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Research Paper

LQR Tuning by Particle Swarm Optimization of Full Car Suspension System

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Abstract

This paper attempts to examine the potential value in showing the performance of Particle Swarm Optimization (PSO) in order to produce diagonal components of matrix Q, R. The linear model was used in this system, because it has ability to describe all basic performance that exist in full car vehicle suspension system such as roll, pitch, body sprung and each wheel vertical movement. Performance of suspension system measured by range of acceleration arise in automobile body. Drive handling and comfort is an opposite condition. Balancing condition of both define quality of control strategy of suspension system. The disturbances are applied to all tires in testing scenario of applied control algorithm. The simulation result shown better performance of LQR tuning by PSO than passive and LQR without tuning system.

Keywords: *Car, Suspension System, Optimization, Modeling*

1. Introduction

Suspension system is one of the main equipment must exist in automobile system. The main function of suspension system is to support automobile body then isolate the body from road irregular surface disturbances [1]. Conventional suspension system consists of spring and damper. Commonly suspension system classified in three model passives, semi active and active. In active suspension system, the implementation control system hold very important role to balancing trade of drive comfort and handling[2]. Recently many researchers involved to increase the performance of suspension system. Ismail [3] [4] apply composite nonlinear feedback (CNF) to quarter car suspension model. The simulation conducted by MATLAB/Carsim and MATLAB/Simulink.[5, 6] Did optimization for quarter car suspension model by consider 2 –D dynamic model. Several models develop to identify character of full car suspension system [6-8]. The model that used in this study is full car system model [9]. The model proposed has ability to test by different disturbance acting on each wheel. In additional the model must capable to perform 7 DOF as basic character of full car suspension system namely body vertical, roll, pitch and each wheel vertical movements [10]. One of the simple but robust control system is LQR control algorithm. The research about LQR in full car suspension perform by Darus [10], implemented into full car system. Sandhu [11] perform combination skyhook and LQR for semi active suspension system on magnetorheological suspension model. However the system self-derive tuning that shall enhance to get more satisfy condition. [12] Did adaptive and dynamic tuning gain of LQR. The adaptive tuning gain feed from dynamic load and suspension deflection, however the development of K that implement in input calculation is taken from many parameter approaches.

The successor key of the LQR suspension system are Q, R matrix. By defining the proper Q, R matrix, will be increase the performance of LQR control system against disturbance [13]. However, the LQR control system has a deficiency especially if applied in large and complex system. Diagonal matrix Q.R is going larger. By only use self-drive searching base on individual experience will wasting time and less efficient. The solution of that problem solves by applied GA on searching method of Q, R matrix. In additionally GA has inefficient by expensive computational cost [14]. To substitute GA, PSO as the chosen solution to tuning Q R matrix. In this study author apply PSO to define Q, R matrix.

2. Full Car Suspension System Model

Full car suspension model used in this study is full car suspension model with 7 degrees of freedom [9]. Figure 1 shows full car suspension model. The solid mass vibration adopt as a solid form with freely movement on z, x, and y axis. The auto mobile body moving vertical, rolling, pitching and each wheel doing freely up and down vibration. Each component is called z_s,θ,$\emptyset$,f_l. f_r r_l r_r Assume that roll and pitch angle movement are quite small. The linear approach for define each movement of sprung mass is shown in equation 1.

$$z_1 f_l(t) = z_s(t) + l_f\theta(t) + t_f\emptyset(t)$$

$$z_1 f l_r(t) = z_s(t) + l_f\theta(t) - t_f\emptyset(t)$$

$$z_1 r_l(t) = z_s(t) - l_f\theta(t) + t_f\emptyset(t)$$

$$z_1 r_r(t) = z_s(t) - l_f\theta(t) - t_f\emptyset(t) \quad (1)$$

Definition of kinematic relation between x(t) and q(t) is shown in equation 2.

$$x_s(t) = L^T q(t) \quad (2)$$

Where q(t) and x(t) are

$$q(t) = [z_s(t)\ \ \theta_t(t)\ \ \emptyset_s(t)]^T,\ x_s(t) = [z_s f_l(t)\ z_s f_r(t)\ z_s R_l(t) z_s R_r(t)]^T$$

$$L = \begin{bmatrix} 1 & 1 & 1 & 1 \\ l_f & l_f & -l_r & -l_r \\ t_f & t_f & t_f & -t_f \end{bmatrix}$$

In the definition of mass, damping and stiffness matrix, movement equation of full car suspension mode defines as shown in equation 3 and equation 4.

