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Research paper

The Controller Design Using Local Stability Analysis on a Nonlinear Inverted Pendulum

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Abstract

This paper is designed for the purpose of analytic design and numerical simulation of a controller for nonlinear mobile inverted pendulum. It needs to obtain a method to overcome the difficulties in the design problem of controller for nonlinear plant. The method was using local stability analysis of two fixed points in two-dimensional ordinary nonlinear differential equation. The result provides a stable response solution using a special pole placement design, and transient stability is simulated using simulink. The response behavior can be selected according to the desired poles. The results obtained at this work are different from the optimal control problem. Indeed, our results have been summarized in the design method, target response and simulation process.

Keywords: *analytic design, special pole placement, simulation, stable response*

1. Introduction

A nonlinear inverted pendulum model has dynamics with two movements, its angle and its wheel movements, as shown in Fig. 1. These dynamics[1]–[4] can be elaborated by two nonlinear second order differential equations shown with its arrangement as can be seen in equations (1) and (2).

$$\ddot{\theta}(t) = \frac{W\left(M_p\, g\, L\, \sin\theta - F\right) - Q\, \cos\theta \left(M_p\, L\, \dot{\theta}^2\, \sin\theta + \frac{1}{r}F\right)}{W\, P - Q^2\, \cos\theta^2} \quad (1)$$

$$\ddot{x}(t) = \frac{\left(M_p\, L\, \dot{\theta}^2\, \sin\theta + \frac{1}{r}F\right) P - \left(M_p\, g\, L\, \sin\theta - F\right) Q\, \cos\theta}{W\, P - Q^2\, \cos\theta^2} \quad (2)$$

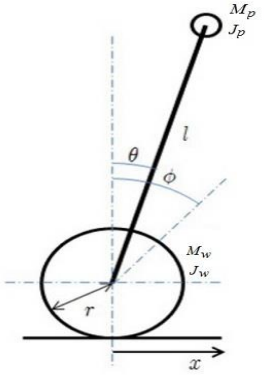


Fig. 1. Model of Inverted Pendulum

Both equations use three parameters can be seen in equation (3).

$$W = M_w + M_p + \frac{J_w}{r^2}, = J_p + M_p\, L^2\, , Q = M_p\, L \quad (3)$$

θ is the position of pendulum; x is the position of the wheel. The initial position of pendulum was $\theta = \theta_0$ in the unit (rad), the initial position of the wheel with $x = x_0 = 0$ in the unit (m). The parameters M_w and M_p are respectively by the mass of the wheel and the pendulum in the unit (kg), and g = 9.81 (m/s^2) is the graffiti acceleration. To do the simulation it needs the parameters: L = 0.6 m, M_p = 1.6 kg, M_w = 1.551 kg, J_w = 0.005 kg-m^2, J_p = 0.027 kg-m^2, r = 0.2 m[1].

The equations (1) and (2) can be modeled in the state space equation. It chooses four states, where, $x_1 = \theta, x_2 = \dot{\theta}, x_3 = x, x_4 = \dot{x}$. Based on equations (1) and (2) have been formed the state space equation (4) below[5]:

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{W\, Q\, g\, \sin x_1 - Q^2\, {x_2}^2 \sin x_1 \cos x_1 - \left(W + \frac{Q}{r}\cos x_1\right)F}{W\, P - Q^2\, \cos {x_1}^2} \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \frac{PQ\, {x_2}^2 \sin x_1 - Q^2 g\, \sin x_1 \cos x_1 + \left(\frac{P}{r} + Q\cos x_1\right)F}{W\, P - Q^2\, \cos {x_1}^2}
\end{aligned} \quad (4)$$

The behavior of the open loop inverted pendulum has been simulated using matlab-simulink[6]–[8], as shown in Fig. 2.

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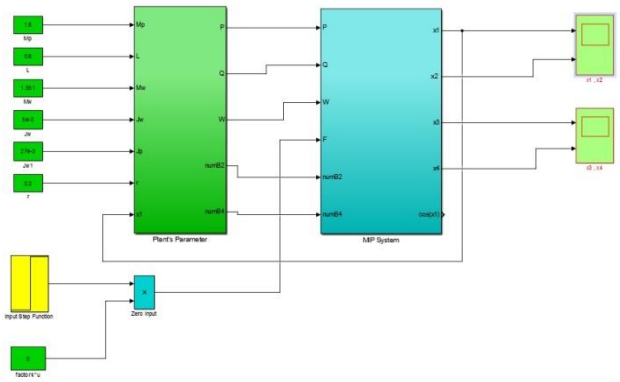


Fig 2. The Simulation of Open Loop System

The responses can be seen on Figs. 3 and 4. All the responses are oscillation and make the system unstable. The characteristic of the open loop system equation (4) include damping factor and natural frequency cannot be explained analytically because both these dynamics are nonlinear.

