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Research paper

Calculation of The Stability of the Form of Equilibrium of Discrete Systems

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Abstract

The paper presents an algorithm for calculating the stability of the equilibrium form of the first kind of compressed discrete systems by the displacements method in combination with the methods of iterations and bisection. The use of the methods makes it possible to effectively determine the minimum critical stress or strain at the first bifurcation and their corresponding form of stability loss, both for statically determined and statically undetermined systems. This approach, using matrix forms, makes it possible to significantly simplify the calculations of the analytical condition for the stability loss of compressed discrete systems (the stability loss equation), which has high orders, as well as to construct the form of stability loss corresponding to a critical load, that is, to solve the problem of loss of equilibrium stability. The calculation actually leads to solving a nonlinear transcendental equation, which is the equation of stability loss. The difficulty lies in the absence of an analytical solution of such an equation due to the presence of complex of Zhukovsky functions, which have transcendental functions in their structure. Such solution can be performed only with the use of numerical methods. This algorithm for calculating the loss of equilibrium of the first kind of compressed discrete systems by displacement in combination with the methods of iteration and bisection is implemented in the software complex "Persist" for PC in Windows OS. The program was approbated and implemented in the educational process at the Department of Structural and Theoretical Mechanics of Poltava National Technical Yuri Kondratyuk University during the training of specialists in engineering specialties.

Keywords: *stability, critical state, critical load, critical stress, stability loss of the equilibrium form (deformation), bifurcation, form of stability loss, iteration, bisection.*

1 Introduction

There are various methods for calculating the stability of the shape of the equilibrium of discrete systems, due to the large volume of computations associated with the solution of the analytical condition for the loss of equilibrium stability. The solution of the analytical condition for the stability loss of the equilibrium of compressed discrete systems, which has high orders, and the definition of the critical load of the form of stability loss, is one of the topical problems. The solution of the problem of calculating the analytical condition for stability loss of the equilibrium of compressed discrete systems, which have high orders, and the determination of the corresponding critical load of the form of stability loss, generated a large number of methods by many mathematicians (Krylov, Laverier, Danilevsky, Jacobi (iterations) [1 – 10] etc.). The calculation actually leads to solving a nonlinear transcendental equation, which is the equation of stability loss. The difficulty lies in the absence of analytical solution of such equation due to the presence of complex functions of Zhukovsky, which have transcendental functions in their structure. Such solution can be performed only with the use of numerical methods. This problem of calculating the analytical condition for stability loss of the equilibrium of compressed discrete systems, as well as the determination of the critical load of the form of stability loss, is proposed to solve by displacement, iteration and bisection methods, which makes it possible to significantly simplify calculations. The purpose of the work is to develop an algorithm and software for PC in Windows OS, which will enable students and engineers to automate calculations of stability of equilibrium forms of compressed discrete systems.

2 Main Body

Here is an algorithm for calculating the stability loss of the equilibrium form of the first kind of compressed discrete systems on the basis of the displacement method in expanded form in combination with the methods of inertia, bisection, chord in conjunction with refinement on iterations by methods of tangent [11].

1. Determine t_i values for each j *-th* compressed rod.

$$t_j^2 = \frac{N_j \cdot l_j}{i_j} = \frac{N_j \cdot l_j^{\,2}}{EI_j}, \quad (j = 1,2,3,...,m) \tag{1}$$

where t - number of compressed rods.

2. Accepted as a baseline (t_0) any of the parameters t_j and express the relationship between the parameters t.

$$K_0 = \frac{t_0}{t_0} = 1; \ldots; \; K_j = \frac{t_j}{t_0}; \ldots; K_n = \frac{t_n}{t_0} \tag{2}$$

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3. All parameters t_j are expressed through basic t_0.

$$t_j = K_j \cdot t_0 \quad (3)$$

4. We set the initial values of the basic parameter $t_0 = 0$, and determine the values of parameters in other rods, expressed in terms of the base.
5. We substitute in equations of stability loss the corresponding values of t_j, expressed in t_0, and calculate its value in the first iteration.
6. We set Δt_0 the step of change the base parameter t_0, that is

$$t_0 = t_0 + \Delta t_0 \quad (4)$$

7. We determine the value of the equation of stability loss on the next iteration in accordance with step 6.
Iterations are performed until there is a change in the sign of the equation of stability loss (D), that is, as it is depicted in the graph (Fig.1).
Execution of steps 5, 6, 7 allows separating the value of the critical parameter with the error to the value of step Δt_0.

Using the iteration method, we determine the subinterval in which the first real root will be of the transcendental equation of stability loss of equilibrium forms.
The iterative process of solving a complex nonlinear equation of loss of equilibrium stability can also be implemented using the chord method in conjunction with the refinement on iterations by methods of tangential, bisection, and others. In this case, the search for the interval of existence of the first real root is carried out by the method of selection as for the method of dichotomy.
The process of iterative refinement of the root continues until the root value reaches the specified accuracy.