$$M_s\ddot{q}(t) = LB_s\big(x_u(t) - x_s(t)\big) + LK_s\big(x_u(t) - x_s(t)\big) - LF(t) \quad (3)$$

$$M_u\ddot{x}_u(t) = B_s\big(\dot{x}_s(t) - \dot{x}_u(t)\big) + LK_s\big(x_s(t) - x_u(t)\big) - K_t(w(t) - x_u(t)) + F(t) \quad (4)$$

$$x_u(t) = [z_2 f_l(t)\ z_2 f_r(t)\ z_2 r_l(t) z_2 r_r(t)]^T$$

$$w(t) = \big[w_{fl}(t)\ \ w_{fr}(t)\ \ w_{rl}(t)\ \ w_{rr}(t)\ \big]^T$$

$$F(t) = \big[F_{fl}(t)\ \ F_{fr}(t)\ \ F_{rl}(t)\ \ F_{rr}(t)\ \big]^T$$

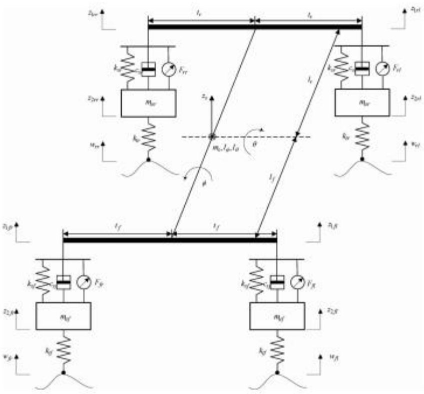

Fig.1: Full car suspension model

The given matrix is shown in equation 5. Subtitution equation 2 to 3 gives equation 6.

$$M_s = \begin{bmatrix} m_s & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_s \end{bmatrix} \qquad K_t = \begin{bmatrix} k_{tf} & 0 & 0 & 0 \\ 0 & k_{tf} & 0 & 0 \\ 0 & 0 & k_{tr} & 0 \\ 0 & 0 & 0 & k_{tr} \end{bmatrix}$$

$$M_u = \begin{bmatrix} m_{sf} & 0 & 0 & 0 \\ 0 & m_{sf} & 0 & 0 \\ 0 & 0 & m_{sf} & 0 \\ 0 & 0 & 0 & m_{sf} \end{bmatrix} \qquad B_s = \begin{bmatrix} c_{sf} & 0 & 0 & 0 \\ 0 & c_{sf} & 0 & 0 \\ 0 & 0 & c_{sf} & 0 \\ 0 & 0 & 0 & c_{sf} \end{bmatrix}$$

$$K_s = \begin{bmatrix} k_{sf} & 0 & 0 & 0 \\ 0 & k_{sf} & 0 & 0 \\ 0 & 0 & k_{sr} & 0 \\ 0 & 0 & 0 & k_{sr} \end{bmatrix} \quad (5)$$

$$M_m\ddot{z}_m(t) + B_m\dot{z}_m(t) + K_m z_m(t) = K_{mt}w(t) + L_m F(t) \quad (6)$$

Where

$$z_m(t) = [q^T(t)\ \ x_u^T(t)]^T \qquad B_m = \begin{bmatrix} LB_sL^T & -LB_s \\ -B_sL^T & B_s \end{bmatrix}$$

$$K_{mt} = \begin{bmatrix} 0 \\ K_t \end{bmatrix}$$

$$L_m = \begin{bmatrix} -L \\ I \end{bmatrix}$$

The state – space form of full car suspension model can be expressed in equation 7.

$$\dot{x}_s(t) = \hat{A}x_s(t) + \hat{B}_1 w(t) + \hat{B}_2 F(t) \quad (7)$$

Where

$$x_g(t) = [z_m^T(t)\ \ \ \dot{z}_m^T(t)]^T$$

$$\hat{A} = \begin{bmatrix} 0 & I \\ -M_m^{-1}K_m & -M_m^{-1}B_m \end{bmatrix}$$

$$\hat{B}_1 = \begin{bmatrix} 0 \\ M_m^{-1}K_{mt} \end{bmatrix} \quad \hat{B}_2 = \begin{bmatrix} 0 \\ M_m^{-1}L_m \end{bmatrix}$$

The table parameter used in this control system is shown in Table 1.

