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Research paper

Liquid Induced Vibrations of Truncated Elastic Conical Shells with Elastic and Rigid Bottoms

Yury V. Naumenko[1], Vasyl I. Gnitko[1], Elena A. Strelnikova[1,2]*

[1] A.N. Podgorny Institute For Mechanical Engineering Problems, 2/10 Pozharsky St., Kharkiv, 61046, Ukraine
[2]V.N. Karazin Kharkiv National University, Svobody Sq., 4, Kharkiv, 61022, Ukraine
**Corresponding Author E-Mail: Elena15@Gmx.Com*

Abstract

A method of estimating natural modes and frequencies of vibrations for elastic shells of revolution conveying a liquid is proposed. The vibration modes of the liquid-filled elastic shells are presented as linear combinations of their own vibration modes without liquid. The explicit expression for fluid pressure is defined using Bernoulli's integral and potential theory suppositions. Non-penetration, kinematic, and dynamic boundary conditions are applied at the shell walls and on a free liquid surface, respectively. The solution of the hydro-elasticity problem is found out using an effective technique based on coupled finite and boundary element methods. Computational vibration analysis of elastic truncated conical shells with different fixation conditions is accomplished. Sloshing and elastic walls frequencies and modes of liquid-filled truncated conical tanks are estimated. Both rigid and elastic bottoms of shells are considered. Some examples of numerical estimations are provided to testify the efficiency of the developed method

***Keywords**: Boundary element method, liquid sloshing, truncated conical shells, rigid and elastic bottoms.*

1. Introduction

Thin-wall structural elements are widely used nowadays in chemistry, aerospace and transport industries, wind power engineering, gas and oil producing and other engineering areas. Such facilities often operate under extremely process loading, furthermore, they are usually filled with dangerous toxic and explosive liquids. Among these reservoirs there are spacecraft fuel tanks, storage tanks and containers for oil and propellant. If such storage sites are subjected to surface shots caused by a terrorist act, an airplane crash or a seismic shockwave, this would be an ecological catastrophe with dangerous repercussions. Not only assessing the safety conditions of operating storage tanks, but also advanced design of new highly effective machines and structures requires the qualified evaluation of their strength characteristics. These data allow us to estimate the critical strength of structures at shockwave or seismic action, to isolate dangerous resonance frequencies, and to identify at the design stage the most critical zones from the viewpoint of stress concentration. Therefore, it is topical to develop refined mathematical and computational models and structural design methods that account for intricate shape, the impact of fluid or gas on the stress-strained state, modes and frequencies of their natural oscillations. However, studying the oscillation processes of structures interacting with a liquid is a challenging design problem. The experimental research of sloshing process in elastic containers is very difficult and expensive. So mathematical modeling for describing these physical processes using advanced computational technologies is the most powerful tool for solution of these problems. Numerical methods are successfully applied when containers are of complicated forms, so the processes of sloshing cannot be described analytically. Both experimental and analytical research were successfully performed in hydro-elasticity of shells and plates in last decade [1-6]. Most of these works are devoted to the hydro-elasticity problems of flat and curved plates, circular cylinder shells [1-3]. The dynamics of elastic conical containers without liquid has been considered by analytical, numerical, and experimental methods in [4–6]. The fluid-filled conical shell has been successfully studied by Lakis in [7]. Noted, that the sloshing effects were ignored in [7]. In [8] the coupled problem was considered for vibrations of the liquid-filled elastic shell simultaneously with liquid vibrations in the rigid shell with the same geometrical characteristics and filling levels. It was demonstrated here that one cannot consider sloshing and walls vibrations separately, because this problem is essentially coupled. Spectrums of sloshing and elastic wall vibrations are not separated, moreover, they are alternated, at least in presence of baffles. So here the method is proposed where the unknown velocity potential is given as a sum of two potentials. First one describes the liquid sloshing in the rigid tank, and second one deals with vibrations of the liquid-filled elastic shell without considering the gravity force. The method allows us to obtain vibration modes and frequencies for different fuel tanks taking into account effects of sloshing, gravity, and elasticity.

