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Research paper

Normal Sections Calculation of Bending Reinforced Concrete and Fiber Concrete Elements

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Abstract

The basic principles of the normal sections calculation of reinforced concrete and fiber reinforced concrete bending elements are considered. In the article the power and deformation methods of calculation of reinforced concrete and fiber concrete elements of rectangular cross-section are presented. The deformation model of the calculation of reinforced concrete and fiber concrete elements is presented in the framework of the method of calculation resistance of the section. This method makes possible from the common methodological positions to perform calculations of reinforced concrete and fiber concrete elements. Namely, to select reinforcement and to determine the carring capacity. The proposed deformation model for calculating fiber concrete elements is based on generally accepted preconditions. A hypothesis of plane cross sections is accepted as fair. The deformation diagram of compressed concrete is described by a nonlinear function with established parametric points. Distribution of stresses in stretched concrete is taken rectangular with corresponding coefficients which are taken depending on the type of deformation diagram. Determination of the carring capacity of fiber concrete elements occurs under extreme deformation criteria. Two cases of destruction of the investigated elements are considered. The first case is the destruction due to the achievement of limiting deformations in the concrete of the compressed zone with the simultaneous achievement of the fluidity limit in the working reinforcement. The second case is the destruction due to the achievement of limiting deformations in the concrete of the compressed zone without reaching the fluidity limit in the working reinforcement. Both cases of calculation are reduced to one functional dependence. This avoids the delimitation of different calculation cases. The main no dimensional modifier is the mechanical coefficient of reinforcement. According to the developed method, examples of calculations of reinforced concrete, fiber reinforced concrete elements and fiber concrete elements with longitudinal reinforcement are executed. The possibility of a spread variant design of reinforced concrete and fiber concrete elements is shown.

Keywords: *reinforced concrete elements, fiber concrete elements, bending elements, beams, strength, stress, deformation model.*

1. Introduction

Fiber concrete elements and structures have a number of advantages over heavy concrete structures. It is well-known that concrete has strong compressive strength properties and is rather low under tensile. The use of fibers from various materials leads to an increase in its strength characteristics. According to various normative sources [1, 2, 3, 4], the increase in the strength of fiber concrete samples occurs up to 20% in compression compared with similar reinforced concrete ones and 2-7 times in stretching. In addition, the introduction of fiber in concrete significantly reduces the appearance of shrinkage cracks and significantly increases its dynamic characteristics. This makes it possible to use fiber concrete not only to create new building elements and structures but also to strengthen the existing ones.
At the same time, there are a number of issues related to the calculation of such elements. Existing power models of their calculation do not allow for the accurate account of their deformation characteristics. This prevents the effective design of such elements. Therefore, the improvement of the deformation model for the calculation of such elements is an urgent task.

2. Methods of Normal Sections Calculation of Reinforced Concrete and Fiber Concrete Elements

2.1. Reinforced Concrete Elements

It is generally accepted to consider the power model calculation of reinforced concrete elements [5, 6]. The power model is taken from the assumption of rectangular stress distribution in a compressed concrete zone. The replacement of the curvilinear circuit of stresses on a rectangular in the section of reinforced concrete elements occurs by introducing coefficients λ, η (Fig. 1). The resistance of concrete areas stretched is neglected.
Value of coefficients λ, η, that determine the actual height and strength of the compressed zone of concrete are determined by expressions:

$$\lambda = \begin{cases} 0{,}8 \; within \; f_{ck} \le 50 \; MPa; \\ 0{,}8 - \dfrac{f_{ck} - 50}{400} \; within \; 50 < f_{ck} \le 90 \; MPa; \end{cases} \quad (1)$$

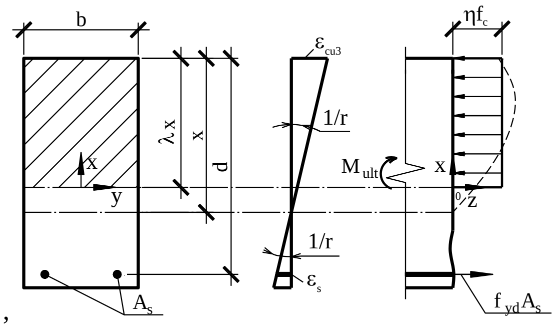

Fig. 1: The scheme of forces and the diagram of stress in the normal section to the longitudinal axis of the bending reinforced concrete element of the rectangular section within calculating its strength

$$\eta = \begin{cases} 1{,}0 \; within \; f_{ck} \le 50 \; MPa; \\ 1{,}0 - \dfrac{f_{ck} - 50}{200} \; within \; 50 < f_{ck} \le 90 \; MPa. \end{cases} \tag{2}$$

In the expressions (1), (2) f_{ck} – characteristic value of concrete strength on compression.