Table 1: Parameter Values of the Full Car Suspension Model Disturbance High and Time Set

Parameter	Value	Unit	Parameter	Value	Unit
m_s	1400	kg	C_{sf}	1100	Nm/s
I_θ	2100	$kg.m^2$	C_{sr}	1100	Nm/s
$I_\emptyset$	460	$kg.m^2$	K_{sf}	23500	N/m
$m_{uf}m_{ur}$	40	kg	K_{sr}	25500	N/m
l_f	0.96	m	K_{tf}, K_{tr}	190000	N/m
l_r	1.44	m	t_f, t_r	0.71	m

3. The Control Strategy

Assume that cost function is shown in equation 8.

$$J = \frac{1}{2}\int_0^t (x^TQx + u^TRu)\,dt \quad (8)$$

The equation used to solve above equation is shown in equation 9.

$$PA + A^TP - PBR^{-1}B^TP + Q = 0 \quad (9)$$

The matrix R and P use calculate the final value of matrix K by as follow equation 10.

$$K = -R^{-1}B^TP \quad (10)$$

and linear control law expression is shown in equation 11.

$$u(t) = -Kx(t) \quad (11)$$

The equation of state space then change as shown in equation 12.

$$\dot{x} = Ax - BKx + Bw$$

$$\dot{x} = (A - BK)x + Bw$$
$$y = Cx \quad (12)$$

4. The Optimization Strategy

The particle swarm optimization is a highly efficient method if compare with another [15]. PSO mechanism started with population initialization to do random search and tracking. PSO consist of each particle that searching final solution on the determining range. Each particle will move consistent on their track on the surface of solution area. The best finding solution that encountered called global best, while the best position that have visited by the best particle is called local best. Global best connected with all individual indicated as ability to remember success experience in the past as shown in equation 13.

$$v_i^{k+1} = h \times v_i^k + c_1 \times rand \times \left(x_i^b - x_i^k\right) c_2 \times rand \times \left(x_i^g - x_i^k\right) \quad (13)$$

Optimization involving step, velocity, personal best and global best. The best solution as the result of the algorithm expression is shown in equation 14.

$$x_i^{k+1} = x_i^k + v_i^{k+1} \quad (14)$$

For implementation into the Q R diagonal matrix searching, the population of particles are set equal to 50 (N) and the searching dimensions are 7 (D). The particle weighting inertia 1.0 and the parameter of personal best and global best constant 2.0. The population will doing random searching over the solution surface in 30 iteration. The solution value range is 1 - 1000. The range value for Q and R are the same. Q must be semi definite matrix ($Q \geq 0$) and (R > 0) positive definite. Where $Q > R$ the best solution of q_{ii} and r_{ii} chosen from the minimum value of equation that expressed below as shown in equations 15, 16, 17, and 18.

$$q_{ii}^{k+1} = x_i^k + v_i^{k+1} \quad (15)$$

$$q_{ii} = \min(q_{ii}^{k+1}) \quad (16)$$

and

$$r_{ii}^{k+1} = x_i^k + v_i^{k+1} \quad (17)$$

$$r_{ii} = \min(r_{ii}^{k+1}) \quad (18)$$

The PSO algorithm in this study is working independently. No feedback from the plant connected and affected the PSO tuning system.

5. Simulation

The model used in the study has advantage by ability of doing individual disturbance on each wheel. The scenario design to expose the automobile body position respect to x, y, z axis, and drive comfort indicated by body acceleration when disturbances impact to particular or all wheel. Assumption used in the simulation scenario is the disturbances form arise in front left will same with rear left but different in time because delay factor. The disturbance sets are shown in Table 2 and Figure 2.

Table 2: Disturbance High and Time Set

Part of Suspension	Disturbance	
	High(cm)	Arise Time(t)
Front left	6	2.1 – 2.2
Front right	3	1.5 – 1.7
Rear left	6	3.2 – 3.4
Rear right	3	4.2 – 4.4

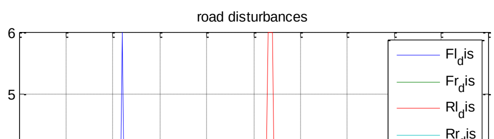

Fig. 2: Disturbance scenario of full car suspension model

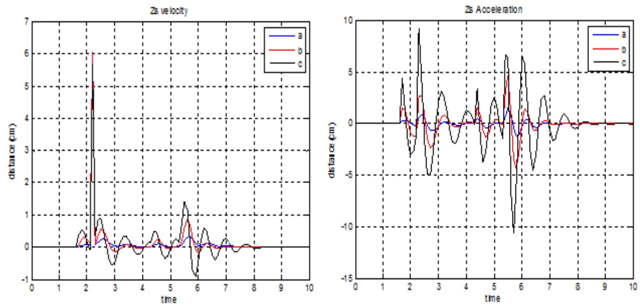

Fig. 3: Automobile body vertical velocity and acceleration

Figure 3 shows the movement of the automobile body on the vertical axis ($\dot{z}_s, \ddot{z}_s$,). Vertical velocity ($\dot{z}_s$) beginning to increase when disturbance arise at 1,5 s, The highest position when the front left wheel hit by bump disturbance. Less effect influence the body when disturbance happen in rear wheel. Body acceleration swinging from the highest to the low peak several times. The highest peak is between 9-10 cm while the lowest peak is 10-11. cm. PSO LQR gave the best performance in damping process of automobile body respect to z axis.