2. Problem Statement and Method of Mode Superposition in Coupled Dynamic Problems

Free harmonic vibrations of conical elastic shells are considered. The shell is supposed to be made of homogeneous, isotropic material. The following shell parameters are involved: thickness h, height L, Poisson's ratio ν, elasticity modulus E, and mass density ρ_s.

Denote by σ the wetted shell surface, and the liquid free surface by S_0. Let S_{bot} be the tank bottom surface. Denote as $\mathbf{U}=(U_1,U_2,U_3)$ the shell displacement vector. Consider at first free oscillations of the elastic empty shell. Suppose that shell displacements are expressed by following formula:

$$\mathbf{U}=\mathbf{u}\exp(i\Omega t);\quad \mathbf{u}=(u_1,u_2,u_3).$$

Here Ω is the frequency of elastic empty shell vibrations; the time-dependant factor $\exp(i\Omega)$ will be separated further on. Then the next system of differential equations in partial derivatives describes the empty shell vibrations, Levitin *et al* [9]

$$\sum_{i=1}^{3}L_{ij}u_i=\Omega^2 u_j,\quad j=1,2,3.$$

Here L_{ij} are linear differential operators corresponding to Kirchhoff - Love shell theory.

The finite element method developed in [8] is in use here for estimating own frequencies Ω_k and modes $\mathbf{u}_k$, $k=\overline{1,N}$ of vibrations for the elastic empty shell of revolution. The next equation of movement for the shell containing a fluid is obtained in [8]:

$$\mathbf{LU}+\mathbf{M\ddot{U}}=p_d\mathbf{n},$$

where $\mathbf{L}$ and $\mathbf{M}$ are global stiffness and mass matrices, $\mathbf{n}$ is an unit outward normal vector to the shell surface, the term $p_d\mathbf{n}$ is for the dynamical component of fluid pressure, perpendicular to the shell surface. The next vector equalities are valid for each eigenvalue Ω_k and eigenmode $\mathbf{u}_k$ of the elastic empty shell:

$$\mathbf{Lu}_k=\Omega_k^2\mathbf{Mu}_k,\quad (\mathbf{Mu}_k,\mathbf{u}_j)=\delta_{kj}\,. \tag{1}$$

The mathematical model is developed for modeling the fluid-structure interaction. It is based on the next suppositions: the liquid is an incompressible and inviscid one, its movement is irrotational, the only small vibrations are considered. It allow us to introduce a scalar velocity potential $\Phi(x,y,z,t)$. Its gradient presents the fluid velocity components. The liquid pressure $p=p(x,y,z,t)$ acting on the wetted surface is defined for a potential flow from Bernoulli's equation

$$p=-\rho_l\left(\frac{\partial\Phi}{\partial t}+gz\right)+p_0,\;\; p_s=-\rho_l gz,\;\; p_d=-\rho_l\frac{\partial\Phi}{\partial t}.$$

Here ρ_l is the liquid density, g is for the acceleration of gravity, z is the liquid point vertical coordinate, p_d and p_s are dynamical and static fluid pressure components, p_0 is for atmospheric pressure.

The velocity potential $\Phi(x,y,z,t)$ could be defined at any instant from the next boundary value problem:

$$\nabla^2\Phi=\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}=0,\;\; \left.\frac{\partial\Phi}{\partial\mathbf{n}}\right|_{\sigma}=\frac{\partial w}{\partial t},$$
$$\left.\frac{\partial\Phi}{\partial\mathbf{n}}\right|_{S_0}=\frac{\partial\zeta}{\partial t};\;\; \left.\frac{\partial\Phi}{\partial t}+g\zeta\right|_{S_0}=0, \tag{2}$$