The carring capacity value of bending reinforced concrete elements of a rectangular section is determined by the expressions:

a) within $\xi = x / d \le \xi_R$

$$M_{ult} = \lambda\eta f_c bx(d - 0{,}5\lambda x), \tag{3}$$

б) within $\xi = x / d > \xi_R$

$$M_{ult} = \lambda\eta f_c bx_R (d - 0{,}5\lambda x_R). \tag{4}$$

In the formulas (3), (4): M_{ult} – the limiting point that can be taken by the section; f_{ck} – estimated value of concrete strength on compression; d, b – working section's height and width of the reinforced concrete element, respectively.

The height of the compressed zone of the reinforced concrete element is determined by the formula

$$x = \frac{1}{\lambda\eta} \frac{f_{yd} A_s}{f_c b}, \tag{5}$$

where f_{yd} – the calculated value of the reinforcement strength at the fluidity limit; A_s – the area of stretched reinforcement.

The boundary value of the height of the compressed zone is determined by the condition of the strain linearity in the section of the element

$$\xi_R = \frac{x_R}{d} = \frac{\varepsilon_{cu,3}}{\varepsilon_{cu,3} + \varepsilon_{s0}}, \tag{6}$$

where $\varepsilon_{s0} = \dfrac{f_{yd}}{E_s}$ – the estimated value of the boundary relative deformations of the reinforcement; $\varepsilon_{cu,3}$ – Limit values of compressed concrete under using a simplified deformation diagram.

The proposed expressions allow solving the following tasks: checking the carring capacity of the reinforced concrete element section for known reinforcement; determination of the carring capacity of the element section for known reinforcement; determination of the sectional area of the working reinforcement for the known values of the acting efforts.

2.2. Fiber Concrete Elements without Longitudinal Reinforcement

The power model for calculating the fiber concrete elements is also taken from the assumptions of rectangular stress distribution in compressed and stretched concrete zones (Fig. 2).

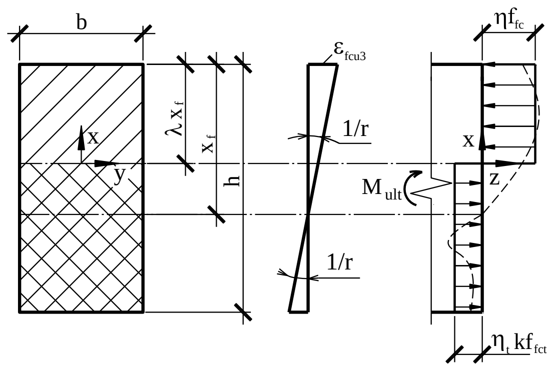

Fig. 2: The scheme of strength and stress in the section normal to the longitudinal axis of the bending fiber concrete element of the rectangular section within calculating its strength according to the power model

The carring capacity value of the bending fiber concrete elements of a rectangular section without longitudinal reinforcement is determined by the expression

$$M_{ult} = 0{,}5\eta f_{fc} b\lambda x_f h, \tag{6}$$

where x_f – the height of the compressed zone of fiber concrete element without longitudinal reinforcement is determined by the formula

$$x_f = \frac{\eta_t k f_{fct} h}{\eta_t k\lambda f_{fct} + \lambda\eta f_{fc}}. \tag{7}$$

$k = 0{,}56$; η_t – a coefficient of completeness of the stresses diagram in the stretched zone in the section of the fiber concrete element.

The coefficient η_t is taken depending on the ratio of the resistance of the fiber concrete to the tensile strength at bending, to the final strength of the fiber reinforced concrete on the tensile bending (Fig. 3)

$$\eta_t = \begin{cases} 0{,}75 \; within \; 0{,}5 \le f_{fct3} / f_{fct} < 0{,}7; \\ 0{,}85 \; within \; 0{,}7 \le f_{fct3} / f_{fct} < 0{,}9; \\ 0{,}95 \; within \; 0{,}9 \le f_{fct3} / f_{fct} < 1{,}1; \\ 1{,}05 \; within \; 1{,}1 \le f_{fct3} / f_{fct} < 1{,}3; \\ 1{,}15 \; within \; 1{,}3 \le f_{fct3} / f_{fct}. \end{cases} \tag{8}$$

