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Research paper

Smooth Optimization Technique for Denoising Image

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Abstract

Noise in images can be characterized as a random variation of brightness information. It can be a by-product of image capture that conceals the desired information. All digital images are subject to different types of noise. Noise is the result of errors during acquisition process which affects the original pixel values in the image. Noise can be introduced in several ways depending on how the image is created. This paper suggests a mathematical model for representing noise in the image and solves the model for denoising. A comparison of smooth optimization algorithm with Fourier transform is presented with test image.

Keywords: *Image denoising, smooth optimization, gradient descent, regularization, fourier transform*

1. Introduction

Removing noise from image is a major challenge in the field of digital image processing. Digital images play significant role in medical field, remote sensing, pattern recognition, transmission and encoding, video processing, microscopic imaging and robot vision. The acquisition techniques in digital imaging introduce various types of noises in images. If the image is scanned from a photograph, noise can be the result of damage to the photograph film or may be introduced by the scanner. The mechanism to gather the image data can introduce noise. Noise may result during electronic transmission of image.

More importance is given to denoising in image processing since it predominantly affects image analysis and applications. Preserving significant details of an image and reducing random noise as far as possible is the main goal of image denoising techniques. A noisy image depresses the visibility of low contrast objects and produces disagreeable visual quality. Hence removal of noise is an essential step in image processing to enhance and recover finer details hidden in the image data. In many applications, noise in images are additive in nature with Gaussian probability distribution. It is difficult to remove Gaussian noise since it corrupts all pixels in the image. The tradeoff between noise suppression and preserving the details of image always exist in denoising algorithms.

An image is normally represented as a matrix of grayscale or color values. Each value is denoted as (v, p), where v is the value present at position p, termed as pixel. The light intensity measurement produced by a charge coupled device connected with a light focusing system represents the values of pixel in an image. The number of incoming photons received by the portion of CCD is counted for a fixed period corresponding to observation time. According to central limit theorem, the number of photons received at each pixel fluctuates in its average when the light source is assumed constant. These fluctuations are expected to be in the order of √m where m is the number of incoming photons. Additionally, if the area receiving photons are not cooled sufficiently, then heat photons may be received. These factor are termed as "noise".

The general Noise model for any denoising algorithm are represented as

$$v = u + n$$

v is the observed value, u is the actual value, n is the noise value

2. Related Work

Image denoising normally preserves the edges and finer structural information in the image while reducing the noise. Researchers have proposed models removing noise while preserving the significant details [1-2]. Partial Differential equation and variational based methods have been applied for image denoising [3-4]. The best among these methods considered is TV model [5]. Though the second-order PDE preserves edges, it introduces staircase effect in the smooth regions. A modified model to prevent staircase effect was proposed to automatically adapt gradient-based exponent to fit the data [6]. But this model is non-convex and hence difficult to solve. Higher order derivatives are introduced into the energy function [7] but numerical computation becomes a challenge [8-9]. Hybrid models combining the second order PDE models and fourth order PDE have been suggested [10-11]. A weighted combination of Tikhonov regularization and total variation regularization for developing the image denoising model has been discussed [12].

Image denoising can be considered as discrete ill-posed problem. The problem can be considered as computation of approximate solutions of a linear system and represented as

$$Ax=b \tag{1}$$

where $A \in R^{mxn}$is a full rank matrix. The vector $b \in R^m$ in (1) represents data corrupted by error, where b is denoted as

$$b = b_f + e \tag{2}$$

where $b_f \in R^m$ is the error-free data and e is the introduced error normally assumed to be white Gaussian noise.

The vector b is dominated by the propagated error because of ill-conditioning of A and hence good approximation cannot be expected. To handle this problem, regularization methods are used since it is found that their solution are less sensitive to the error e in b.

