

# A Comprehensive Study of Vehicle Routing Problem With Time Windows Using Ant Colony Optimization Techniques

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## Abstract

Ant Colony Optimization (ACO) is a nature-inspired swarm intelligence technique and a metaheuristic approach which is inspired by the foraging behavior of the real ants, where ants release pheromones to find the best and shortest route from their nest to the food source. ACO is being applied to various optimization problems till date and has been giving good quality results in the field. One such popular problem is known as Vehicle Routing Problem (VRP). Among many variants of VRP, this paper presents a comprehensive survey on VRP with Time Window constraints (VRPTW). The survey is presented in a chronological order discussing which of the variants of ACO is used in each paper followed by the advantages and limitations of the same.

**Keywords:** Metaheuristics, Combinatorial Optimization, Ant Colony Optimization, Vehicle Routing Problem.

## 1. Introduction

ACO is first proposed in the early 1990's by M. Dorigo [1] [2]. Ants while going to find the food source from their nest choose different paths initially and deposit a chemical substance known as pheromone on their way. The quantity of the pheromones deposited by the previous ants influences the other ants and act as an indicator regarding the goodness of the path that was chosen by the former ants. In this way, the ants indirectly establish a form of communication among themselves which eventually ends up in the discovery of the optimum route from the nest to the food source [3]. This natural phenomenon is mathematically modeled to develop efficient algorithms in the field of optimization to solve problems which are NP-hard in nature.

A high-level abstraction of the ACO technique is enumerated below.

Step 1: The pheromone levels are initialized at first.

Step 2: The following steps are performed until the termination condition is reached:

- Construct Ant Solutions – A set of ant construct solutions with each ant starting with an empty solution and extending it one component at a time.
- Apply Local Search – This is an optional step which is used to further improve the results of the complete candidate solutions.
- Update pheromones – This involves pheromone deposit and pheromone evaporation that facilitate search exploration and exploitation.

Step 3: Stop

Now in a real-life scenario, in the applications of ACO, ants are treated as artificial ants which are used to optimize the cost function

of a given optimization problem. In the domain of hard optimization problems, ACO is found to give promising results.

VRP is a combinatorial optimization problem that requires the construction of optimal routes for a set of vehicles to serve the customers located at various places in a geographical sector. VRP is first introduced by G. B. Dantzig and J. H. Ramser [4]. It is studied in-depth in the literature and many novel heuristics and metaheuristics are developed. VRP finds great utility in the discipline of Supply Chain Management System.

This paper introduces ACO variants i.e. the various types of ACO algorithms which are developed over the years and then discusses in brief about the VRP variants as well. Then, it goes on to elaborately put forward the research that is reported in the literature pertaining to VRPTW using the ACO techniques.

## 2. Major Aco Variants

There are various types of ACO algorithms which have been designed to meet the various optimization needs as well as to improve upon the previously reported results. The paper presents here three major ACO variants.

### 2.1. Ant System (AS)

The first ACO algorithm reported in the literature is the Ant System which was proposed by Dorigo [5].

It requires the update of the pheromone levels by all the ants who have completed a tour of their food-finding process. It was used to solve the instances of TSP [6].

The pheromone update rule is as follows.

$$\tau_{ij} (1-\rho) + \sum_{k=1}^m \Delta \tau_{ij}^k \quad (1)$$

where  $m$  is the number of ants,  $\tau_{ij}$  is the pheromone associated with the edges connecting nodes  $i$  and  $j$ ,  $\rho \in (0,1]$  is the rate of evaporation and  $\Delta\tau_{ij}^k$  is the amount of pheromone released on edge  $(i, j)$  by the  $k^{\text{th}}$  ant.

$$\Delta\tau_{ij}^k = \begin{cases} \frac{1}{L_k} & \text{if ant } k \text{ used edge } (i, j) \text{ in its tour} \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

where  $L_k$  is length of  $k^{\text{th}}$  ant.

## 2.2. Ant Colony System(ACS)

The ACS algorithm is rather a popular variant of ACO which is developed to improve the efficiency of the AS [7]. In this method, a pseudorandom proportional rule is used to determine the probability of an ant to travel from city  $i$  to city  $j$  based on a random variable  $q$  (which is uniformly distributed over  $[0, 1]$ ) and a parameter  $q_0$ . If the value of  $q$  is less than that of  $q_0$ , then, among the feasible components, the component that maximizes the product of  $\tau_{ij}$  and  $\eta_{ij}$  is chosen, otherwise, the same equation as applied in Ant System is used. This rule favors exploitation of pheromone information. Other differences between ACS and AS are that ACS allows local pheromone update for exploring more search spaces and only the best ant is chosen for offline pheromone update.