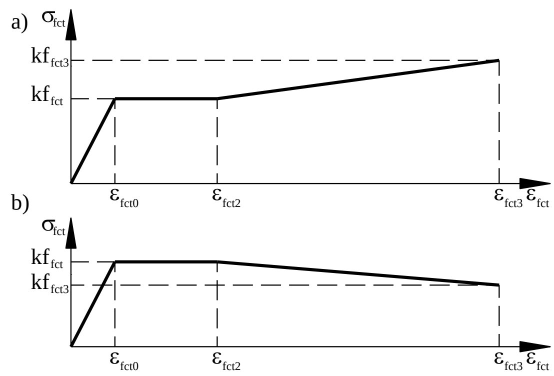

Fig. 3: Deformation diagrams of stretched fiber concrete

2.3. Fiber Concrete Elements with Longitudinal Reinforcement

The value of the carring capacity of bending fiber concrete elements of a rectangular section with a longitudinal reinforcement is determined by the expression (Fig. 4):

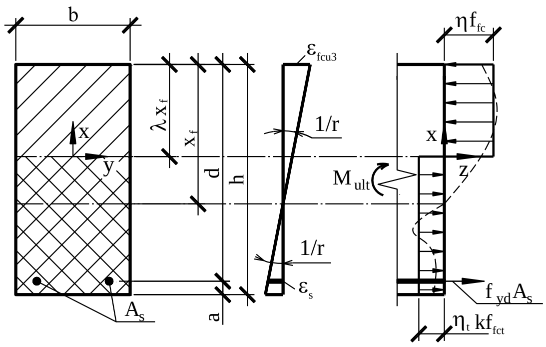

Fig. 4: The scheme of strength and stress in the section normal to the longitudinal axis of the bending fiber concrete element of the rectangular section with longitudinal reinforcement within calculating its strength according to the power model

a) within $\xi = x_f / d \le \xi_R$

$$M_{ult} = \eta f_{fc} b \lambda x_f \left(d - \frac{\lambda x_f}{2} \right) + \eta_t k f_{fc} b \left(h - \lambda x_f \right) \left(\frac{h - \lambda x_f}{2} - a \right). \quad (9)$$

where x_f – the height of the compressed concrete zone of a fiber concrete element with a longitudinal reinforcement is determined by the formula

$$x_f = \frac{f_{yd} A_s + \eta_t k f_{fct} bh}{\left(\eta_t k \lambda f_{fct} h + \eta \lambda f_{fc} \right) b}. \quad (10)$$

б) $\xi = x_R / d > \xi_R$

$$M_{ult} = \eta f_{fc} b x_R \left(d - \lambda x_R \right) + \eta_t k f_{fc} b \left(h - x_R \right) \left(\frac{h - x}{2} - a \right). \quad (11)$$

The boundary value of the height of the compressed zone is determined by the expression

$$\xi_R = \frac{\varpi \varepsilon_{fcu,3}}{\varepsilon_{fcu,3} + \varepsilon_{s0}}, \quad (12)$$

where $\varepsilon_{s0} = \frac{f_{yd}}{E_s}$ – the estimated value of the boundary relative deformations of reinforcement; $\varepsilon_{fcu,3}$ – boundary values of compressed fiber concrete using a simplified deformation diagram; ϖ – the characteristic of the compressed zone of fiber concrete which is accepted for fiber concrete of heavy concrete class up to B60 equal to 0,8, and for fiber concrete fibers from heavy concrete classes B70 - B100 equal to 0,7.

3. Normal Sections Calculation by the Method of Settlement Resistance of Reinforced Concrete and Fiber Concrete

3.1. Reinforced Concrete Elements

Considered in the previous section, power models include a lot of coefficients due to the use of rectangular stress distribution in compressed and stretched zones. The adoption of nonlinear deformation diagrams of concrete and the hypothesis of the linearity of longitudinal deformations in the element section allows losing this disadvantage. The sections of elements calculation with a nonlinear stresses distribution in a compressed zone involves the solution of nonlinear equations and the use of destruction criteria. In the general, destruction criterion is determined by such a system of equations:

$$\begin{cases} \frac{dM_d}{d\varepsilon} = 0 \ within \ \varepsilon_c \le \varepsilon_{cu}; \\ \sigma_s \le f_{yd}; \\ \varepsilon_s \le \varepsilon_{su}. \end{cases} \quad (13)$$

In the system (13): ε_{cu}, ε_{su} – boundary deformations of concrete and reinforcement, respectively.

For further researches for the concrete of a compressed zone, it can be accepted a function which is laid down in the norms of a reinforced concrete elements designing [7]

$$\frac{\sigma_c}{f_c} = \frac{k\eta - \eta^2}{1 + (k-2)\eta}, \quad (14)$$

where $\eta = \varepsilon_c / \varepsilon_{c1}$, $k = 1{,}05 E_c \varepsilon_{c1} / f_c$.

