (3-b)e_x + 5e_y + (3-a)e_z + c(\sin y_2 - \sin y_1) \\ & & +x_2 y_2^2 - x_1 y_1^2 - x_2^3 + x_1^3 + u\end{array}\right. \tag{40}$$

Substituting the value of u from (24) into Eq. (40), we get

$$\left\{\begin{array}{lll}S & = & -[a - A(t)]e_z - [b - B(t)]e_x \\ & & +[c - C(t)](\sin y_2 - \sin y_1) - k\xi_3\end{array}\right. \tag{41}$$

Using the definition of parameter estimation errors given in Eq. (22), we can simplify Eq. (41) as follows:

$$S = -e_B e_x - e_a e_z + e_c(\sin y_2 - \sin y_1) - k\xi_3 \tag{42}$$

Substituting the value of S from Eq. (42) into Eq. (38), we get

$$\left\{\begin{array}{lll}\dot{V} & = & -\xi_1^2 - \xi_2^2 - (1+k)\xi_3^2 + e_a[-\xi_3 e_z - \dot{A}] \\ & & +e_b[-\xi_3 e_x - \dot{B}] \\ & & +e_c[\xi_3(\sin y_2 - \sin y_1) - \dot{C}]\end{array}\right. \tag{43}$$

Substituting the parameter update law from Eq. (26) into Eq. (43), we get

$$\dot{V} = -\xi_1^2 - \xi_2^2 - \xi_3^2 - (1+k)\xi_4^2 \tag{44}$$

which is a negative semi-definite function on $\mathbf{R}^8$.
Thus, by Barbalat's lemma in Lyapunov stability theory [70], we conclude that $\mathbf{e}(t)$ is globally exponentially stable. Hence, it is consequent that the master and slave chaotic systems represented by the new jerk systems (18) and (19) are globally and exponentially synchronized for all initial conditions.
Hence, the proof is complete. ■ □

For numerical simulations, we take the parameter values of the new jerk systems (18) and (19) as in the chaotic case, *i.e.*

$$a = 4.0, \quad b = 1.0, \quad c = 1.2 \tag{45}$$

We take the positive gain constant k as $k = 10$.
We take the initial state of the master system (18) as

$$x_1(0) = 1.7, \quad y_1(0) = 2.4, \quad z_1(0) = -1.8 \tag{46}$$

We take the initial state of the slave system (19) as

$$x_2(0) = -3.6, \quad y_2(0) = -1.1, \quad z_2(0) = 5.2 \tag{47}$$

The initial conditions of the parameter estimates are taken as

$$A(0) = 7.2, \quad B(0) = 3.8, \quad C(0) = 5.6 \tag{48}$$

Figures 12-14 show the complete synchronization of the chaotic jerk systems (18) and (19). Figure 15 shows the time-history of the chaos synchronization error $\mathbf{e} = (e_x, e_y, e_z)$.

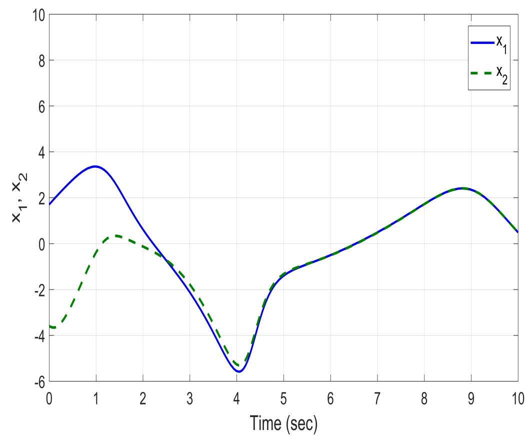

Figure 12: Synchronization of the states x_1 and x_2 for the chaotic jerk systems (18) and (19)

4. Conclusion

In this paper, based on mathematical models and numerical simulations, we have described the dynamics and chaos control of a new jerk system with two cublic nonlinearities and a sinusoidal nonlinearity. Its chaotic features are fully examined by eigen value structure, Lyapunov exponents and Kaplan-Yorke dimension. We have also carried out an extensive bifurcation analysis and discovered new features such as point reflection symmetry and bistability for the new chaotic jerk system. In addition, an adaptive backstepping controller is introduced to achieve global chaos synchronization for the new chaotic jerk system. As future work, the new chaotic jerk system can be used in applications such as secure communications and encryption devices.

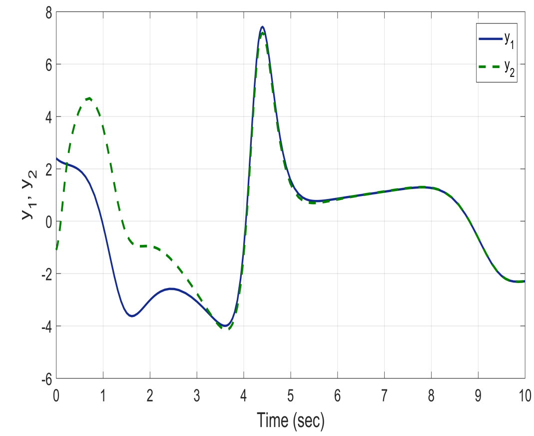

Figure 13: Synchronization of the states y_1 and y_2 for the chaotic jerk systems (18) and (19)

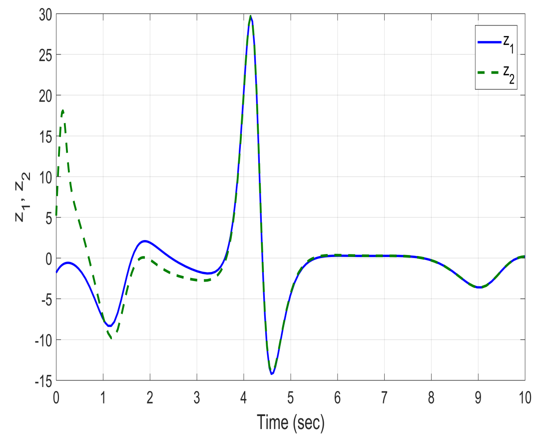

Figure 14: Synchronization of the states z_1 and z_2 for the chaotic jerk systems (18) and (19)

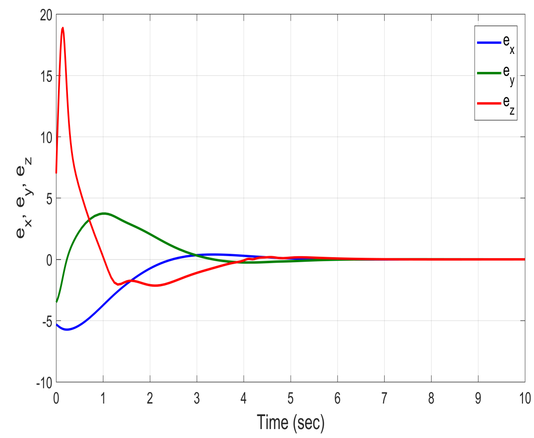

Figure 15: Time-history of the synchronization error $\mathbf{e}$ between the chaotic jerk systems (18) and (19)

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