## 2.3. Max-Min Ant System(MMAS)

The MMAS [8] is introduced by Stützle and Hoos which is based on AS. MMAS and AS differ in the following two aspects i.e., the pheromone trails are added only by the best ant and the maximum and minimum values are limited explicitly whereas in AS and ACS, these values are limited implicitly. The pheromone update rule is given as follows.

$$\tau_{ij} \leftarrow (1-\rho) \tau_{ij} + \sum_{k=1}^m \tau_{ij}^k \quad (3)$$

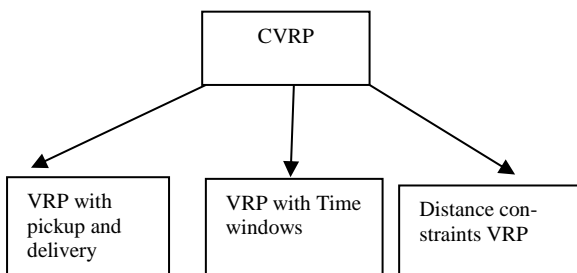
where  $\Delta\tau_{ij}^{\text{best}} = 1/L_{\text{best}}$  when the edge  $(i, j)$  is used by the best ant in its tour, otherwise  $\Delta\tau_{ij}^{\text{best}} = 0$ .  $L_{\text{best}}$  is the length of the tour of the best ant.

## 3. VRP Variants

VRP is an NP-hard optimization problem that deals with transportation of goods between the depot and a set of customers within a given set of constraints by a fleet of vehicles [9]. Some of the popular variants of this problem are discussed below

### 3.1. Capacitated Vehicle Routing Problem (CVRP)

CVRP the most common variant of VRP in which a fixed set of vehicles of predefined uniform capacity, is used to distribute goods to the customers from a single depot travelling an optimum distance within an optimum time period. CVRP is mainly categorized as follows.



In the next section, we will discuss the Vehicle Routing Problem with Time Windows in details.

### 3.2. Vehicle Routing Problem With Time Windows (VRPTW)

VRPTW is one of the extensively studied variants of CVRP in the literature [10]. In VRPTW, the other objective that comes into consideration along with minimization of the total travel time is the minimization of the number of vehicles servicing the customers. Hence, it is a bi-objective optimization problem which can be extended to a multi-objective problem by incorporating other real-life parameters. In this scenario, there is an associated time window with each customer being serviced. The vehicles are only permitted to service the customers within that period of time for that particular customer. In other words, vehicles arriving too early for the service have to wait or vehicles arriving after the closure of the time window also cannot render the service. VRPTW finds real-life applications in supermarket delivery, customer delivery, and many more.

The constraints for solving the VRPTW are as follows.

1. Mainly all the customers requesting service must be visited.
2. Every vehicle should begin and end at the same depot (Not necessarily in case of Open VRP).
3. Every vehicle has to arrive within the stipulated time i.e., it should not come too early or it should not be late.

The mathematical model for VRPTW is presented as follows [15]:

Inputs:

$b_j^k$ : The time at which the vehicle  $k$  starts its service at node  $j$ ,  $b_j^k = \max\{e_j, b_i^k + t_{ij}\}$

$c_{ij}$ : Travel distance between node  $i$  and node  $j$

$d_i$ : The demand of the customer at node  $i$ ,  $i \in N$

$e_i$ : The earliest start time of the vehicle for node  $i$

$l_i$ : The latest start time of the vehicle for node  $i$

$N$ : The total number of service nodes

$Q$ : The capacity of Vehicle

$V$ : The fleet of vehicles

$t_{ij}$ : the travel time of edge  $(i,j)$  and which also includes the service time at node  $i$

Decision variable:

$$x_{ijk} = \begin{cases} 1, & \text{if vehicle } k \text{ goes from node } i \text{ to } j \\ 0, & \text{otherwise} \end{cases}$$

Model: Equations.