The expression (14) is written in the form of a functional dependence

$$\sigma_c = f_c \times f(k, \eta) = f_c \times f(f_c). \quad (15)$$

Putting equilibrium equation and conducting simple transformation with taking into account the hypothesis of flat sections, there will be:

- for not re-reinforced:

$$\frac{x}{d} = \frac{\rho_f f_{yd}}{f_{cd}} \frac{\varepsilon_c}{\int_0^{\varepsilon_c} f(f_c) d\varepsilon_c}; \quad (16)$$

$$\left(\frac{x}{d} \right)^2 \frac{\int_0^{\varepsilon_c} f(f_c) \varepsilon_c d\varepsilon_c}{\varepsilon_c^2} + \frac{f_{yd} A_s}{f_{cd}} \left(1 - \frac{x}{d} \right) = \frac{M_{Ed}}{f_{cd} bd^2}; \quad (17)$$

- for re-reinforced:

$$\frac{x}{d} = \frac{E_s \varepsilon_s}{f_{yd}} \frac{\rho_f f_{yd}}{f_{cd}} \frac{\varepsilon_c}{\int_0^{\varepsilon_c} f(f_c) d\varepsilon_c}; \quad (18)$$

$$\left(\frac{x}{d} \right)^2 \frac{\int_0^{\varepsilon_c} f(f_c) \varepsilon_c d\varepsilon_c}{\varepsilon_c^2} + \frac{E_s \varepsilon_s}{f_{yd}} \frac{f_{yd} A_s}{f_{cd}} \left(1 - \frac{x}{d} \right) = \frac{M_{Ed}}{f_{cd} bd^2}. \quad (19)$$

Taking into account notations

$$\omega = \frac{\rho_f f_{yd}}{f_{cd}}; \ \eta_s = \frac{E_s \varepsilon_s}{f_{yd}}. \quad (20)$$

equations (16), (17) and (18), (19) will take on this form:

- for not re-reinforced

$$\frac{x}{d}=\omega\frac{\varepsilon_c}{\int_0^{\varepsilon_c} f(f_c)d\varepsilon_c};\quad (21)$$

$$\left(\frac{x}{d}\right)^2\frac{\int_0^{\varepsilon_c} f(f_c)\varepsilon_c d\varepsilon_c}{\varepsilon_c^2}+\omega\left(1-\frac{x}{d}\right)=\frac{M_{Ed}}{f_{cd}bd^2};\quad (22)$$

- for re-reinforced

$$\frac{x}{d}=\eta_s\omega\frac{\varepsilon_c}{\int_0^{\varepsilon_c} f(f_c)d\varepsilon_c};\quad (23)$$

$$\left(\frac{x}{d}\right)^2\frac{\int_0^{\varepsilon_c} f(f_c)\varepsilon_c d\varepsilon_c}{\varepsilon_c^2}+\eta_s\omega\left(1-\frac{x}{d}\right)=\frac{M_{Ed}}{f_{cd}bd^2}.\quad (24)$$

Introduce notations to equations (22), (24)

$$k_z=f(f_c,\omega)=\frac{6M_{Ed}}{f_c bd^2}=\frac{M_{Ed}}{f_c W_c}.\quad (25)$$

Finally, get

- for not re-reinforced

$$k_z=6\left(\frac{x}{d}\right)^2\frac{\int_0^{\varepsilon_c} f(f_c)\varepsilon_c d\varepsilon_c}{\varepsilon_c^2}+6\omega\left(1-\frac{x}{d}\right);\quad (26)$$

- for re-reinforced

$$k_z=6\left(\frac{x}{d}\right)^2\frac{\int_0^{\varepsilon_c} f(f_c)\varepsilon_c d\varepsilon_c}{\varepsilon_c^2}+6\eta_s\omega\left(1-\frac{x}{d}\right).\quad (27)$$

An introduced parameter k_z in general depends on the mechanical coefficient of reinforcement ω and deformation characteristics of concrete. Let's represent the graph of the parameter dependence k_z in depends on the mechanical coefficient of reinforcement ω for different classes of concrete (Fig. 4).

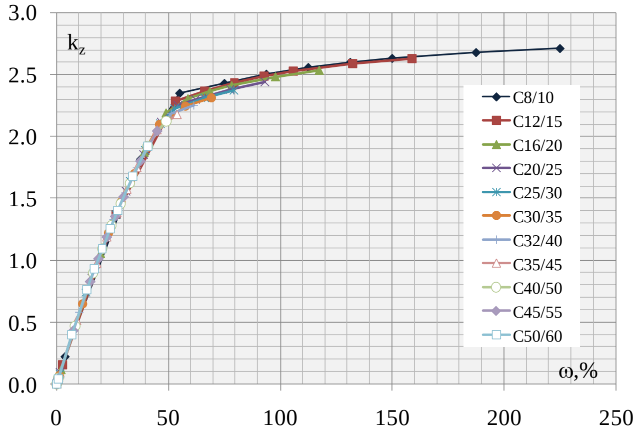

Fig. 4: – The parameter dependence k_z of concrete reinforcement on the mechanically reinforced coefficient with unit reinforcement