3. Regularization methods

Truncated Singular Value Decomposition (TSVD), Minimal residual iterative method and Tikhonov regularization are some of the popular regularization methods. In the TSVD method the solution of modified least-squares problem is obtained by setting k largest singular values of the matrix A to zero. The parameter k controls the level of modification of the given problem and also determines the how sensitive the solution of the modified problem to the error e. k is termed as a regularization parameter. Tikhonov regularization replaces the problem least-square problem by a penalized least-squares problem, and the regularization parameter determines the level of modification in the given problem. Lastly, the truncated iteration regularization carries out few iterations with iterative solution method to provide regularized solutions. The number of iterations is considered as regularization parameter. The following section provides an overview of the TSVD, Tikhonov, and minimal residual methods for computing regularized solution that can be applied to discrete ill-posed problems.

3.1. TSVD

The Singular Value Decomposition (SVD) of a matrix $A \in R^{mxn}$is a factorization of the form

$$A = U\Sigma V^T \tag{3}$$

where $\Sigma = \mathrm{diag}\left(\sigma_{1,}\sigma_{2,}, \dots, \sigma_{n,}\right) \in R^{mxn}$ is a (possibly rectangular) diagonal matrix whose diagonal entries $\sigma_{1,} \geq \sigma_2 \geq \dots \geq \sigma_n \geq 0$ are the singular values of A, $U = [u_1, u_2, \dots, u_n] \in R^{mxm}$and $V = [v_1, v_2, \dots, v_n] \in R^{nxn}$ are orthogonal matrices.

The matrix A is substituted by its best rank-k approximation which is involves the computation of 2-norm

An optimal rank-p approximation A_p of matrix A in norm k2 is given by

$$A_k = \sum_{i=1}^{k} \sigma_i u_i v_i^T \ \ with\ ||A - A_k||_2 = \sigma_{k+1} \tag{4}$$

The approximate solution x_k is given by

$$x_k = \sum_{i=1}^{k} \frac{u_i^T b}{\sigma_i} v_i + \ \varepsilon \sum_{i=1}^{k} \frac{u_i^T e}{\sigma_i} v_i \tag{5}$$

The integer k is a regularization parameter. If A has very small singular values, then the terms with these singular values may dominate other terms and the propagated error increases. Hence, Regularization by TSVD omits the terms associated with the smallest singular values. The value of k must be selected small such that x_k is not dominated by propagated error.

TSVD method is mostly preferable for rank-deficient problems. The ideal approximation of x_k is found when the value of k reaches the numerical rank r of A.

Though the filtering strategy is not appropriate for the gradually decaying singular values of A, TSVD method can be useful for solving discrete ill-posed problems.

3.2. Tikhonov regularization

Tikhonov regularization replaces the general least-square problem by the penalized least-squares problem. Tikhonov regularization has been formulated as

$$\min\left||Au - b|\right| subject\ to\ \left||Lu|\right| \leq \delta \tag{6}$$

where δ is a positive constant.

The above representation is a linear least square problem with a quadratic constraint, and uses the Lagrange multiplier formulation

$$\mathcal{L}(u, \lambda) = ||Au - b||^2 + \ \lambda(\left||Lu|\right|^2 - \delta^2) \tag{7}$$

If u_L denotes the least square solution of $\left||Au - b|\right| = min$ and $\delta \leq ||u_L||$, then the solution $u_\delta$is identical to the solution $u_\lambda$for an appropriately chosen λ.

Tikhonov regularization exhibits much smoother filter factors than truncated SVD which is favorable for discrete ill-posed problems.

3.3. A minimal residual iterative method

Assuming the matrix $A \epsilon R^{nxn}$to be symmetric. For any symmetric matrix A, it is the known fact that iterative methods for the solution of minimization problems should produce an approximate solution in the range of A. Starting with the initial iterate $x_0 = 0$,, the p^{th} iterate x_p satisfies

$$\left||Ax_p - b|\right|_2 = \min_{x \epsilon K_p(A, A_b)} ||Ax - b||_2 \tag{8}$$

where $K_p(A, A_b) = span\{Ab, A^2 b, \dots, A^p b$ is a Krylov subspace in the range of A.

The number of iterations p should be specified for this iterative method. Here the parameter p is the regularization parameter. When p is very small, the number of iterations may not be sufficient to approximate the desired solution accurately. At the same time, when p is too large, then the propagated error may dominate the computed approximate solution.

4. Fourier transform and smooth optimization algorithm

In this paper we have discussed solving image denoising problem using Tikhonov model and compared Iterative algorithm with Fourier transform.