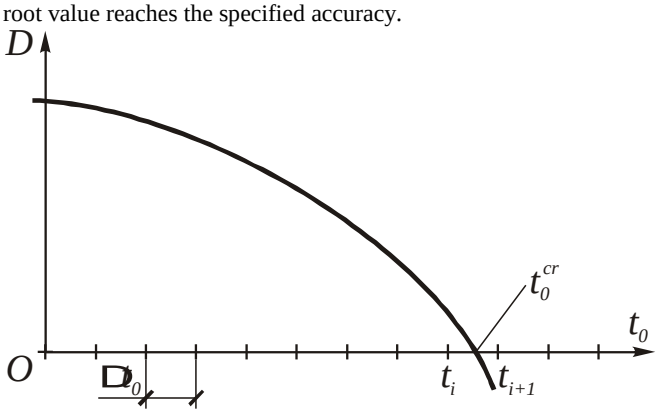


Fig. 1: Graph of the solution to the equation of stability loss

8. In the interval $t_i < t_0 < t_{i+1}$ specify the value of the critical parameter $t_{0,cr}$ by the methods of bisection, tangent, and so on. The calculations are carried out as long as

$$\left| t_{0,i} - t_{0,i+1} \right| \le \varepsilon \quad (5)$$

where ε - the pre-specified accuracy of calculation of basic critical parameter $t_{0,cr}$.

9. It should be kept in mind that when selecting step Δt_0, you can skip the first critical root (the first least critical force). Therefore, when choosing it is necessary to accept as little as possible its initial value, ie $\Delta t_0 \le 0{,}5$.

To solve this engineering problem, the Persist software was developed and implemented on PC on the Department of Structural and Theoretical Mechanics of Poltava National Technical Yuri Kondratyuk University. This software is implemented in a modern compiler, contains several subroutines, utilities, which are combined and presented in the form of the same software package.

General algorithm for calculating stability by the software "Persist".

The calculation of the compressed frame on the stability of the equilibrium form involves forming a loss-stability equation independently.
Only after completing and checking the equation of stability loss as a nontrivial solution of a homogeneous system of canonical equations, a user uses a PC to solve stability of the equilibrium form according to the "Persist" software.
The calculation of the compressed discrete system on the stability of the equilibrium form actually reduces to the solution of the difficultly described nonlinear transcendental equation, which is the equation of stability loss. The difficulty lies in the absence of an analytical solution of such equation due to the presence of complex of Zhukovsky functions, which have transcendental functions in their structure. The arguments of the Zhukovsky functions are the values of the parameters of the compressed rods parameter t.

Generalized structural scheme of solving equation of stability loss:

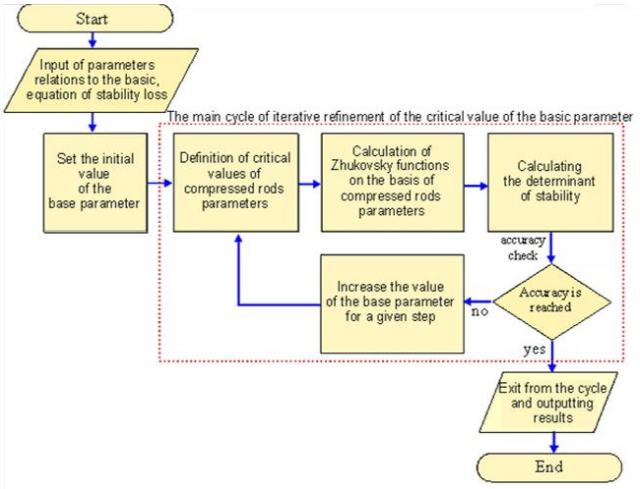


Fig. 2: Structural diagram of the algorithm for solving the equation of stability loss

The main points of describing the source data in the program «Persist».
1. Description of the compressed rods parameters.
The dialog for describing the parameters of the compressed rods is activated by pressing the corresponding button, Fig.3. In this case, it should be noted that only non-zero parameters are described, that is, the parameters of only compressed rods, rods with zero value of longitudinal force are not part of the ones described.

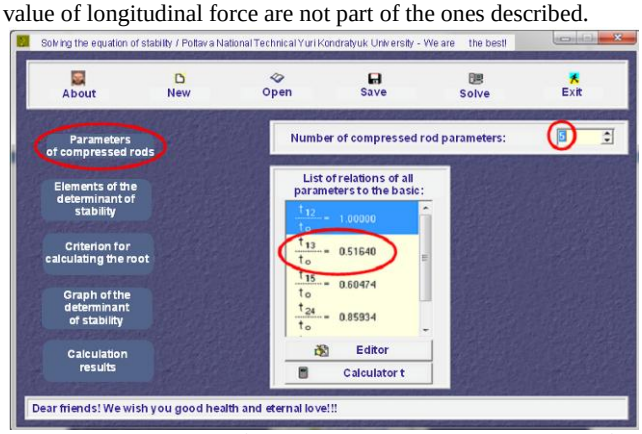


Fig. 3: Visual components for managing the description parameters of the compressed rods

2. Introduction of elements of equation of stability loss of the equilibrium form.
Determination of the first real root of the equation of equilibrium of stability loss.
3. Criteria setting for iterative calculation.
As a precision criterion, the "Persist" program uses the so-called absolute criterion when the process of iterative refinement of the root stops when the absolute value of the desired root does not exceed the value of the given error, which is recommended to set in the range 10^{-6} … 10^{-4}. Fig.7 shows an activated menu item for adjusting the criteria for an iterative calculation.