Influence of deformation properties of concrete (parameters $\eta=\varepsilon_c/\varepsilon_{c1}$, $k=1{,}05E_c\varepsilon_{c1}/f_c$) on the carring capacity of bending reinforced concrete elements for the considered classes of concrete varies with a difference of 10%. Such an error is completely absorbed by the normative variation coefficients of the strength of such elements. Then it can be accepted a single functional dependence $k_z=f(\omega)$ for all classes of concrete, and the estimated resistance of reinforced concrete can be determined by the formula

$$f_{zM}=k_z f_c.\quad (28)$$

Finally, the condition of the normal sections strength of bending reinforced concrete elements will be:

$$M_{Ed}\le W_c f_{zM},\quad (29)$$

where f_{zM} – the estimated resistance of the reinforced concrete to the bend, which depends on the classes of concrete and reinforcement, the shape of the section, and the percentage of section reinforcement is determined by the expression (28). The parameter k_z in expression (28) is calculated depending on the mechanical reinforcement coefficient ω by Table 1.
The expression (29) allows solving a number of tasks in determining the reinforcement and strength.
A more detailed derivation of the basic formulas of this method is shown in [10].

Table 1: Functional dependence of parameters $k_z=f(\omega)$

No.	Parameters for calculating bending elements	
	k_z	Mechanical reinforcement coefficient ω
1	0,000	0,00
2	0,568	0,10
3	0,828	0,15
4	1,071	0,20
5	1,299	0,25
6	1,511	0,30
7	1,706	0,35
8	1,885	0,40
9	2,028	0,45
10	2,070	0,50
11	2,140	0,60
12	2,195	0,70
13	2,310	1,00
14	2,476	2,00
15	2,542	3,00

3.2. Fiber Concrete Elements without Longitudinal Reinforcement

The deformation model of the normal sections calculation of fiber concrete elements also involves the use of nonlinear diagrams of deformation of concrete. The deformation diagram of a fiber concrete during compression is taken as (14) or in the form of two and three linear functions. The general view of the deformation diagram of fibro concrete during stretching depends on many factors: the kind, type and number of fibers, the type of fillers and binder, and so on. Such diagrams can be of two types (Fig. 3). The type of the diagram is established by experimental testing of reinforced concrete beams with a cut to the bend according to the method described in the standards [8, 9]. Because the zone is stretched concrete that has a rather high variability of expediency in the description of the nonlinear functions diagram it disappears. Therefore, a valid deformation diagram of the stretched concrete zone will be replaced for a straight line with the corresponding coefficients k and η_t (Fig. 5).

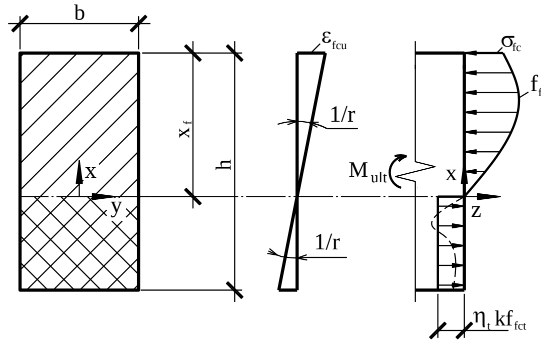

Fig. 5: The scheme of strength and stress in the section normal to the longitudinal axis of the bending fiber concrete element of a rectangular section in the calculation of its strength according to the proposed strain model

In this case, the equation of the static equilibrium of the section of the fiber concrete element, taking into account the notation

$$\omega = \frac{f_{fct}k\eta_t}{f_{fc} + f_{fct}k\eta_t}. \tag{30}$$

will be:

$$\frac{x_f}{h} = \omega \frac{\varepsilon_c}{\int_0^{\varepsilon_c} f(f_{fc})d\varepsilon_c}; \tag{31}$$

$$\left(\frac{x_f}{h}\right)^2 \frac{\int_0^{\varepsilon_c} f(f_{fc})\varepsilon_c d\varepsilon_c}{\varepsilon_c^2} + \omega\left(1 - \frac{x_f}{h}\right) - \\ - \frac{f_{fct}k\eta_t}{2(f_{fc} + f_{fct}k\eta_t)} = \frac{M_{Ed}}{(f_{fc} + f_{fct}k\eta_t)bh^2}. \tag{32}$$