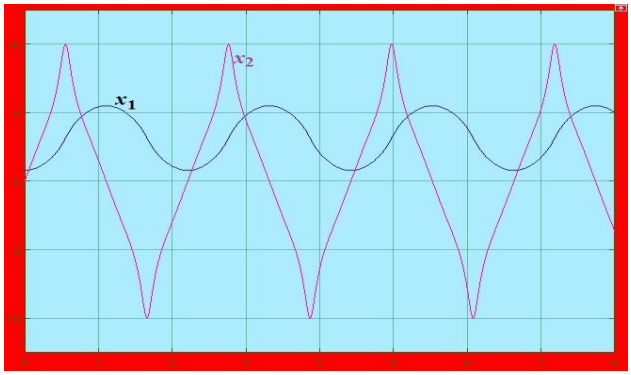


Fig 3. Response of $\theta(t)$ and $\dot{\theta}(t)$ on Open Loop System

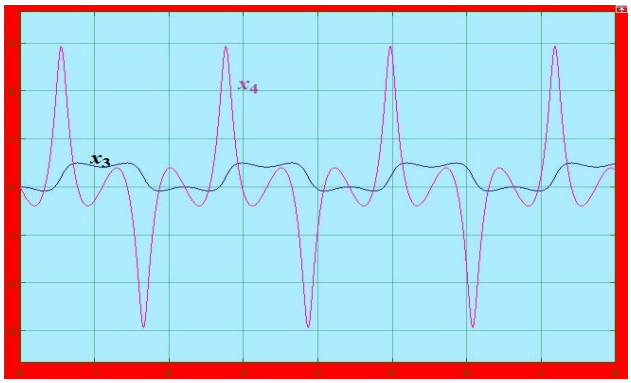


Fig 4. Response of $x(t)$ and $\dot{x}(t)$ on Open Loop System

To stabilize all the responses of the open-loop system it needs the state feedback. It is difficult to design feedback as a linear plant. First it needs a way to do the process for linearization of system dynamics. The local stability analysis method can be developed in model of open loop system; furthermore it needs to apply the pole assignment or pole placement to the local stability analysis.

2. Local Stability Analysis

The local stability analysis of fixed points in two-dimensional nonlinear ordinary differential equations such as equations (1) and (2), it is based on approximating the nonlinear equation with linear equation (5) in the following below[9]–[11]:

$$\frac{dx}{dt} = f_1(x,u), \frac{du}{dt} = f_2(x,u) \tag{5}$$

Let is assumed that there is a fixed point (x^*, u^*) for which $f_1(x^*, u^*) = f_2(x^*, u^*) = 0$, occurs on the plant. The linear analysis involves to carry out a Taylor expansion of the nonlinear functions in $f_1(x,u)$ and $f_2(x,u)$ in the neighborhood of form $f(x^*, u^*)$, as can be seen in the following below:

$$f(x,u) = f(x^*,u^*) + \frac{\partial f}{\partial x}|_{(x^*,u^*)}(x-x^*) + \frac{\partial f}{\partial u}|_{(x^*,u^*)}(u-u^*) + \dots \tag{6}$$

Based on equation (6), it can be defined:

$$X = x - x^*, U = u - u^* \tag{7}$$

Then it can expand equation (6) become,

$$\frac{dX}{dt} = A\,X + B\,U + \cdots\ , \quad \frac{dU}{dt} = C\,X + D\,U + \cdots \tag{8}$$

The equation (8) needs two functions f_1 and f_2 related to equations (1) and (2) in the form of equation (6), then it found:

$$A = \frac{df_1}{dx}|_{(x^*,u^*)}, B = \frac{df_1}{dy}|_{(x^*,u^*)}, C = \frac{df_2}{dx}|_{(x^*,u^*)},$$
$$D = \frac{df_2}{dy}|_{(x^*,u^*)} \tag{9}$$

In the first order of differential equation, the nonlinear form of equation (8) can be approximated by a linear equation (10),

$$\frac{dX}{dt} = A\,X + B\,Y\ , \quad \frac{dU}{dt} = C\,X + D\,Y \tag{10}$$

The poles of the linear approximation in equation (10) will be found with the formula below[5]:

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \tag{11}$$

It found the poles μ_1 and μ_2 in equation (11), these are,

$$|\mu I - M| = \begin{vmatrix} \mu - A & -B \\ -C & \mu - D \end{vmatrix} = 0 \tag{12}$$

After taking determine of equation (12), both of the poles have equation as can be seen in the following below:

$$\mu_1 = \frac{A+D}{2} + \frac{\sqrt{(A-D)^2+4\,BC}}{2}$$
$$\mu_2 = \frac{A+D}{2} - \frac{\sqrt{(A-D)^2+4\,BC}}{2} \tag{13}$$

If the result of equation (13) has the real part with $\mu_1 < 0$ and $\mu_2 < 0$, the system is called stable and the responses of the system has steady-state value.

3. The Special Pole Placement

Based on the dynamics of nonlinear plant equation (1) and (2), it needs equation (9) and (10) to generate the linear form of nonlinear plant. Furthermore, it found the dynamics of nonlinear plant has changed into the linear equation, and the process of pole placement is explained with some steps in the following below. First, the four states have defined in equation (4), those are:

$$x_1 = \theta, x_2 = \dot{\theta}, x_3 = x, x_4 = \dot{x} \tag{14}$$

Second, refers to equation (6), it needs to make both equations (1) and (2) become f_1 and f_2. By using equations (9) and (10), it found all parameters for equation (11) as shown below:

$$A_{11} = \frac{\partial f_1}{\partial x_1}|_{(x_1,x_2,x_3,x_4,F)}, A_{12} = \frac{\partial f_1}{\partial x_2}|_{(x_1,x_2,x_3,x_4,F)},$$

$$A_{13} = \frac{\partial f_1}{\partial x_3}|_{(x_1,x_2,x_3,x_4,F)}, A_{14} = \frac{\partial f_1}{\partial x_4}|_{(x_1,x_2,x_3,x_4,F)},$$
$$A_{21} = \frac{\partial f_2}{\partial x_1}|_{(x_1,x_2,x_3,x_4,F)}, A_{22} = \frac{\partial f_2}{\partial x_2}|_{(x_1,x_2,x_3,x_4,F)}$$
$$A_{23} = \frac{\partial f_2}{\partial x_3}|_{(x_1,x_2,x_3,x_4,F)}, A_{24} = \frac{\partial f_2}{\partial x_4}|_{(x_1,x_2,x_3,x_4,F)},$$
$$C_{11} = \frac{\partial f_3}{\partial x_1}|_{(x_1,x_2,x_3,x_4,F)}, C_{12} = \frac{\partial f_3}{\partial x_2}|_{(x_1,x_2,x_3,x_4,F)},$$
$$C_{13} = \frac{\partial f_3}{\partial x_3}|_{(x_1,x_2,x_3,x_4,F)}, C_{14} = \frac{\partial f_3}{\partial x_4}|_{(x_1,x_2,x_3,x_4,F)},$$
$$C_{21} = \frac{\partial f_4}{\partial x_1}|_{(x_1,x_2,x_3,x_4,F)}, \ C_{22} = \frac{\partial f_4}{\partial x_2}|_{(x_1,x_2,x_3,x_4,F)}$$
$$C_{23} = \frac{\partial f_4}{\partial x_3}|_{(x_1,x_2,x_3,x_4,F)}, C_{24} = \frac{\partial f_4}{\partial x_4}|_{(x_1,x_2,x_3,x_4,F)}$$
$$B_1 = \frac{\partial f_1}{\partial F}|_{(x_1,x_2,x_3,x_4,F)}, B_2 = \frac{\partial f_2}{\partial F}|_{(x_1,x_2,x_3,x_4,F)}$$
$$D_1 = \frac{\partial f_1}{\partial F}|_{(x_1,x_2,x_3,x_4,F)}, D_2 = \frac{\partial f_2}{\partial F}|_{(x_1,x_2,x_3,x_4,F)} \quad (15)$$

Third, if equation (15) is applied to equation (10) then it found equation (16) in the following below:

$$\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\\ \dot{x_3}\\ \dot{x_4}\end{bmatrix} = \begin{bmatrix}A_{11} & A_{12} & A_{13} & A_{14}\\ A_{21} & A_{22} & A_{23} & A_{24}\\ C_{11} & C_{12} & C_{13} & C_{14}\\ C_{21} & C_{22} & C_{23} & C_{24}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}B_1\\ B_2\\ D_1\\ D_2\end{bmatrix}F \quad (16)$$

To stabilize the system equation (16) it needed the feedback $F(t)$ with equation (17) in the following below[5], [12]–[14]:

$$F(t) = v(t) - K\,x(t) \quad (17)$$

Where $K = [K_1\ K_2\ K_3\ K_4]$, and $v(t)$ is the reference input to the system

This process didn't use the Riccati's equation to design the optimal feedback, but it used special pole placement method and chooses four the stable desired poles.