where the function w is the normal component of shell displacements, namely, $w=(\mathbf{U},\mathbf{n})$; an unknown function $\zeta=\zeta(x,y,t)$ describes the free liquid surface shape and position. The second item in relations (2) is the impermeability condition on the shell wetted surfaces, the third equality over there defines the kinematics boundary condition, that assumes that any fluid particle being at the free liquid surface in initial time, will be remain there in all subsequent times. Forth equation in (2) defines the boundary dynamical condition, consisting in equality of liquid and atmospheric pressure on the free liquid surface. So the considered problem of fluid-structure interaction is here reduced to the next system of ordinary differential equations:

$$\mathbf{LU}+\mathbf{M\ddot{U}}=p_d\mathbf{n};\qquad \Delta\Phi=0 \tag{3}$$

with boundary conditions (2), relative to the potential Φ, and shell fixation conditions for the displacements $\mathbf{U}$. In this paper clamped-free (C-F), clamped–clamped (C-C), simply supported - clamped (SS-C), and clamped- simply supported (C-SS) truncated conical tanks are studied.

Consider fluid-filled tank vibration modes as following:

$$\mathbf{U}=\sum_{k=1}^{N}c_k(t)\mathbf{u}_k\,. \tag{4}$$

Here $c_k(t)$ are unknown coefficients, and $\mathbf{u}_k$ are shapes of the empty shell natural vibrations. Thus the vibration modes of the liquid-filled elastic shell are linear combinations of its own modes of vibrations without the liquid. Due equalities (1) one can obtain

$$(\mathbf{Lu}_k,\mathbf{u}_j)=\Omega_k^2\delta_{kj},$$

where Ω_k, $k=\overline{1,N}$ are natural frequencies of the empty elastic shell. Let the potential Φ be a sum $\Phi=\Phi_1+\Phi_2$, as it was proposed by Degtyarev in [10]. The series for potential Φ_1 is following

$$\Phi_1=\sum_{k=1}^{N}\dot{c}_k(t)\varphi_{1k},$$

where the time-dependant coefficients $c_k(t)$ are given in eqn (4). To determine functions φ_{1k} the next boundary value problems are formulated:

$$\Delta\varphi_{1k}=0,\;\left.\frac{\partial\varphi_{1k}}{\partial\mathbf{n}}\right|_{\sigma}=w_k,\;\varphi_{1k}\big|_{S_0}=0,\;w_k=(\mathbf{u}_k,\mathbf{n}),\;k=\overline{1,N} \tag{5}$$

Noted, that problems (5) were solved numerically in [8].

So the fluid-structure interaction problem for elastic liquid-filled shells of revolution, without gravity, is formulated using the unknowns $\mathbf{U}$ and Φ_1. These functions satisfy differential equations (3) with the shell fixation conditions. The conditions of impermeability and pressure lack on the free liquid surface are also satisfied. Represent solutions of boundary problems (5) in the operator form as $\varphi_{1k}=i\Omega\mathbf{H}(\mathbf{u}_k)$, where $\mathbf{H}(\mathbf{u}_k)$ is the corresponding inverse operator [8]. Suppose that $c_k(t)=C_k\exp(i\omega t)$, and ω is a natural frequency of the liquid-filled shell. From eqns (1), (4), (5) we have

$$\left(\Omega_k^2\delta_{kj}+\delta_{kj}\right)C_j=\omega^2\rho_l\sum_{k=1}^{N}C_k\left(\mathbf{H}(\mathbf{u}_k),\mathbf{u}_j\right). \tag{6}$$

This equation describes a generalized eigenvalue problem. Solution of problem (6) provides the natural vibration frequencies ω of the considered elastic tank with the liquid, but without considering the gravity effects.