Equations (31) and (32) obtained by considering the following assumptions

$$\frac{f_{fct}k\eta_t\varepsilon_c}{\int_0^{\varepsilon_c} f(f_{fc})d\varepsilon_c} \approx f_{fct}k\eta_t, \quad \frac{f_{fct}k\eta_t\varepsilon_c^2}{2\int_0^{\varepsilon_c} f(f_{fc})\varepsilon_c d\varepsilon_c} \approx f_{fct}k\eta_t. \tag{33}$$

Introduce the notation to the equation (32)

$$k_z = f(f_{fc}, \omega) = \left(\frac{x_f}{h}\right)^2 \frac{\int_0^{\varepsilon_c} f(f_{fc})\varepsilon_c d\varepsilon_c}{\varepsilon_c^2} + \omega\left(1 - \frac{x_f}{h}\right). \tag{34}$$

Finally, the expression (32) can be represented as follow

$$k_z - 3\omega = \frac{6M_{Ed}}{(f_{fc} + f_{fct}k\eta_t)bd^2} = \frac{M_{Ed}}{(f_{fc} + f_{fct}k\eta_t)W_c}. \tag{35}$$

The value of the estimated resistance of the fiber concrete during bending can be determined by the formula

$$f_{fzM} = (k_z - 3\omega)(f_{fc} + f_{fct}k\eta_t). \tag{36}$$

The strength condition of the normal sections of the bending fiber concrete elements will take this form:

$$M_{Ed} \le W_c f_{fzM}. \tag{37}$$

The parameter k_z in the expression (37) is calculated depending on the mechanical reinforcement coefficient ω and also by Table 1.

3.3. Fiber Concrete Elements with Longitudinal Reinforcement

For fiber concrete elements with longitudinal reinforcement, the distribution of stresses in the compressed zone in the form of non-linear dependence is taken (14). In a stretched zone, as in the previous case, the distribution of stresses equal coefficients k and η_t is accepted (Fig. 6).

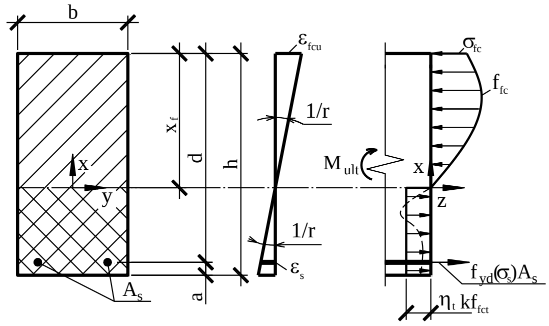

Fig. 4: The scheme of strength and stress in the section normal to the longitudinal axis of the bending fiber concrete element of a rectangular section with longitudinal reinforcement in the calculation of its strength according to the proposed strain model

Introduce the notation

$$\omega = \frac{\rho_f f_{yd} + f_{fct}k\eta_t \frac{h}{d}}{f_{fc} + f_{fct}k\eta_t}, \quad \eta_s = \frac{E_s\varepsilon_s}{f_{yd}}. \tag{38}$$

Equations of static equilibrium will take this form:
- for not re-reinforced:

$$\frac{x_f}{d} = \omega \frac{\varepsilon_c}{\int_0^{\varepsilon_c} f(f_{fc})d\varepsilon_c}; \tag{39}$$

$$\left(\frac{x_f}{d}\right)^2 \frac{\int_0^{\varepsilon_c} f(f_{fc})\varepsilon_c d\varepsilon_c}{\varepsilon_c^2} + \omega\left(1 - \frac{x_f}{d}\right) - \\ - \frac{f_{fct}k\eta_t}{2(f_{fc} + f_{fct}k\eta_t)} = \frac{M_{Ed}}{(f_{fc} + f_{fct}k\eta_t)bd^2}; \tag{40}$$

- for re-reinforced

$$\frac{x_f}{d} = \omega\eta_s \frac{\varepsilon_c}{\int_0^{\varepsilon_c} f(f_{fc})d\varepsilon_c}; \tag{41}$$

$$\left(\frac{x_f}{d}\right)^2 \frac{\int_0^{\varepsilon_c} f(f_{fc})\varepsilon_c d\varepsilon_c}{\varepsilon_c^2} + \omega\eta_s\left(1 - \frac{x_f}{d}\right) - \\ - \frac{f_{fct}k\eta_t}{2(f_{fc} + f_{fct}k\eta_t)} = \frac{M_{Ed}}{(f_{fc} + f_{fct}k\eta_t)bd^2}. \tag{42}$$

Equations (39) - (42) are obtained taking into account the assump tions (33) and

$$\frac{\rho_f E_s\varepsilon_s + f_{fct}k\eta_t}{\rho_f f_{yd} + f_{fct}k\eta_t} \approx \frac{E_s\varepsilon_s}{f_{yd}} = \eta_s. \tag{43}$$