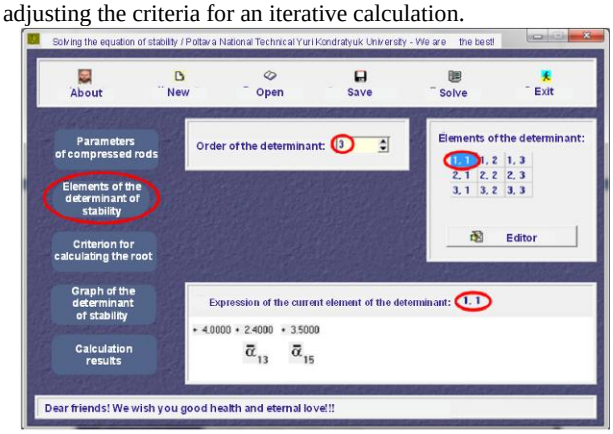


Fig. 4: Visual components of control description of equation elements of stability loss of the equilibrium form

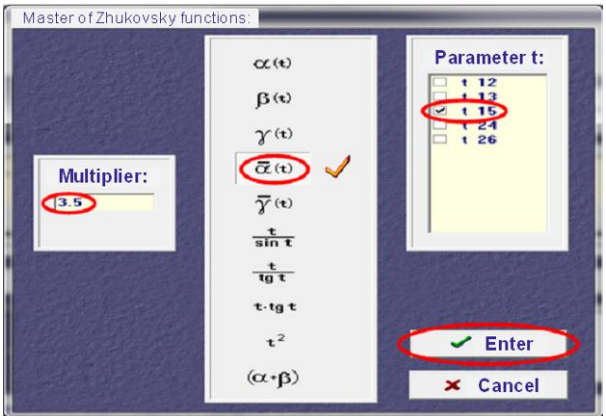


Fig. 5: Visual components of description control element 1.1 of equation of stability loss of the equilibrium form

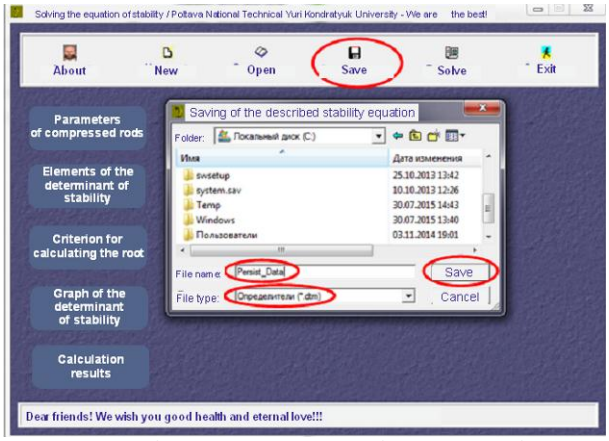


Fig. 6: Saving of the described equation of stability loss of the equilibrium form

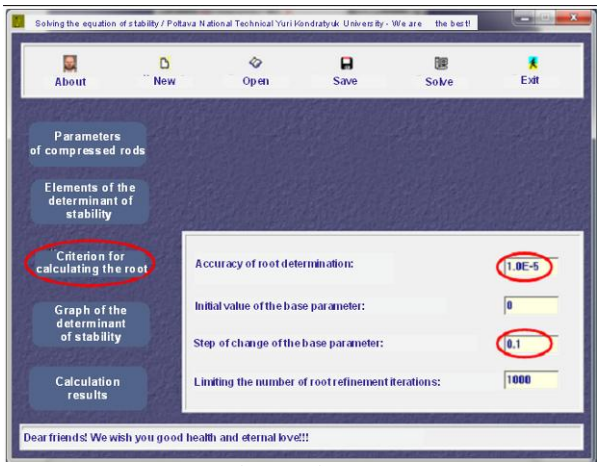


Fig. 7: Settings of criteria for iterative calculation

4. Directly solving the equation of stability in «Persist».
As noted above, solving the equation of stability loss of the equilibrium form involves the use of numerical methods. Thus, an iterative process of solving a complex nonlinear equation using the chord method in conjunction with refinement on its iterations by the method of bisection is organized in "Persist". The search for the interval of existence of the first root is carried out by the method of selection. The process of iterative refinement of the root continues until the root value reaches the given accuracy.
For a solution already described in the program "Persist" equation of stability loss of the equilibrium form, the user must use the item in the main menu "Solve", Fig.3.
5. Analysis and verification of calculation results.
As is known, transcendental equations can have not one root due to the presence of periodic functions. According to the physical content of the equation of stability loss, we are interested in the first root, which corresponds to the minimum critical value of the basic parameter of the compressed rod t_0^{cr}. On its basis, the critical values of the parameters of all compressed rods are determined as: $t_i^{cr} = t_0^{cr} \cdot k_i$, where k_i – is the coefficient of reduction of compressed rods lengths.
When numerically iterative solving of such an equation, it is necessary to focus attention on preventing the passage of the first root of the equation. To do this, one needs to analyze the results. As a rule, it is enough to visually assess the type of graph of the dependence of the determinant of stability of the equilibrium form from the value of the basic parameter t_{0cr} (Fig.8).
Fig.9 and Fig.10 show the results of the solution of the equation of stability loss of the equilibrium form in table and graph forms.