If potential Φ_2 is known, the sloshing modes will be obtained. To define potential Φ_2, the problem of the liquid oscillations in the rigid tank including gravity effects is formulated. Consider the expansion

$$\Phi_2=\sum_{k=1}^{M}\dot{d}_k(t)\varphi_{2k},$$

where $d_k.(t)$ are unknown coefficients, and functions φ_{2k} are sloshing eigenmodes. To determine these modes the next boundary value problems are considered:

$$\Delta\varphi_{2k}=0,\;\left.\frac{\partial\varphi_{2k}}{\partial\mathbf{n}}\right|_{\sigma}=0;\;\left.\frac{\partial\varphi_{2k}}{\partial t}+g\zeta\right|_{S_0}=0;\;\left.\frac{\partial\varphi_{2k}}{\partial\mathbf{n}}\right|_{S_0}=\frac{\partial\varsigma}{\partial t},$$
$$k=\overline{1,N} \tag{7}$$

Suppose $\varphi_{2k}(t,x,y,z)=e^{i\chi_k t}\varphi_{2k}(x,y,z)$ and obtain from (7) the next relations along the free liquid surface for each sloshing mode φ_{2k} with frequency χ_k:

$$\frac{\partial\varphi_{2k}}{\partial\mathbf{n}}=\frac{\chi_k^2}{g}\varphi_{2k},\quad k=1,M\ . \tag{8}$$

It brings us to the next eigenvalue boundary problems

$$\Delta\varphi_{2k}=0\,;\ \left.\frac{\partial\varphi_{2k}}{\partial\mathbf{n}}\right|_{\sigma}=0;\ \frac{\partial\varphi_{2k}}{\partial\mathbf{n}}=\frac{\chi_k^2}{g}\varphi_{2k},\quad k=1,M \tag{9}$$

Solving these problems provides the unknown sloshing modes φ_{2k} and frequencies χ_k .

Thus, for the sum of potentials $\Phi=\Phi_1+\Phi_2$ the next expression is found out:

$$\Phi=\sum_{k=1}^{N}\dot{c}_k(t)\varphi_{1k}+\sum_{k=1}^{M}\dot{d}_k(t)\varphi_{2k}\ . \tag{10}$$

The unknown function ζ becomes

$$\zeta=\sum_{k=1}^{N}c_k(t)\frac{\partial\varphi_{1k}}{\partial n}+\sum_{k=1}^{M}d_k(t)\frac{\partial\varphi_{2k}}{\partial n}\ . \tag{11}$$

To define harmonic vibration coupled modes assume that $c_k(t)=C_k\exp(i\omega t); d_l(t)=D_k\exp(i\omega t)$. Substituting these time-dependant coefficients into (10)-(11) and then into equations

$$\mathbf{LU}+\mathbf{M}\ddot{\mathbf{U}}=p_d\mathbf{n}\ ,\ \left.\frac{\partial\Phi}{\partial t}+g\zeta\right|_{S_0}=0$$

brings us to generalized eigenvalue problem. Note that both gravity and elasticity effects here are taken into account.

3. Boundary Singular Integral Equations

To determine unknown functions φ_{1k} and φ_{2k} the boundary element method (BEM) in its direct formulation is in use, Brebbia *et al* [11]. Dropping for simplicity indexes $1k$ and $2k$ the basic relation is presented as

$$2\pi\varphi(P_0)=\iint_S q\frac{1}{|P-P_0|}dS-\iint_S\varphi\frac{\partial}{\partial\mathbf{n}}\frac{1}{|P-P_0|}dS\,, \tag{12}$$

where $S=\sigma\cup S_0$. The functions φ and q, given on σ, are the liquid pressure on the wetted shell surface and the flux, $q=\partial\varphi/\partial\mathbf{n}$. Module $|P-P_0|$ is Cartesian distance between points P and P_0. We start with boundary integral equation for potential (12), then transform Cartesian coordinates (x, y, z) into cylindrical ones (r, θ, z), and integrate in (12) with respect to variables z and θ. In the cylindrical coordinate system unknown functions are represented as Fourier series expansion by the coordinate θ