Furthermore, equations (15) and (16) has been completed with the fixed point $x_1 = \pi/4$, $x_2 = 0$, $x_3 = 0$, and $x_4 = 0$, it found the parameters,

$$A_{11} = 0\ ,\ A_{12} = 1\ ,\ A_{13} = 0\ ,\ A_{14} = 0$$
$$A_{21} = \frac{W\,M_p\,g\,L\sin(\frac{\pi}{4})}{W\,P - Q^2\,(\cos(\frac{\pi}{4}))^2}\ , A_{22} = 0\ \ , A_{23} = 0\ ,\ A_{24} = 0$$
$$A_{11} = 0\ ,\ A_{12} = 0\ ,\ A_{13} = 0\ ,\ A_{14} = 1$$
$$A_{41} = \frac{-M_p\,g\,L\,Q\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right)}{W\,P - Q^2\,(\cos(\frac{\pi}{4}))^2}\ , A_{42} = 0\ \ , A_{43} = 0\ ,\ A_{44} = 0\ B_1 =$$
$$0, B_2 = \frac{W + Q\cos(\frac{\pi}{4})/r}{W\,P - Q^2(\cos(\frac{\pi}{4}))^2},\ B_3 = 0, B_4 = \frac{P/r + Q\cos(\frac{\pi}{4})}{W\,P - Q^2(\cos(\frac{\pi}{4}))^2} \quad (18)$$

Now for the linearization system, equation (18) to equation (16) can be applied as written below:

$$\dot{x} = \begin{bmatrix}0 & 1 & 0 & 0\\ A_{21} & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ A_{41} & 0 & 0 & 0\end{bmatrix}x + \begin{bmatrix}0\\ B_2\\ 0\\ B_4\end{bmatrix}F, x = \begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} \quad (19)$$

Fourth, the next step was to perform the controller needed to stabilize the system. Define the desired closed loop system with four poles; those are μ_1, μ_2, μ_3 and μ_4. The characteristic equations to all poles are given in equation (20).

$$L(\mu) = (\mu + \mu_1)\,(\mu + \mu_2)\,(\mu + \mu_3)\,(\mu + \mu_4) \quad (20)$$

If the controller in equation (17) is substituted in equation (19), the closed loop equation can be found as below:

$$\dot{x} = \begin{bmatrix}0 & 1 & 0 & 0\\ A_{21} & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ A_{41} & 0 & 0 & 0\end{bmatrix}x + \begin{bmatrix}0\\ B_2\\ 0\\ B_4\end{bmatrix}[v - K_1\,x_1 - K_2\,x_2 - K_3\,x_3 - K_4\,x_4] \quad (21)$$

Then the equation of closed loop system can be written down with equation (22) below:

$$\dot{x} = \begin{bmatrix}0 & 1 & 0 & 0\\ A_{21} - B_2\,K_1 & -B_2\,K_2 & -B_2\,K_3 & -B_2\,K_4\\ 0 & 0 & 0 & 1\\ A_{41} - B_4\,K_1 & -B_4\,K_2 & -B_4K_3 & -B_4\,K_4\end{bmatrix}x + \begin{bmatrix}0\\ B_2\\ 0\\ B_4\end{bmatrix}r \quad (22)$$

In general form, the equation (22) can be written with equation (23) below:

$$\dot{x} = A_{CL}\,x + B_{CL}\,r \quad (23)$$

Where,

$$A_{CL} = \begin{bmatrix}0 & 1 & 0 & 0\\ A_{21} - B_2\,K_1 & -B_2\,K_2 & -B_2\,K_3 & -B_2\,K_4\\ 0 & 0 & 0 & 1\\ A_{41} - B_4\,K_1 & -B_4\,K_2 & -B_4\,K_3 & -B_4\,K_4\end{bmatrix},$$
$$B_{CL} = \begin{bmatrix}0\\ B_2\\ 0\\ B_4\end{bmatrix} \quad (24)$$