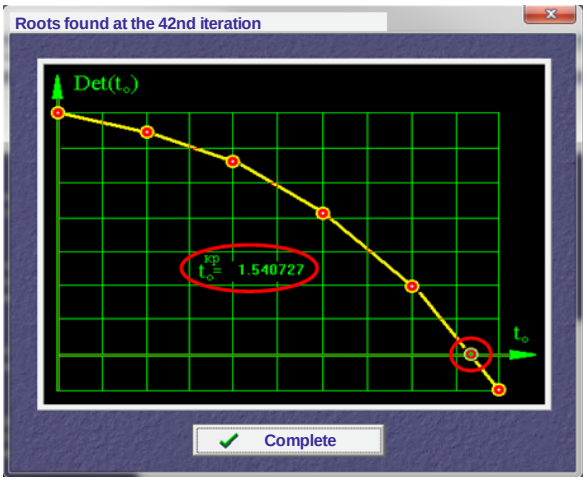


Fig. 8: Chart of the dependence of the determinant of resistance on the value of the base parameter

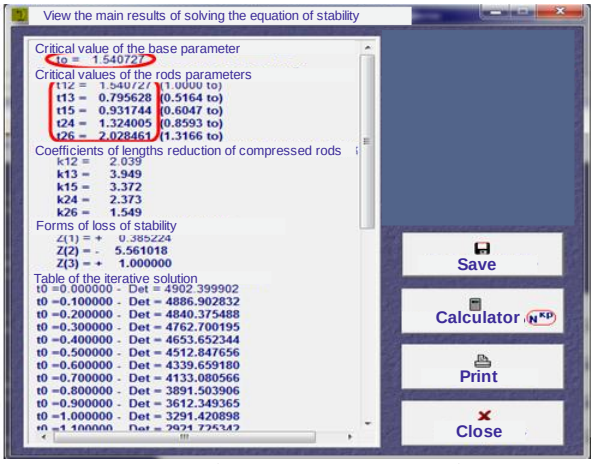


Fig. 9: View the results of the solution the equation of stability loss of the equilibrium form



Fig. 10: Graphic representation of the solution of the equation of stability loss

The user is able to adjust the accuracy of the graphical display of the solution of the equation of stability loss (Fig.11). A calculator is available for the user which allows to calculate the values of critical longitudinal forces (Fig.12).

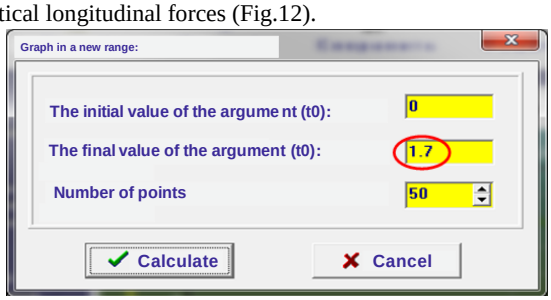


Fig. 11: Settings of the graphical representation of the solution of the equation of stability loss of the equilibrium form

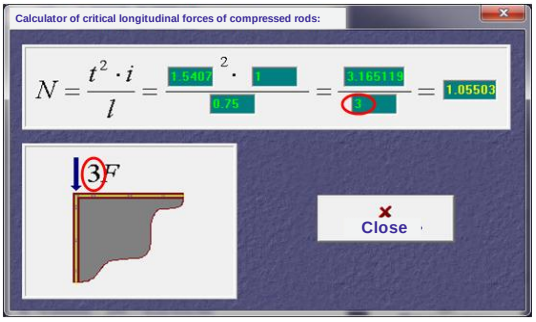


Fig. 12: Calculator for calculating critical longitudinal forces

6. Saving the results and getting a copy of them.

For the possibility of further analysis and processing of the results of calculations, "Persist" provides the possibility of saving them in a text file. To do this, one should use the appropriate menu item "Save" in the viewport of the main results, see Fig.3. In this case, all actions of saving file are absolutely similar to those used in any Windows program (Fig.13).

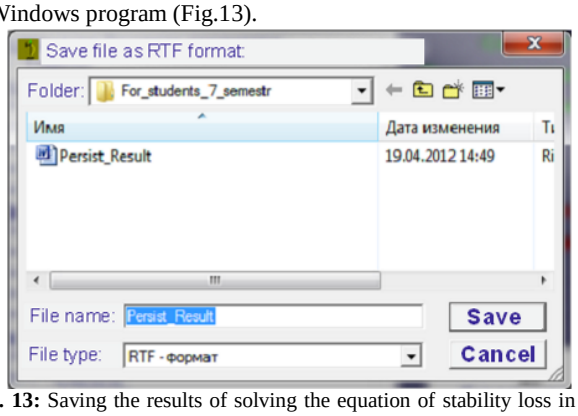


Fig. 13: Saving the results of solving the equation of stability loss in a text file

A text file with results can be downloaded to any text editor to prepare for printing in a more user-friendly format.

7. Main properties of "Persist" software

Requirements for PC system resources. The program uses the Windows operating system. For ease of use, we recommend using monitors with a resolution of 800x600 pixels.

Software package Persist consist of files:

- Persist.exe (mandatory) – main program file;
- Persist.hlp (recommended) –help file for using "Persist";
- Persist.ini (optional) – initializing the values of the search criterion for the root equation of stability, can be created automatically;
- Persist.txt (optional) – file with a short description of the program;
- Demo*.dtm (optional) – demo files of the described equations of stability, which can be downloaded for calculation and analysis.

Copyrights. The program can be distributed freely, and authors are not liable for any losses incurred as a result of using this program (data loss, financial loss, etc.).

Due to the constant improvement of the program, authors recommend that you follow the release number in the About menu.