4.1. Image denoising model

Consider our data v is of the form

$$v = u + b \tag{9}$$

where $u \epsilon R^{N^2}$is the image we are looking for, $v \epsilon R^{N^2}$is the data at our disposal and $b \epsilon R^{N^2}$is the realization of a Gaussian white noise. The model we are considering is to minimize the energy

$$E(w) = \sum_{i,j=0}^{N-1} |\nabla_{w_{i,j}}|^2 + \ \lambda \sum_{i,j=0}^{N-1} (w_{i,j} - v_{i,j})^2 \tag{10}$$

where $w \epsilon R^{N^2}$and λ≥0, with

$$\nabla_{w_{i,j}} = \begin{pmatrix} (D_x w)_{i,j} \\ (D_y w)_{i,j} \end{pmatrix} = \begin{pmatrix} w_{i+1,j} - w_{i,j} \\ w_{i,j+1} - w_{i,j} \end{pmatrix}$$

$$\text{For } (i, j) \in \{0, \dots, N - 1\}^2$$

4.2. Fourier transform

Reducing noise can be accomplished in the image space domain, or in the image transform domain [13,14].

Denoising methods on spatial domain cannot remove plenty of noise information but the image transform domain can often work wonders. Fourier Transforms in signal processing is one of the most important and widely used transform [15]. For two-dimensional discrete-time signal$f(x, y), x = 0, 1, \dots, X - 1, y = 0, 1, \dots, Y - 1$,, whose Fourier transform definitions are as follows:

$$F(u, v) = \frac{1}{XY} \sum_{x=0}^{X-1} \sum_{y=0}^{Y-1} f(x, y) e^{-j2\pi\left(\frac{ux}{X} + \frac{vy}{Y}\right)} \tag{11}$$

where u=0,1,…,X-1,v=0,1,…,Y-1, u and v are called the spatial frequency and j is the imaginary unit. The inverse transform definition is as follows:

$$f(x,y)=\sum_{u=0}^{X-1}\sum_{v=0}^{Y-1}F(u,v)e^{j2\pi\left(\frac{ux}{X}+\frac{vy}{Y}\right)} \qquad (12)$$

where x=0,1,…,X-1,y=0,1,…,Y-1.
The discrete Fourier transform of h can also be represented as

$$h_{k,l}=\lambda+4-2\left(\cos\left(\frac{2\pi k}{N}\right)+\cos\left(\frac{2\pi l}{N}\right)\right) \qquad \forall k,l=0\ldots N-1. \ (13)$$

Application of Fourier transform on noisy images is experimented with sample image by varying the values of λ.

4.3. Gradient algorithm

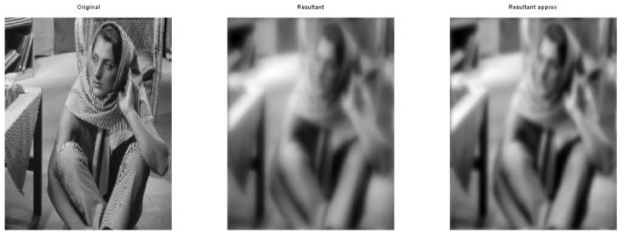
Smooth Optimization using Gradient Algorithm is shown in Fig. 1. The algorithm uses steepest descent step size rule for convergence.

```
Algorithm: Gradient Algorithm
Entry: Entry needed for computing E and ∇E
Output: Approximation of a minimizer : w∗
Initialize w
While Not converged Do
        Compute d = ∇E(w)
        Compute a step-size t ≥ 0
        Update : w ← w − t d
End while
```

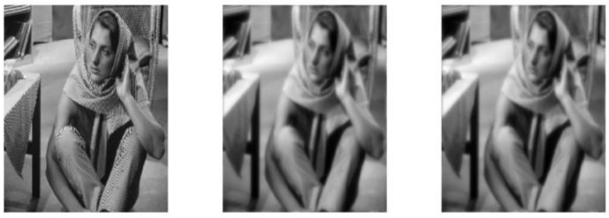
Fig. 1: Gradient algorithm

5. Experimental Results

FFT and Smooth optimization based on gradient descent are applied on the image by varying λ and results obtained are depicted in Fig.2. Smooth optimization algorithm performs well compared to Fourier Transform with less complexity. The performance is judged for 100 iterations with various values of λ. Results show that for the lower λ value denoising produces blur image and when λ=1, the effect of denoising is appreciable.