$$\begin{gathered} w_k^i(r,z,\theta)=w_k^i(r,z)\cos n\theta; \\ \varphi_{jk}^i(r,z,\theta)=\varphi_{jk}^i(r,z)\cos n\theta;\ i=1,2;\ j=1,2;\ k=1,2,... \end{gathered}\ , \tag{13}$$

where n is the wave number. It would be noted, that solution is independent now of the angular coordinate θ, so 3D problem has been reduced to a two-dimensional one in radial and axial coordinates r and z. The singular boundary integral equations for mixed boundary value problems (5),(7) have been obtained in [8,10]. Numerical simulation procedures built upon the one-dimensional BEM are depicted in [3],[8],[10]. Numerical solutions are obtained using BEM with constant approximations of unknown functions φ and q inside boundary elements.

4. Numerical Simulation and Discussion

The empty and liquid-filled truncated isotropic conical tanks are considered. The sketch of the liquid-filled truncated conical tank is given in Figure 1. Here R_1 and R_2 are cone radii at its large and small edges, α is the semivertex angle, and H is the cone height. The conical tank is refereed to the cylindrical coordinate system (x,θ,z). Hereinafter in the next numerical simulation, the shell thickness and Poisson's ratio are taken as h/R_1=0.01 and ν=0.3, the semivertex angle α=45°, H/R_2=1, Young's modulus E = 2,11·10^6 MPa, ρ_s=8000 kg/m^3, ρ_l=1000 kg/m^3 , R_1=1m, [6]. Here the next fixation conditions are applied at the shell edges: clamped–clamped (CC), clamped–simply supported (C-SS), simply supported –clamped (SS-C), and clamped –free (C-F).

4.1. Hollow Conical Shell without Bottom

To testify the developed here method the hollow conical shell without bottom is considered. The first step was to define the requisite number of finite elements for evaluating the own frequencies with given accuracy. The convergence is established when numbers of finite elements on the tank walls was equal to 30.

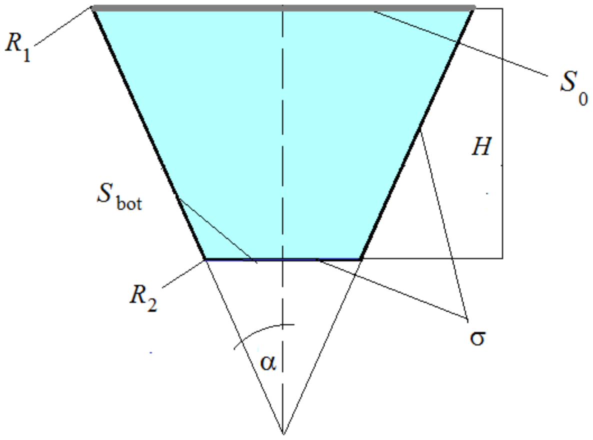

Fig. 1: Truncated conical shell.

The comprehensive analysis of our results and those of Shu et al. [6] is given in Table 1. The abovementioned boundary conditions are included here, with wave numbers n=0,1,2,3,4,5, 6,7.

Table 1: Dimensionless frequencies Ω_1 for the conical tank

n	Boundary conditions					
	SS-C		C-SS		C-C	
	present	Shu [6]	present	Shu [6]	present	Shu [6]
0	0,8701	0,8700	0,7157	0,7151	0.8730	0.8732
1	0,8118	0,8118	0,7098	0,7090	0,8120	0,8120
2	0,6614	0,6613	0,6479	0,6475	0,6694	0,6696
3	0.5247	0.5245	0,5204	0,5201	0,5427	0,5428
4	0,4319	0,4319	0,4166	0,4161	0,4563	0,4566
5	0,3827	0,3826	0.3596	0.3592	0,4087	0,4089
6	0,3739	0,3737	0,3458	0,3450	0,3960	0,3964
7	0,3983	0,3981	0,36518	0,3648	0,4141	0,4143

Here we compare dimensionless frequencies obtained for truncated isotropic elastic conical tanks at different fixation conditions with results obtained by Shu et al. [6] (m = 1, α = 45°) with usage the frequency parameter

$$\Omega_1=\lambda\Omega,\quad \lambda=R_1\sqrt{\rho\left(1-\nu^2\right)/E}$$

for different circumferential number n. The results are in good agreement.