Installing software package «Persist» on PC. For correct operation of the program "Persist" you should install it on the hard disk of the PC. It is not desirable to launch the program directly from the CD. This is due to the need in certain cases saving data on hard disk. To install the "Persist" program on a PC, you need to copy the "Persist" subdirectory to a PC hard disk.

Consider an example of calculating the stability of the equilibrium form of a compressed discrete system by the displacements method in expanded form. The relationship between the geometric dimensions of the rods and the stiffness on the bend:

$$\frac{l}{h}=\frac{3}{4};\quad \frac{i_1}{i_0}=1;\quad \frac{i_2}{i_0}=1,3;\quad \frac{i_3}{i_0}=2,4;\quad \frac{i_4}{i_0}=3,5;$$
$$i_1=i_0;\quad i_2=1,3i_0;\quad i_3=2,4i_0;\quad i_4=3,5i_0. \qquad (6)$$

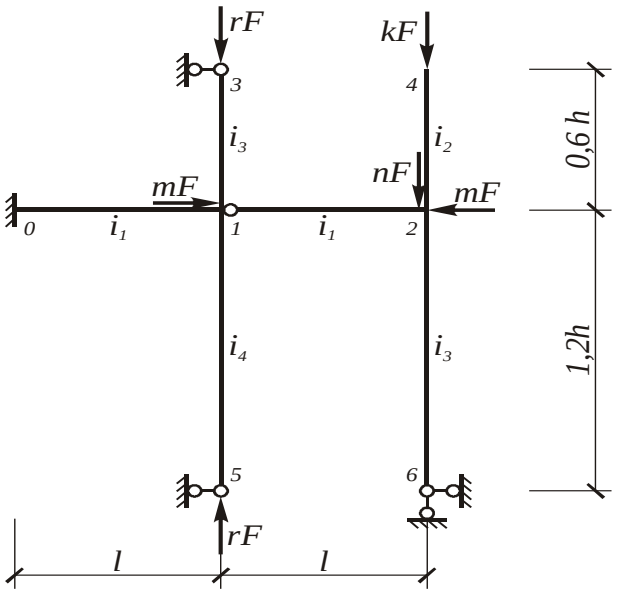


Fig. 14: Design scheme of the frame

1. Determination of the degree of kinematic uncertainty of the system

$$n = n_v + n_l = 2 + 1 = 3 \tag{7}$$

2. Determination of longitudinal forces in compressed frame rods

$$\begin{aligned} &N_{01} = 0; \quad N_{12} = 5F; \quad N_{31} = N_{15} = 4F; \\ &N_{42} = 6F; \quad N_{26} = 6F + 7F = 13F. \end{aligned} \tag{8}$$

3. The basic system of the method of displacement

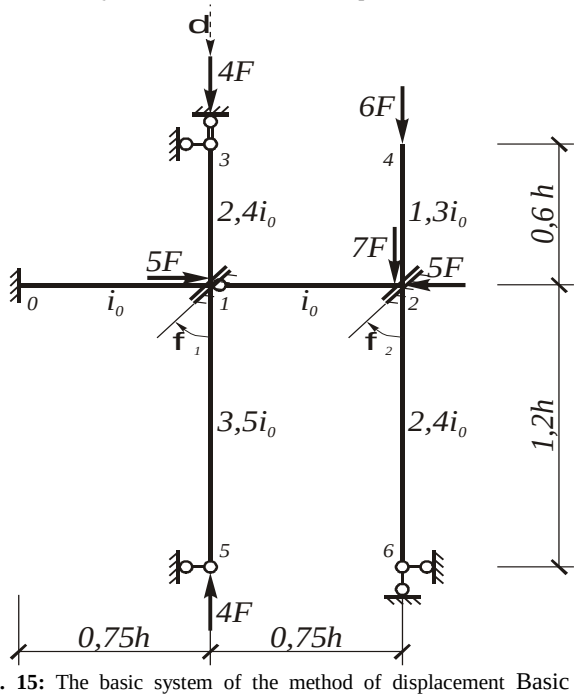
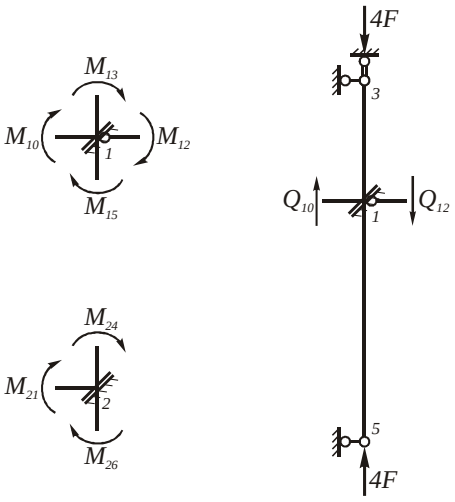


Fig. 15: The basic system of the method of displacement Basic unknowns: φ_1, φ_2 i δ

4. Basic and additional equations of the method of displacement (expanded form)



Basic equations:

$$\sum M_1 = 0; \quad M_{10} + M_{13} + M_{12} + M_{15} = 0 \tag{9}$$

$$\sum M_2 = 0; \quad M_{21} + M_{24} + M_{26} = 0 \tag{10}$$

Additional equations:

$$\sum Y = 0; \quad Q_{10} - Q_{12} = 0 \tag{11}$$

System of equation of the displacements method in expanded form:

$$\begin{cases} M_{10} + M_{13} + M_{12} + M_{15} = 0; \\ M_{21} + M_{24} + M_{26} = 0; \\ Q_{10} - Q_{12} = 0. \end{cases} \tag{12}$$