$$\text{Min } \sum_{k \in V} \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ij}^k \quad (4)$$

$$\text{such that } \sum_{k \in V} \sum_{i \in N} x_{ij}^k = 1, \forall j \in N \setminus \{0\} \quad (5)$$

$$\sum_{k \in V} \sum_{i \in N} x_{ij}^k = 1, \forall i \in N \setminus \{0\} \quad (6)$$

$$\sum_{i \in N} d_i \sum_{j \in N} x_{ij}^k \leq Q, \forall k \in V \quad (7)$$

$$\sum_{i \in N} x_{ih}^k - \sum_{j \in N} x_{hj}^k = 0, \forall h \in N \setminus \{0\}, \forall k \in V \quad (8)$$

$$\sum_{j \in N \setminus \{0\}} x_{0j}^k = 1, \forall k \in V \quad (9)$$

$$\sum_{i \in N \setminus \{0\}} x_{i0}^k = 1, \forall k \in V \quad (10)$$

$$x_{ij}^k (b_i^k + t_{ij}) \leq b_j^k, \forall i \in N, \forall j \in N, \forall k \in V \quad (11)$$

$$e_i \leq b_i^k \leq l_i, \forall i \in N, \forall k \in V \quad (12)$$

$$x_{ij}^k \in \{0,1\}, \forall i \in N, \forall j \in N, \forall k \in V \quad (13)$$

Equation (4) minimizes the total distance of travel. Each node being serviced only once by one vehicle is described by equations (5) and (6). Equation (7) models that the total demand for a particular route of a vehicle should not exceed its capacity. Equation (8) states that the vehicle which has entered the node must leave the same.

Equations (9) and (10) denote that each vehicle must return to the depot after servicing the customers. (11) expresses that vehicle  $k$  cannot arrive at  $j$  before  $b_j^k + t_{ij}$ , if it has to travel from node  $i$  to node  $j$ . (12) states that start of every vehicle must be within the time windows. (13) is integrality constraint.

#### 4. Solving VRPTW Using ACO

This section discusses the various ACO algorithms applied on VRPTW to date.

Gambardella et al. [11] were the first to introduce the concept of VRP with time windows by designing a novel algorithm namely, MACS-VRPTW. It was designed to optimize multiple objective functions. At first, it minimized the number of vehicles and subsequently, it also minimized the total travel distance for a set of vehicles. The algorithm was implemented on Solomon's benchmark problems [12]. The quality of the solution was found to be promising among the other techniques available during that period. The limitation of the method was that it used two ant colonies to optimize the two objective functions.

Baran et al. [13] improved upon the results of Gambardella et al. [11] by considering only one ant colony for generating a set of Pareto optimal solutions to optimize the three objective functions, viz., the number of vehicles, total travel time and total delivery time. The algorithm found promising utility in different multi-objective problems. The work could be further extended to develop a parallel version of the algorithm for a network of personal computers, using PVM (Parallel Virtual Machine) to solve bigger problems within reasonable time.

Tan et al. [14] used the algorithm developed by Gambardella et al. [11] and applied ACS to solve VRPTW. The paper described the pseudocode in a lucid way and applied the algorithm for three groups of benchmark problems, i.e. for clustered customers, randomly placed customers and a combination of both.

Chen et al. [15] developed a hybrid algorithm namely, IACS-SA that used improved ACS technique coupled with simulated annealing approach (which has been applied in many NP-hard problems). The algorithm had been tested on 56 Solomon benchmark problems and gave good quality results which were competitive with other methods during that time.

Zhang et al. [16] introduced a mathematical model for VRPTW and Reused vehicles. A new algorithm viz., ACO-VRPTWRV was presented by incorporating the good aspects of ACS, MMAS and ASRank (Rank-based Ant System) by optimizing the three objectives namely, vehicle travel time, customer time window and service time. The algorithm was tested on modified Solomon C2 instances. The performance of the algorithm was proved to be quite satisfactory and it could efficiently avoid stagnation at a local optimum.

Tan et al. [17] used sequential insertion heuristic method in ACS and Nearest Neighborhood Search to generate the initial solution. It also used two ant colonies for achieving multiple objective optimizations. The results had shown that ACS was competitive for solving VRPTW with respect to other methods.

Favaretto et al. [18] introduced Ant Colony System for vehicle routing problem with multiple time windows, taking into account periodic constraints. The design of the algorithm was similar to that of MACS-VRPTW. The performance was satisfactory, however, there were possibilities of getting trapped in local extremum. By using hybridization of the algorithm with efficient local search procedure, it could have produced better results.

Gong et al. [19] used two-generation (children and father) ant colony algorithm. The children generation was used in the construction

of sub tours and in the father generation, the sub tours were composed of feasible solutions. These feasible routes were constructed using MMAS. The limitation of the proposed approach was that it was unable to solve problems with larger city instances.