(a)λ = 0.01



(b) λ = 0.1

(c) λ = 1

(d) λ = 10

Fig. 2: Results of denoising using fourier transform and smooth optimization technique

6. Conclusion

Denoising is an important step in image processing applications. Removal of noise helps to understand the finer details without losing significant image information. This paper discusses the performance of Fourier transform over the noisy image. Considering the complexity of the algorithm a smooth optimization technique is suggested which performs well with less computation. The comparative results are shown on the noisy image for various λ values which is a regularization parameter. The optimal λ value for which denoising performance is appreciable has been demonstrated.

References

[1] Buades A, Coll B & Morel JM, "A review of image denoising algorithms, with a new one", *Multiscale Modeling and Simulation*, Vol.4, No.2, (2005), pp.490–530,

[2] Dabov K, Foi A, Katkovnik V & Egiazarian K, "Image denoising by sparse 3-D transform-domain collaborative filtering", *IEEE Transactions on Image Processing*, Vol.16, No.8, (2007), pp.2080–2095.

[3] Chen D, Chen Y & Xue D, "Three fractional-order TVL2 models for image denoising", *Journal of Computational Information Systems*, Vol.9, No.12, (2013), pp.4773–4780.

[4] Hajiaboli MR, "An anisotropic fourth-order diffusion filter for image noise removal", *International Journal of Computer Vision*, Vol.92, No.2, (2011), pp.177–191.

[5] Rudin LI., Osher S & Fatemi E, "Non linear total variation based noise removal algorithms", *PhysicaD: Non linear Phenomena*, Vol.60, No.1–4, (1992), pp.259–268.

[6] Blomgren P, Chan TF & Mulet P. "Extensions to total variation denoising", *Advanced Signal Processing: Algorithms, Architectures and Implementations VII, Proceedings of SPIE*, (1997), pp.367–375.

[7] Lysaker M, Lundervold A & Tai XC, "Noise removal using fourth-order partial differential equation with applications to medical magnetic resonance images in space and time", *IEEE Transaction son Image Processing*, Vol.12, No.12, (2003), pp.1579–1589.

[8] Khare AS, Mohan R & Sharma S, "An efficient image denoising method based on fourth-order partial differential equations", *International Journal of Advanced Computer Research*, Vol.3, No.9, (2013), pp.126–131.

[9] Youand YL & Kaveh M, "Fourth-order partial differential equations for noise removal", *IEEE Transactions on Image Processing*, Vol.9, No.10, (2000), pp.1723–1730.

[10] Zheng S, Pan Z, Wang G & Yan X, "A variational model of image restoration based on first and second order derivatives and its split Bregman algorithm", *Proceedings of the 3rd IEEE/IET International Conference on Audio, Language and Image Processing (ICALIP '12)*, (2012), pp.860–865.

[11] Oh S, Woo H, Yun S & Kang M, "Non-convex hybrid total variation for image denoising", *Journal of Visual Communication and Image Representation*, Vol.24, No.3, (2013), pp.332–344.

[12] Liu K, Tan J & Su B, "An Adaptive Image Denoising Model Based on Tikhonov and TV Regularizations", *Advances in Multimedia*, (2014), pp.1-10.

[13] Buades A, Coll B & Morel JM, "A non-local algorithm for image denoising", *Proc. IEEE CVPR*, Vol.2, (2005), pp.60–65.

[14] Liu Y, Wang J, Chen X, "A robust and fast non-local means algorithm for image denoising", *Journal of Computer Science and Technology*, Vol.23, No.2, (2008), pp.270–279.

[15] Wang J, Guo Y, Ying Y, Y. Liu, and Q. Peng, "Fast Non-Local Algorithm for Image Denoising", *Proc. IEEE ICIP*, (2006), pp.1429-1432.