We express the bending moments of the first equation through the basic unknowns:

$$\begin{aligned} &M_{10} = 2 \cdot i_{10} \cdot [\alpha_{10} \cdot \varphi_1 + \beta_{10} \cdot \varphi_0 - (\alpha + \beta)_{10} \cdot \psi_{10}] = \\ &= 2 \cdot i_0 \cdot [\alpha_{10} \cdot \varphi_1 - (\alpha + \beta)_{10} \cdot \delta / (0,75h)] \end{aligned} \tag{13}$$

Since $N_{01} = 0$, then $\alpha_{10} = 2$, and $\beta_{10} = 1$.

Consequently $M_{10} = 2 \cdot i_0 \cdot [2 \cdot \varphi_1 - 4 \cdot \delta / h]$ (14)

$M_{13} = i_{13} \cdot \bar{\alpha}_{13} \cdot [\varphi_1 - \psi_{13}] = 2,4 \cdot i_0 \cdot \bar{\alpha}_{13} \cdot \varphi_1$, and $M_{12} = 0$ (15)

$M_{15} = i_{15} \cdot \bar{\alpha}_{15} \cdot [\varphi_1 - \psi_{15}] = 3,5 \cdot i_0 \cdot \bar{\alpha}_{15} \cdot \varphi_1$ (16)

$(4 + 2,4 \cdot \bar{\alpha}_{13} + 3,5 \cdot \bar{\alpha}_{15}) \cdot \varphi_1 \cdot i_0 + (-8) \cdot \delta / h \cdot i_0 = 0$ (17)

We express the bending moments of the second equation through the basic unknowns:

$M_{21} = i_{21} \cdot \bar{\alpha}_{21} \cdot [\varphi_2 - \psi_{21}] = i_0 \cdot \bar{\alpha}_{21} \cdot [\varphi_2 + \delta / (0,75h)]$ (18)

$M_{24} = -i_{24} \cdot t_{24} \cdot tgt_{24} \cdot \varphi_2 = -1,3 \cdot i_0 \cdot t_{24} \cdot tgt_{24} \cdot \varphi_2$ (19)

$M_{26} = i_{26} \cdot \bar{\alpha}_{26} \cdot [\varphi_2 - \psi_{26}] = 2,4 \cdot i_0 \cdot \bar{\alpha}_{26} \cdot \varphi_2$ (20)

$(\bar{\alpha}_{21} - 1,3 \cdot t_{24} \cdot tgt_{24} + 2,4 \cdot \bar{\alpha}_{26}) \cdot \varphi_2 \cdot i_0 + (1,333 \cdot \bar{\alpha}_{21}) \cdot \delta / h \cdot i_0 = 0$ (21)

We express the transverse forces of the third equation through the main unknowns:

$$Q_{10} = -2 \cdot i_{10} / l \cdot [(\alpha + \beta)_{10} \cdot (\varphi_1 + \varphi_0) - \gamma_{10} \cdot \psi_{10}] = \\ = -2 \cdot i_0 / (0,75h) \cdot [3 \cdot \varphi_1 - 6 \cdot \delta / (0,75h)] \quad (22)$$

$$Q_{12} = -i_{12} / l \cdot [\bar{\alpha}_{12} \cdot \varphi_2 - \bar{\gamma}_{12} \cdot \delta / l] = \\ = -i_0 / (0,75h) \cdot [\bar{\alpha}_{12} \cdot \varphi_2 + \bar{\gamma}_{12} \cdot \delta / (0,75h)] \quad (23)$$

$$(-8) \cdot \varphi_1 / h \cdot i_0 + (1,333 \cdot \bar{\alpha}_{21}) \cdot \varphi_2 / h \cdot i_0 + \\ + (21,333 + 1,778 \cdot \bar{\gamma}_{21}) \cdot \delta / h^2 \cdot i_0 = 0 \quad (24)$$

The system of equations of the method of displacement in expanded form:

$$\begin{cases} (4 + 2,4 \cdot \bar{\alpha}_{13} + 3,5 \cdot \bar{\alpha}_{15}) \cdot \varphi_1 + 0 \cdot \varphi_2 - 8 \cdot \delta / h = 0; \\ 0 \cdot \varphi_1 + (\bar{\alpha}_{21} - 1,3 \cdot t_{24} \cdot tgt_{24} + 2,4 \cdot \bar{\alpha}_{26}) \cdot \varphi_2 + (1,333 \cdot \bar{\alpha}_{21}) \cdot \delta / h = 0; \\ -8 \cdot \varphi_1 + (1,333 \cdot \bar{\alpha}_{21}) \cdot \varphi_2 + (21,333 + 1,778 \cdot \bar{\gamma}_{21}) \cdot \delta / h = 0. \end{cases} \quad (25)$$