Donati et al. [20] presented an algorithm for Time Dependent Vehicle Routing Problem(TDVRPTW). It took the motivation from MACS-VRPTW algorithm where the objective functions were the minimization of the number of tours and total travel time. The algorithm also introduced time-dependent local search procedures to improve the efficiency. In MACS-VRPTW, the depot could be replicated depending on the number of tours, whereas in TDVRPTW only one depot existed. The model was useful for scenario having variable traffic conditions. It was also integrated with a robust shortest-path algorithm for computation of time-dependent paths between each customer pair of the time-dependent model.

Zhen et al. [21] proposed two novel techniques namely, dynamic sweep algorithm and multiple ant colonies techniques which led to the development of hybrid Ant Colony system(DSACA-VRPTW). The dynamic sweep algorithm improved each ant's solution by grouping the customers and the improved ant colonies improved the optimization search speed. The algorithm was finally tested on Solomon's benchmark instances.

Oliveira et al. [22] proposed an architecture for solving Dynamic Vehicle Problem with Time Windows(DVRPTW). Here, the demands of the customers were dynamic in nature and could take place anytime within the time frame. There were two main components in the architecture viz., the Event Manager element and the ACS element. The results obtained were promising, however, they were not compared with other results due to lack of normative datasets.

Qi et al. [23] proposed a hybrid ant colony system (RPACS-VRPTW) along with randomized algorithm and Pareto local search procedures. The algorithm was instrumental in reducing the time complexity of the construction procedure of the ACS. The algorithm was tested on partial benchmark problems and it was found to outperform many contemporary techniques.

Belmecheri et al. [24] modeled a novel Ant Colony Optimization technique(HVRPMBTW) for Heterogeneous fleet VRP with mixed linehauls and backhauls having time window constraints. The results were compared with the results reported by Rieck et al. [25] with Multi-start heuristic (MS) using savings heuristic approach. It was found that in many cases ACO obtained feasible solutions where MS failed to obtain the same. The limitation of the proposed technique was its application for smaller problem instances. As a future extension, it could be coupled with other local search techniques to improve the efficiency.

Guiyun et al. [26] proposed an improved ACO algorithm for Open VRPTW (OVRPTW). Along with that, it also discussed an integer programming mathematical model to solve OVRPTW. In Open VRP, the vehicle need not return to the depot after servicing the last customer. The improved algorithm could overcome the limitations of premature and slow convergence of the conventional ACO technique.

Ortega et al. [27] introduced VRPTW with scheduled loading(VRPTWSL). The mathematical formulation was based on the model developed by Toth et al. [28], where every vehicle would not depart from the zero<sup>th</sup>-time instant from the depot. The algorithm was based on multi-ant colony system in which the number of vehicles and tour length were minimized. A time update procedure was added and constraint programming was used for determining feasible moves to a new customer node. As an improvement to the algorithm, a local search could be incorporated into the time update procedure.

Carabetti et al. [29] had put forward a technique for solving VRPTW with pickup and delivery (VRPPDTW). The proposed technique was divided into two phases i.e. the construction phase and the refinement phase. In the construction phase, the elitist strategy of ACO was applied for generating the initial solution. These solutions were then refined using the local search procedure with First Improvement Descent Method and three neighborhood structures. The algorithm was tested for its efficiency using standard datasets.

Ma et al. [30] presented an improved algorithm to solve VRPTW. The new technique not only accelerated the convergence speed by quickly solving the initial solution but also employed pheromone adjustment strategies for preventing the solution to get trapped at a local optimum. A path construction strategy was employed and new exchange law was introduced for speeding up the overall search procedure. The results obtained were quite satisfactory.

Balseiro et al. [31] proposed an ACS coupled with Insertion heuristic for time-dependent VRPTW, i.e. TDVRPTW. The new approach overcame the limitations of conventional Ant Colony algorithms of producing infeasible solutions during the final stages. The novelty was introduced by the inclusion of local search procedure with a minimum delay metric. The results confirmed the benefits of using the insertion heuristics.

Yu et al. [32] introduced an improved ACO (IACO) to solve periodic VRPTW, where the planning period could be extended to several days from a single day and each customer should be served within the time windows. In that paper, a multi-dimensional pheromone information and two-crossover operations were proposed. The multi-dimensional pheromone information was the most suitable extension and the two-crossover further improved the quality of the solution. However, the limitation of this approach was in its greater time complexity due to the introduction of the two-crossover approach.