4.2. Hollow Conical Tank with Elastic Clamped Bottom

Next simulation is concern with comparison of dynamical characteristics of hollow conical tanks with rigid bottoms and with clamped elastic bottoms. The fixation conditions at edges are the clamped-free ones. The results for n=0 are shown in Table 2.

Table 2: Axisymmetric natural frequencies Ω_1 for conical shell, Hz

m	Type of shell				
	Hollow shell without bottom			Conical shell with elastic clamped bottom	
	Frequency	Dominant		Frequency	Dominant
1	559,46	wall		101,67	bottom
2	675, 85	wall		393,43	bottom
3	707,27	wall		559,48	wall
4	824,91	wall		675,85	wall
5	1001,1	wall		707,68	wall
6				824,91	wall

The own frequencies of the elastic shells with rigid bottoms and hollow shells are coincided. The dynamical characteristics are changed in the case of elastic bottom. Comparing the above results from Table 2 one can conclude that the lowest axisymmetric frequencies belong to the bottom vibrations.

4.3. Sloshing Process in Conical Shells

Linear liquid sloshing in the rigid Λ-shape conical tank with R_2 = 0.5m and R_1 = 1.0m, H=0.5m and $\theta=\pi/4$ is considered. The sloshing frequencies are calculated accordingly to Degtyarev [12]. Below we testify that sloshing frequencies of rigid cylindrical and conical shells with equal heights (H=1m) and large radius of cone R_1=1m equals to cylinder radius are differed. Comparison of results for n=0 is given in Table 3.

Table 3: Sloshing axisymmetric natural frequencies, Hz

m	Type of shell		
	Cylindrical shell		Conical shell
1	6,1309		5,8368
2	8,3007		8,1042
3	9,9975		9,8394
4	11,.434		11,3082
5	12,7261		12,6069

The difference between frequencies of conical and cylindrical shells is essential at the lowest m.

4.4. Vibrations of Elastic Fluid-Filled Truncated Conical Tanks

The main purpose of this research is to estimate natural modes and frequencies of the elastic cone coupled with liquid sloshing. Below the elastic conical tank with CF edges is considered. The results are obtained for 7 wave numbers $n=\overline{0,6}$ and for $m=\overline{1,4}$. The frequencies of liquid sloshing, vibrations of empty and liquid-filled tanks are considered. The numerical simulation results are given in table 4.

Table 4: Own frequencies of elastic truncated conical shell, Hz

n	*m*	Frequencies		
		Sloshing	Empty	Fluid-filled
0	1	5.836	101.07	41.67
	2	8.300	393.49	214.06
	3	9.997	559.52	257.91
	4	11.443	675.88	471.43
1	1	3.659	210.34	113.56
	2	7.001	327.90	126.64
	3	8.979	601.83	425.00
	4	10.577	649.99	438.22
2	1	4.819	193.05	96.57
	2	7.897	345.07	224.03
	3	9.729	605.43	346.52
	4	11.236	764.39	500.66
3	1	5.707	128.30	64.217
	2	8.661	504.88	281.69
	3	10.397	519.84	327.95
	4	11.837	723.61	474.97
4	1	6.460	100.89	58.200
	2	9.340	436.93	467.25
	3	11.005	689.26	265.86
	4	12.394	693.20	506.67
5	1	7.1288	101.85	56.908
	2	9.9581	385.15	232.28
	3	11.568	671.56	452.47
	4	12.915	897.75	686.67
6	1	7.736	123.20	78.861
	2	10.529	368.32	241.18
	3	12.094	663.03	458.88
	4	13.406	952.83	688.34

As a result of these numerical simulations three basic systems are built. The elastic empty shell modes are the first one. The second system presents the free vibration modes of the elastic tank without gravity effects, and the third one consists of sloshing modes including gravity effects. In the considered case the mutual influence of sloshing and elastic shell vibrations is negligible. The separation of frequency spectrums for the liquid-filled elastic cone tank and sloshing in the rigid tank is observed.