Determination of the relationship between the parameters t of the compressed rods:

$$t_i = t_{ab} = \sqrt{\frac{N_{ab}}{i_{ab}} \cdot l_{ab}}; \quad t_0 = t_{12} = \sqrt{\frac{5F}{i_0} \cdot 0,75h}; \\ t_1 = t_{13} = \sqrt{\frac{4F}{2,4i_0} \cdot 0,6h}; \quad t_2 = t_{15} = \sqrt{\frac{4F}{3,5i_0} \cdot 1,2h}; \quad (26) \\ t_3 = t_{24} = \sqrt{\frac{6F}{1,3i_0} \cdot 0,6h}; \quad t_4 = t_{26} = \sqrt{\frac{13F}{2,4i_0} \cdot 1,2h};$$

$$K_i = \frac{t_i}{t_0}; \quad K_0 = 1; \quad K_1 = \sqrt{\frac{4F \cdot 0,6h \cdot i_0}{2,4i_0 \cdot 5F \cdot 0,75h}} \cong 0,516398; \\ K_2 = \sqrt{\frac{4F \cdot 1,2h \cdot i_0}{3,5i_0 \cdot 5F \cdot 0,75h}} \cong 0,604743; \quad (27) \\ K_3 = \sqrt{\frac{6F \cdot 0,6h \cdot i_0}{1,3i_0 \cdot 5F \cdot 0,75h}} \cong 0,859338; \\ K_4 = \sqrt{\frac{13F \cdot 1,2h \cdot i_0}{2,4i_0 \cdot 5F \cdot 0,75h}} \cong 1,316561.$$

5. Output data for calculating the equation of stability loss on a PC on the given algorithm.
The relationship between the parameters t:

$$t_0 = t_{12}; \quad t_1 = t_{13}; \quad t_2 = t_{15}; \quad t_3 = t_{24}; \quad t_4 = t_{26}; \\ K_0 = 1; \quad K_1 = 0,516398; \quad K_2 = 0,604743; \quad (28) \\ K_3 = 0,859338; \quad K_4 = 1,316561.$$

Equation the loss of equilibrium stability (upper determinant triangle):

$$\begin{vmatrix} 4 + 2,4 \cdot \bar{\alpha}_{13} + 3,5 \cdot \bar{\alpha}_{15} & 0 & -8 \\ & \bar{\alpha}_{21} - 1,3 \cdot t_{24} \cdot tgt_{24} + 2,4 \cdot \bar{\alpha}_{26} & 1,333 \cdot \bar{\alpha}_{21} \\ & & 21,333 + 1,778 \cdot \bar{\gamma}_{21} \end{vmatrix} = 0 \quad (29)$$

Results of calculation:
The values of the critical parameters t of the compressed rods are:

$$t_{01} = 2,503462; \quad t_{12} = 1,269506; \quad t_{23} = 1,526861; \\ t_{35} = 4,231602; \quad t_{46} = 2,448636. \quad (30)$$

Forms of stability loss:

$$Z(1) = \phi_1 = 0,385224; \\ Z(2) = \phi_2 = -5,561034; \quad (31) \\ Z(3) = \delta / h = 1,000000.$$

Table 1. Iterative solution the equation of stability loss of the equilibrium form

t_0	Det
0.000000	4902.399902
0.100000	4886.902832
0.200000	4840.375488
0.300000	4762.700195
0.400000	4653.652344
0.500000	4512.847656
0.600000	4339.659180
0.700000	4133.080566
0.800000	3891.503906
0.900000	3612.349365
1.000000	3291.420898
1.100000	2921.725342
1.200000	2491.159424
1.300000	1977.557617
1.400000	1336. 789063
1.500000	469.061646
1.550000	-121.025391
1.525000	192.218048
1.537500	40.719765
1.543750	-38.789200
1.540625	1.295279
1.542187	-18.663136
1.541406	-8.663139
1.541016	-3.678754
1.540820	-1.190446
1.540723	0.052739
1.540771	-0.568773
1.540747	-0.257997
1.540735	-0.102624
1.540729	-0.024941
1.540726	0.013899
1.540727	-0.005521
1.540726	0.004189
1.540727	-0.000666

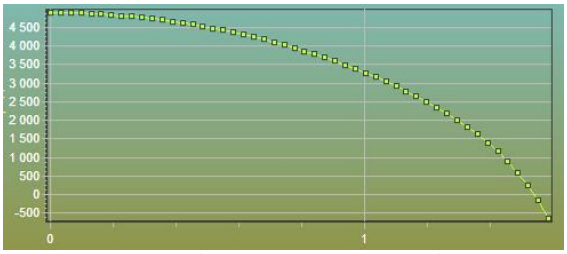


Fig. 16: Graph of the solution the equation of stability loss

6. Determination of longitudinal critical forces in compressed rods

$$N_{ab}^{cr} = \frac{(t_{ab}^{cr})^2 \cdot i_{ab}}{l_{ab}}. \quad (32)$$

$$N_{12}^{cr} = \frac{(t_{12}^{cr})^2 \cdot i_0}{0,75h} = \frac{1,540727^{\,2} \cdot i_0}{0,75h} = 3,165 \frac{i_0}{h} [\kappa N] \quad (33)$$

$$N_{13}^{cr} = \frac{(t_{13}^{cr})^2 \cdot 2,4i_0}{0,6h} = \frac{0,795628^{\,2} \cdot 2,4i_0}{0,6h} = 2,532 \frac{i_0}{h} [\kappa N] \quad (34)$$

$$N_{15}^{cr} = \frac{(t_{15}^{cr})^2 \cdot 3,5i_0}{1,2h} = \frac{0,931744^{\,2} \cdot 3,5i_0}{1,2h} = 2,532 \frac{i_0}{h} [\kappa N] \quad (35)$$

$$N_{24}^{cr} = \frac{(t_{24}^{cr})^2 \cdot 1,3i_0}{0,6h} = \frac{1,324005^{\,2} \cdot 1,3i_0}{0,6h} = 3,798 \frac{i_0}{h} [\kappa N] \quad (36)$$

$$N_{26}^{cr} = \frac{(t_{26}^{cr})^2 \cdot 2,4 i_0}{1,2h} = \frac{2,028461^{\,2} \cdot 2,4 i_0}{1,2h} = 8,229 \frac{i_0}{h} [\kappa N] \quad (37)$$