Introduce the notation to the equations (40), (42)
- for not re-reinforced:

$$k_z = 6\left(\frac{x_f}{d}\right)^2 \frac{\int_0^{\varepsilon_c} f(f_{fc})\varepsilon_c d\varepsilon_c}{\varepsilon_c^2} + 6\omega\left(1 - \frac{x_f}{d}\right); \quad (44)$$

- for re-reinforced

$$k_z = 6\left(\frac{x_f}{d}\right)^2 \frac{\int_0^{\varepsilon_c} f(f_{fc})\varepsilon_c d\varepsilon_c}{\varepsilon_c^2} + 6\eta_s\omega\left(1 - \frac{x_f}{d}\right). \quad (45)$$

Finally, the expressions (40) and (42) can be represented as follow

$$k_z - 3\frac{f_{fct}k\eta_t}{f_{fc} + f_{fct}k\eta_t} = \frac{6M_{Ed}}{(f_{fc} + f_{fct}k\eta_t)bd^2}. \quad (46)$$

The value of the estimated resistance of the fiber concrete with longitudinal reinforcement under bending can be determined by the formula

$$f_{fzM} = \left(k_z - 3\frac{f_{fct}k\eta_t}{f_{fc} + f_{fct}k\eta_t}\right)(f_{fc} + f_{fct}k\eta_t). \quad (47)$$

The condition of the normal sections strength of bending fiber concrete elements with longitudinal reinforcement is expressed by the formula (37).

4. Examples of Reinforced Concrete and Fibro Concrete Elements Calculation

Let's illustrate in the examples the advantages of using the proposed calculation methods.

Example 1. Determine the carring capacity of beams that have a cross-section 20×30 cm and made from:
a) heavy concrete C25/30 (f_c=17 MPa), reinforced in a lower area 3∅18 A400C (f_{yd}=415 MPa, A_s=7,63 cm^2), working high of section d=26,0 cm;
б) fiber concrete C25/30 (f_c=17 MPa), characteristics of fiber concrete on tension are next f_{ct}=3,5 MPa, f_{ct3}=3,1 MPa;
в) fiber concrete C25/30 (f_c=17 MPa), characteristics of fiber concrete on tension are next f_{ct}=3,5 MPa, f_{ct3}=3,1 MPa, reinforced in a lower area 3∅18 A400C (f_{yd}=415 MPa, A_s=7,63 cm^2), working high of section d=26,0 cm;

Solution.
1. Determine the mechanical reinforcement coefficient
a) for reinforced concrete elements

$$\omega = \frac{\rho_f f_{yd}}{f_{cd}} = \frac{7{,}63 \times 415}{20 \times 26 \times 17} = 0{,}358;$$

b) for fiber concrete elements without reinforcement

$$\omega = \frac{f_{fct}k\eta_t}{f_{fc} + f_{fct}k\eta_t} = \frac{3{,}5 \times 0{,}56 \times 0{,}85}{17 + 3{,}5 \times 0{,}56 \times 0{,}85} = 0{,}089;$$

c) for fiber concrete elements with longitudinal reinforcement

$$\omega = \frac{\rho_f f_{yd} + f_{fct}k\eta_t \frac{h}{d}}{f_{fc} + f_{fct}k\eta_t} = \frac{0{,}0147 \times 415 + 1{,}667 \times \frac{30}{26}}{17 + 1{,}667} = 0{,}430.$$

According to Table 1 determine the value k_z:
a) for reinforced concrete elements $k_z = 1{,}735$;
b) for fiber concrete elements without reinforcement $k_z = 0{,}505$;
c) for fiber concrete elements with longitudinal reinforcement $k_z = 1{,}97$.
Determine the estimated resistance of the section
a) for reinforced concrete elements

$$f_z = k_z f_c = 1{,}735 \times 17 = 29{,}50\ \text{MPa};$$

b) for fiber concrete elements without reinforcement

$$f_z = (k_z - 3\omega)(f_{fc} + f_{fct}k\eta_t) = (0{,}505 - 3 \times 0{,}089) \times 18{,}67 = 4{,}44\ \text{MPa};$$

c) for fiber concrete elements with longitudinal reinforcement

$$f_z = \left(k_z - 3\frac{f_{fct}k\eta_t}{f_{fc} + f_{fct}k\eta_t}\right)(f_{fc} + f_{fct}k\eta_t) = (1{,}97 - 3 \times 0{,}089) \times 18{,}67 = 31{,}80\ \text{MPa}.$$