The interesting conclusions of numerical simulations are following. First, the lowest frequency here belongs to the axisymmetric mode with dominant bottom vibrations. The axisymmetric modes for the bottom and wall of the truncated conical tanks are demonstrated in Figures 2-3.

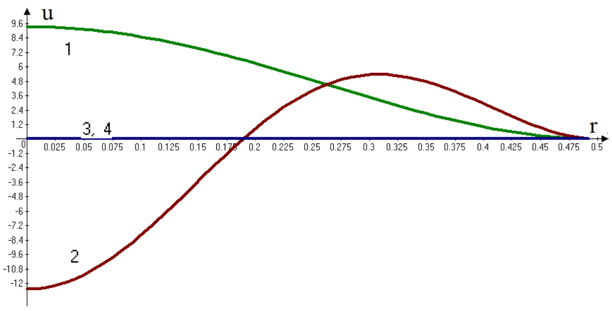

Fig. 2: Axisymmetric modes of bottom vibrations.

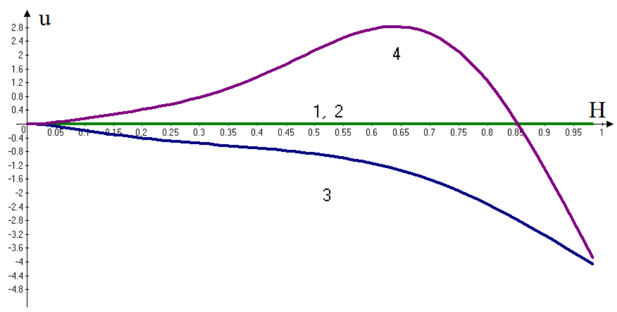

Fig. 3: Axisymmetric modes of wall vibrations.

Here and hereinafter numbers 1,2,3,4 correspond to the vibration number m. Figures 2-3 demonstrate different behavior of the bottom and shell walls vibrations.

One can observe that in this case the bottom and wall vibrations do not affect each other. Note that the frequency ω=41.67 Hz is the lowest one for vibrations of the liquid-filled elastic conical shell with elastic bottom. It corresponds to n=0 and m=1. If the conical shell with the rigid bottom is considered, then the lowest frequency occurs at n=4 and m=1 for the empty shell, and for n=5 and m=1 for the fluid-filled shell.

Figure 4 demonstrates modes that correspond to the bottom vibrations for n=5 and m=1.

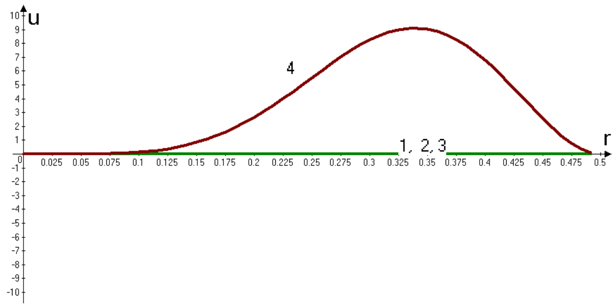

Fig. 4: Modes of bottom vibrations for *n*=5 and *m*=1

Here one can see that the frequencies of bottom vibrations are not the lowest ones.

Figure 5 demonstrates the modes of wall vibrations for *n*=5 and *m*=1. The lowest frequency here responds to the wall vibrations. So, if the truncated elastic conical tank having the rigid bottom is considered then the lowest frequency does not correspond to the axisymmetric mode.