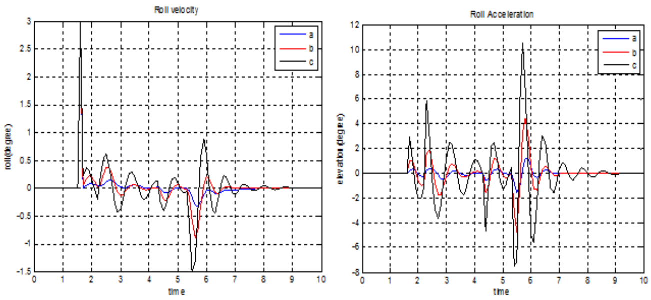

Fig. 4: Automobile body roll velocity and acceleration

Figure 4 express the first impact of disturbance made the body rotation roll velocity $\dot{\theta}$ respect to x get the high rotation 3^0, then going down propagate sinusoidal. When the time 5s the automobile body reach the lowest negative rotation -1.5^0. The automobile acceleration propagate dynamically along the time accumulation of disturbance energy absorb by body made it reach the maximum value after the last disturbance at 5 – 6 s. The PSO LQR display the best result stabilizing the roll movement of automobile body.

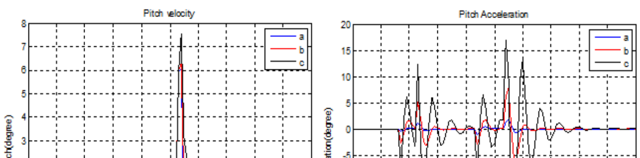

Fig. 5: Automobile body pitch velocity and acceleration

Figure 5 describe pitch movement of the automobile body. The peak of pitch movement arise after all road disturbance hit the wheel of the car, while the acceleration of the body vibrate beginning along with disturbance and get the highest value after all disturbance complete impact the suspension system. LQR PSO give the best result stabilizing the pitch body movement

By the success of PSO LQR damping the acceleration movement of automobile body made it become more comfortable to drive.

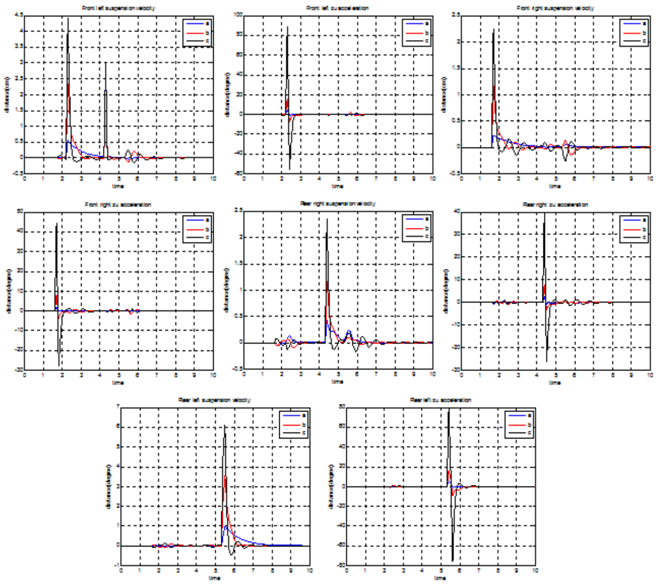

Fig. 6: Automobile body velocity and acceleration

Figure 6 describes all disturbance arise in automobile suspension unit. The figure shown how disturbance force lift the suspension unit. The force made velocity and acceleration of suspension system increase. The figure of each unit suspension system shown how effective PSO LQR to deal with disturbance that arise in each wheel. Decreasing velocity and acceleration in suspension part will bring the automobile in driving comfort zone.

6. Conclusion

Overall results on the numerical experiment by using MATLAB had shown that the PSO LQR algorithm improve the effectiveness in control performance as in figure 3, 4, 5, and 6 especially on reduction of overshoot velocity and acceleration of suspension system. Future work, it recommends that PSO LQR control algorithm can tested by various dynamical road condition and implemented in real condition.

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