Carrying capacity of beams sections is equal:
a) for reinforced concrete elements

$$M_{Ed} = \frac{bd^2}{6} f_z = \frac{20 \times 26^2}{6} 29{,}50 \times 10^{-3} = 66{,}47\ \text{kHm};$$

b) for fiber concrete elements without reinforcement

$$M_{Ed} = \frac{bh^2}{6} f_z = \frac{20 \times 30^2}{6} 4{,}44 \times 10^{-3} = 13{,}32\ \text{kHm};$$

c) for fiber concrete elements with longitudinal reinforcement

$$M_{Ed} = \frac{bd^2}{6} f_z = \frac{20 \times 26^2}{6} 31{,}80 \times 10^{-3} = 71{,}65\ \text{kHm}.$$

Example 2. Determine the necessary reinforcement of the beams section for the perception of the bending moment M_{Ed}=80 kHm, that have cross-section 30×30 cm and made from:
a) heavy concrete C25/30 (f_c=17 MPa), reinforced in a lower area by reinforcement of class A400C (f_{yd}=415 MPa), working high of section d=26,0 cm;
b) fiber concrete C25/30 (f_c=17 MPa), characteristics of fiber concrete on tension are next f_{ct}=3,5 MPa, f_{ct3}=3,1 MPa, reinforced in a lower area by reinforcement of class A400C (f_{yd}=415 MPa, working high of section d=26,0 cm.

Solution.
Determine the required estimated resistance section

$$f_z = \frac{6M_{Ed}}{bd^2} = \frac{6 \times 80}{30 \times 26^2} \times 10^3 = 23{,}67\ \text{MPa}.$$

Set the parameter k_z:
a) for reinforced concrete elements $k_z = \frac{f_z}{f_c} = \frac{23{,}67}{17} = 1{,}39$;
b) for fiber concrete elements with longitudinal reinforcement

$$k_z = \frac{f_z}{(f_{fc} + f_{fct}k\eta_t)} + 3\frac{f_{fct}k\eta_t}{f_{fc} + f_{fct}k\eta_t} = \frac{23{,}67}{18{,}667} + 3 \times 0{,}089 = 1{,}535.$$

According to Table 1 determine the mechanical reinforcement coefficient
a) for reinforced concrete elements $\omega = 0{,}272$;
b) for fiber concrete elements with longitudinal reinforcement $\omega = 0{,}306$.
Set the required percentage of reinforcement:
a) for reinforced concrete elements

$$\rho_f = \frac{\omega f_{cd}}{f_{yd}} = \frac{0{,}272 \times 17}{415} = 0{,}0111;$$

b) for fiber concrete elements with longitudinal reinforcement

$$\rho_f = \frac{\omega(f_{fc} + f_{fct}k\eta_t) - f_{fct}k\eta_t \frac{h}{d}}{f_{yd}} = \frac{0{,}653 \times 18{,}667 - 1{,}92}{415} = 0{,}0091.$$

The final section area:
a) for reinforced concrete elements

$$A_s = \rho_f \times b \times d = 0{,}0111 \times 30 \times 26 = 8{,}658\text{cm}^2;$$

b) for fiber concrete elements with longitudinal reinforcement

$$A_s = \rho_f \times b \times d = 0{,}0091 \times 30 \times 26 = 7{,}098\text{cm}^2.$$

5. Conclusion

The calculation of reinforced concrete and fiber concrete elements from the unified methodological positions by the method of calculation resistance of reinforced concrete and fiber concrete is proposed. The method takes into account the features of the sections of bending reinforced concrete and fiber concrete elements under the load. As a result of the theoretical studies, a generalized model for calculating reinforced concrete and fibro concrete elements is successfully developed. In general, the condition of the normal sections strength of reinforced concrete and fiber concrete elements has the form

$$M_{Ed} \le W_c f_z, \quad (48)$$

where f_z – estimated resistance section:

- for reinforced concrete elements

$$f_z = k_z f_c; \quad (49)$$

- for fiber concrete elements

$$f_z = \left(k_z - 3\frac{f_{fct} k \eta_t}{f_{fc} + f_{fct} k \eta_t} \right)\left(f_{fc} + f_{fct} k \eta_t \right).$$

The value of the parameter k_z is accepted depending on the mechanical reinforcement coefficient ω and by Table 1. The mechanical reinforcement coefficient ω is determined by the formulas:

- for reinforced concrete elements

$$\omega = \frac{\rho_f f_{yd}}{f_{cd}};$$

- for fiber concrete elements without longitudinal reinforcement

$$\omega = \frac{f_{fct} k \eta_t}{f_{fc} + f_{fct} k \eta_t};$$

- for fiber concrete elements with longitudinal reinforcement

$$\omega = \frac{\rho_f f_{yd} + f_{fct} k \eta_t \frac{h}{d}}{f_{fc} + f_{fct} k \eta_t}.$$

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