Ding et al. [33] proposed an improved hybrid algorithm to solve VRPTW namely, HACO. HACO was instrumental in eliminating the premature convergence and low search rates of the traditional ACO techniques. By the introduction of a disaster operator, HACO prevented solutions to fall in a local optimum. Subsequently, the convergence speed was improved by hybridizing ACO with the saving algorithm and  $\lambda$ -interchange mechanism. Also, the paper gave a guideline on how to adjust the parameters to obtain good quality solutions.

Wang et al. [34] introduced the improved version of ACO taking into consideration the real-world traffic situation concentrating on the three aspects namely, people, vehicle and road. A negative feedback strategy was developed during the update of global pheromone to prevent the convergence to local optima. The algorithm produced positive results showing good efficient solutions.

Veen et al. [35] proposed an ACO solution to Dynamic Vehicle Routing Problem with Time Windows namely, MACS-DVRPTW. In that scenario, the customers and time windows could be incorporated any time during the working hours and they had to be considered in the partial solutions. The algorithm was tested on 56 Solomon's benchmark problems.

Bansal et al. [36] presented an Enhanced ACS for solving VRPTW. To prevent premature convergence and obtain a high-quality solution, the upper and lower bounds were considered for pheromone update. The algorithm was tested on Solomon's benchmark problems for 100 customers. The results obtained were promising for R and RC sets with less computational overhead and improved convergence rate.

Ran et al. [37] proposed a hybrid algorithm for VRPTW using a combination of ACO and Genetic Algorithm (GA). It combined the advantages of both the GA and ACO to provide a good quality solution. The initial solution was obtained from the GA strategy. Thus, the final quality of the ACO was dependent on the GA-based results obtained during the initial stages. The ACO algorithm considered was the MMAS to prevent premature convergence as the initial pheromone was asymmetrical. The numerical results verified that the proposed method was very effective for solving VRPTW.

Wang et al. [38] proposed Mixed Ant Colony algorithm (MACO) for solving VRPTW. The pheromone evaporation rate was dynamically adjusted based on the real-world traffic conditions. The algorithm was tested on Solomon's benchmark problems and promising results were obtained.

A Robust Multiple Ant Colony System (RMACS) was proposed by Toklu et al. [39] for generating solution pools for CVRP with time windows and uncertain travel times (CVRPTWU). The uncertain travel times might arise due to various real-life road conditions like a traffic jam, etc. The algorithm produced multiple solutions with distinct robustness against the aforementioned uncertainty. The choice of the solution from the pool would then be determined by analyzing the robustness and cheapness of the same.

Hifi et al. [40] presented a hybrid technique for VRPTW that included hybridization of ACO with Large Neighborhood Search technique (LNS) which was developed on the notion of creating and exploring neighborhoods. Since ACO suffers from the limitation of convergence to local optima, LNS would be beneficial for its non-guided search strategy to explore unknown regions. The objective functions were the same as that of MACS-VRPTW i.e., minimization of the total travel distance and the count of vehicles. The algorithm was tested on standard Solomon's benchmark problems and yielded promising outputs.

Bansal et al. [41] introduced a novel technique for tackling VRPTW by combining the ACO with the fuzzy logic approach. Here, VRPTW is considered with uncertain travel time. A probabilistic constrained model was presented using fuzzy credibility theory and consequently, a stochastic simulation was performed to obtain the fuzzy travel time. In addition, ACO was applied to obtain optimum solution for the problem within a reasonable period of time. The proposed technique was found to perform satisfactorily for both short and long duration of time windows.

A new algorithm for VRPTW was introduced by Zhang et al. [42] which is known as Improved Pareto Ant Colony Algorithm (IPACA). The algorithm improved the global search strategy of the ACO and helped prevent ACO from falling into the local optima. New transition rules were also devised for making the algorithm more effective.

Yang et al. [43] proposed an ant-based solver to solve dynamic vehicle routing problem with time windows and multiple priorities (DVRPTWMP). The customer set was dynamic in nature with varying priority levels of services. The objectives were optimization of total travel time along with the total delay of the customers based on their priorities. The ant solver was applied on a newly generated benchmark (which was based on Van Veen's benchmark [35]). Two strategies were adopted i.e. one with serving the high priority customers with the nearest available vehicles and the other with introducing penalties with a delay in service and minimizing both the penalties and delays. The second approach was found to be superior to the first one.