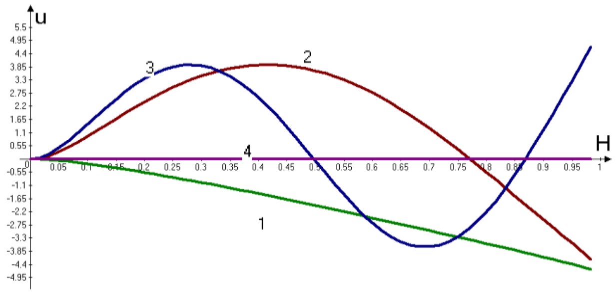

Fig. 5: Modes of wall vibrations for *n*=5 and *m*=1

Here one can observe that the wall vibrations became dominant.

Analyzing figures 2-5 leads to understanding that at low wave numbers the dominant modes of truncated elastic cone vibrations correspond to its bottom. With increasing the number of nodal diameters the wall vibrations become dominant.

Figure 6 shows the modes of lowest frequencies for liquid sloshing in the rigid tank (left) and for the elastic conical tank with the rigid bottom (right).

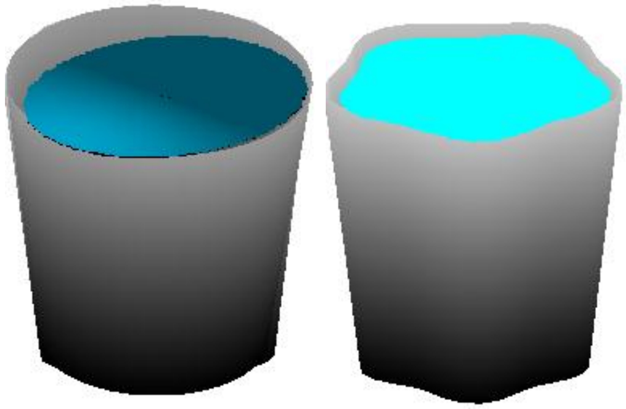

Fig. 6: Modes of lowest frequencies

If the bottom deformation is neglected, than the lowest frequency of elastic fluid-filled shell will be missed.

The frequencies ω near 100Hz are so considered as most dangerous for empty shells. The results given in table 4 testify it. For example, ω=101.07Hz correspond to *n*=0 and *m*=1; ω=100.89Hz correspond to *n*=4 and *m*=1; and ω=101.86Hz correspond to *n*=5 and *m*=1.

It is also important to note that lowest frequencies of the empty and liquid-filled tanks correspond to different circumferential wave numbers.

The frequencies of liquid-filled tank vibrations are drastically differ from frequencies of empty ones. But with increasing the wave number this difference become gradually smaller.

5. Conclusion

The analysis of natural vibrations for the truncated elastic shell in interaction with the liquid sloshing has been accomplished. Coupled one-dimensional finite and boundary element methods are in use. The vibration analysis includes several steps. At the first stage the frequencies and modes of the empty elastic conical tank are obtained. The displacements in the coupled problem are considered as the linear combinations of the empty elastic shell natural modes. So free vibration modes and frequencies for the liquid-filled elastic shell without gravity effects are defined at the second step. The sloshing frequencies and modes in the rigid conical tank including gravity effects are estimated at third stage. The numerical simulation for two latter problems is accomplished with usage of the one-dimensional BEM. The developed method essentially reduces the computer time for numerical analysis and produces new qualitative possibilities in advanced computational modeling the dynamical characteristics of elastic shell structures. The difference in dynamical characteristics between elastic truncated shells with rigid and elastic bottoms is established. The [resented here results can be considered as a basis for further research in the dynamical behaviour of structures subjected to intensive loadings and interacting with fluids. It would be noted that our interpretation and understanding the dynamical processes in elastic shell structures subjected to actions of flowing fluids is nowadays far from completion, and requires additional research.

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