7. Determination of critical load

$$F_{12}^{cr} = \frac{N_{12}^{cr}}{5} = 0,633 \frac{i_0}{h} \quad [\kappa N] \quad (38)$$

$$F_{13}^{cr} = \frac{N_{13}^{cr}}{4} = 0,633 \frac{i_0}{h} \quad [\kappa N] \quad (39)$$

$$F_{15}^{cr} = \frac{N_{15}^{cr}}{4} = 0,633 \frac{i_0}{h} \quad [\kappa N] \quad (40)$$

$$F_{24}^{cr} = \frac{N_{24}^{cr}}{6} = 0,633 \frac{i_0}{h} \quad [\kappa N] \quad (41)$$

$$F_{26}^{cr} = \frac{N_{26}^{cr}}{13} = 0,633 \frac{i_0}{h} \quad [\kappa N] \quad (42)$$

$$F_{mid}^{cr} = \frac{5 \cdot 0,633}{5} \cong 0,633 \frac{i_0}{h} \quad [\kappa N] \quad (43)$$

$$\varepsilon = \left| \frac{0,633 - 0,633}{0,633} \right| \cdot 100\% = 0\% \quad (44)$$

8. Determination of the coefficients of reducing the lengths of the compressed rods

$$\mu_{ab} = \frac{\pi}{t_{ab}^{cr}} \quad (45)$$

$$\mu_{12} = \frac{\pi}{t_{12}^{cr}} = \frac{\pi}{1,540727} = 2,039 > 0,7 \quad (46)$$

$$\mu_{13} = \frac{\pi}{t_{13}^{cr}} = \frac{\pi}{0,795628} = 3,949 > 0,7 \quad (47)$$

$$\mu_{15} = \frac{\pi}{t_{15}^{cr}} = \frac{\pi}{0,931744} = 3,372 > 0,7 \quad (48)$$

$$\mu_{24} = \frac{\pi}{t_{24}^{cr}} = \frac{\pi}{1,324005} = 2,373 > 2 \quad (49)$$

$$\mu_{26} = \frac{\pi}{t_{26}^{cr}} = \frac{\pi}{2,028461} = 1,549 > 0,7 \quad (50)$$

9. Estimated lengths of compressed rods

$$l_0^{ab} = \mu_{ab} \cdot l_{ab} \quad (51)$$

$$l_0^{12} = \mu_{12} \cdot 0,75h = 2,039 \cdot 0,75h = 1,529h \quad [m] \quad (52)$$

$$l_0^{13} = \mu_{13} \cdot 0,6h = 3,949 \cdot 0,6h = 2,369h \quad [m] \quad (53)$$

$$l_0^{15} = \mu_{15} \cdot 1,2h = 3,372 \cdot 1,2h = 4,046h \quad [m] \quad (54)$$

$$l_0^{24} = \mu_{24} \cdot 0,6h = 2,373 \cdot 0,6h = 1,424h \quad [m] \quad (55)$$

$$l_0^{26} = \mu_{26} \cdot 1,2h = 1,549 \cdot 1,2h = 1,859h \quad [m] \quad (56)$$

10. Forms of stability loss and construction of a deformed frame scheme

We accept $\delta / h = 1$. (57)

$$\begin{cases} r_{11} \cdot \varphi_1 + r_{12} \cdot \varphi_2 = R_1; \\ r_{21} \cdot \varphi_1 + r_{22} \cdot \varphi_2 = R_2. \end{cases} \quad (58)$$

$$\begin{cases} (4 + 2,4 \cdot \bar{\alpha}_{13} + 3,5 \cdot \bar{\alpha}_{15}) \cdot \varphi_1 = 8; \\ (\bar{\alpha}_{21} - 1,3 \cdot t_{24} \cdot tgt_{24} + 2,4 \cdot \bar{\alpha}_{26}) \cdot \varphi_2 = 1,333 \cdot \bar{\alpha}_{21}. \end{cases} \quad (59)$$

$$\begin{aligned} Z(1) &= \varphi_1 = 0,385224; \\ Z(2) &= \varphi_2 = -5,561034; \\ Z(3) &= \delta / h = 1,000000. \end{aligned} \quad (60)$$

Conclusions

The algorithm and program complex "Persist" for PC in Windows OS are developed. This computer program is implemented in a modern compiler, contains several subroutines (utilities), which are combined and presented in the form of the same software complex. It will enable students and engineers to automate calculations of the stability of the equilibrium form of compressed discrete systems, calculate critical loads (stresses) and determining the corresponding principal forms of oscillations. The program has been approbated and implemented in the educational process at the Poltava National Technical Yuri Kondratyuk University during the training of specialists in engineering specialties.

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