Brito et al. [44] employed an ACO hybrid algorithm for close-open VRPTW and fuzzy constraints. The parameters and constraints in the real-work scenario for modeling VRPTW were taken into consideration and a fuzzy optimization approach was adopted. This can

be applied to practical situations for estimating the delivery cost for a set of vehicles for serving its customers. The ACO approach was coupled with Greedy Randomized Adaptive Search (GRASP) and Variable Neighborhood Search (VNS) for finding the feasible optimal solutions.

An adaptive ACO algorithm for VRPTW was introduced by Grancy et al. [45] for multiple service workers (VRPTWMS) scenario. At first, the stochastic clustering was done followed by routing. The clustering would take care of scheduling customers to the parking locations which would eventually obtain high-quality results during routing. ACO was employed to form the feedback loop for linking the two stages viz., clustering and routing. That adaptive feedback loop aided in improving the quality of the solutions. The clustering results could, however, be improved by the incorporation of local search techniques which could be applied for multi-modal transportation.

Liao et al. [46] used a two-stage optimization algorithm for VRPTW. During the first stage, the AS version of ACO was applied followed by a plug-in heuristic algorithm in the second stage. The AS was used for finding the pre-path and that pre-path was then segmented based on the distribution of delivery points. Following the pre-path segmentation, sub-path re-optimization used insertion heuristic by respecting the time-windows. The algorithm was tested on the C1 class of Solomon's benchmark problems and quite satisfactory results were obtained. One major advantage of the proposed approach was that the time complexity was less than that of its many counterparts.

Necula et al. [47] introduced a new ACS algorithm for tackling Dynamic VRPTW, namely DVRPTW-ACS. It took the inspiration from the algorithm developed by Veen et al. [35] and incorporated the time slices and node commitment approach from MACS-DVRPTW. There were two parts in the algorithm viz., the planner (responsible for the time slices management and also dealing with the dynamic aspect of the problem) and the optimization part (which used the ACS algorithm). Unlike MACS-DVRPTW, the proposed approach optimized both the distance of travel and the number of vehicles simultaneously by using a single ant colony. The results obtained were better than that of MACS-DVRPTW in terms of both its objectives.

A hybrid method was introduced by Abderrahman et al. [48] by combining ant colony optimization with neighborhood search technique for solving VRPTW with target time i.e. VRPTWTT. A penalty was associated with early service as well as with delay in service. The objectives of the problem were the minimization of transport cost as well as the penalty. The proposed approach was tested on Solomon's benchmark problems and found to give promising results.

Bouyahyious et al. [49] proposed an ACO approach for solving full-truckload selective multi-depot VRPTW (FT-SMDVRPTW). The objective function was the maximization of the profit by constructing a set of feasible routes for the trucks. The ACO was judiciously modified to develop a robust optimization strategy for handling the full truckload scenario. The approach was applied to randomly generated data instances.

Gupta et al. [50] proposed a hybrid approach by coupling Genetic Algorithm and Ant Colony Optimization on Capacitated VRPTW. At first, the ACO algorithm was applied and subsequently, the mutation operators of GA were applied for the tour construction to achieve diversity in the solution sets and to prevent locally optimized results. Finally, the results were compared with the individual ACO and GA techniques and it was found out that combining the two algorithms gave better results.

## 5. Conclusion

From the past two decades, VRPTW has been one of the active research domains in the literature. VRPTW is the heart of distribution management. It is modeled for the various real-life scenario by the distribution companies who look towards reducing the total budget for distribution of goods. The budget, in turn, depends on various aspects like the number of vehicles, total travel time, etc. Various exact, approximate, heuristic and metaheuristic techniques have been devised to tackle the problem. In this paper, a comprehensive study is presented on the applications of ACO metaheuristic on VRPTW. It is observed that along with this particular problem, other variants like Open VRPTW, Dynamic VRPTW, VRPTW with pickup and delivery and many other types are also considered. Also, it is observed that apart from using different variants of ACO like ACS, AS, MMAS, etc., it is coupled with other metaheuristic approaches like GA, Simulated Annealing and local search techniques that significantly improve the results and help overcome the limitations associated with ACO. The proposed methodologies are mainly tested on standard Solomon's benchmark problems. Hopefully, the survey would be helpful for researchers to evaluate the various proposed techniques and also help to understand the future scope in this particular domain.

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