Finally the value for the special pole placement for state feedback should be found as $K = [K_1\ K_2\ K_3\ K_4]$ with the procedure below:

First, it needs to elaborate the equation (20) to find the characteristic equation below:

$$L(\mu) = \mu^4 + m_3\,\mu^3 + m_2\,\mu^2 + m_1\,\mu + m_0 \quad (25)$$

Where,

$m_3 = \mu_1 + \mu_2 + \mu_3 + \mu_4$
$m_2 = \mu_1\,\mu_2 + \mu_1\,\mu_3 + \mu_1\,\mu_4 + \mu_2\,\mu_3 + \mu_2\,\mu_4 + \mu_3\,\mu_4$
$m_1 = \mu_1\,\mu_2\,\mu_3 + \mu_1\,\mu_2\,\mu_4 + \mu_1\,\mu_3\,\mu_4 + \mu_2\,\mu_3\,\mu_4$
$m_0 = \mu_1\,\mu_2\,\mu_3\,\mu_4$

Next, the characteristic equation of the closed loop system in the equation (24) was found with equation below:

$$|\,\mu\,I - A_{CL}\,| = 0 \quad (26)$$

The determinant of equation (26) gave the characteristic equation was shown in equation (27) below:

$$L_{CL}(\mu) = \mu^4 + [B_4\,K_4 + B_2\,K_2]\,\mu^3 + [B_4\,K_3 + B_2\,K_1 - A_{21}]\,\mu^2 + [A_{41}\,B_2 - A_{21}\,B_4]\,K_4\,\mu + [A_{41}\,B_2 - A_{21}\,B_4]\,K_3 \quad (27)$$

To find the special feedback $K = [K_1\ K_2\ K_3\ K_4]$, the equation (27) must be equal to equation (25). If both equations are equal, it found these four equations below:

$$\begin{aligned} B_4\,K_4 + B_2\,K_2 &= m_3\\ B_4\,K_3 + B_2\,K_1 - A_{21} &= m_2\\ [A_{41}\,B_2 - A_{21}\,B_4]\,K_4 &= m_1\\ [A_{41}\,B_2 - A_{21}\,B_4]\,K_3 &= m_0 \end{aligned} \quad (28)$$

By using the analytical process in equation (28), finally the equation to obtain the value of K by the four equations was obtained below:

$$K_3 = \frac{m_0}{A_{41}\,B_2 - A_{21}\,B_4}$$

$$K_4 = \frac{m_1}{A_{41}\,B_2 - A_{21}\,B_4}$$

$$K_2 = \frac{m_3 - B_4\,K_4}{B_2}$$

$$K_1 = \frac{m_2 + A_{21} - B_4\,K_3}{B_2} \quad (29)$$

4. Simulation

Using the parameters of the open loop system in equations (18) and (19) it found the state-space equation below:

$$\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\\ \dot{x_3}\\ \dot{x_4}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 6.2843 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ -0.1200 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ 1\\ 0\\ 1\end{bmatrix}F \quad (30)$$

It needs to choose four the desired poles refer to the equation (25) with the value in the following below:

$[\mu_1\ \mu_2\ \mu_3\ \mu_4] = [-3\ \ -3\ \ -3\ \ -3]$

By using equations (29) and (25) it found the value of state feedback in the following below:

$$K = [-170.2331 - 68.5773 - 99.3821 - 94.4130] \quad (31)$$

The state feedback can be written as in the following below:

$$F(t) = v(t) - [k_1 \quad k_2 \quad k_3 \quad k_4]\,[x_1 \quad x_2 \quad x_3 \quad x_4]^T \quad (32)$$

By inserting *F*(t) from the equation (32) to equation (30), the closed loop system has the result in the following below:

$$\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\\ \dot{x_3}\\ \dot{x_4}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 6.2843 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ -0.1200 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} - \begin{bmatrix}0\\ 1\\ 0\\ 1\end{bmatrix}[k_1\ \ k_2\ \ k_3\ k_4]\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ 1\\ 0\\ 1\end{bmatrix}v(t) \quad (33)$$

Finally, with the input *v*(t) = 0, the equation (33) can be applied to the nonlinear system in equations (1) and (2) as in the following below:

$$\ddot{\theta}(t) = \frac{WM_pgL\sin\theta - QM_pL\,\dot{\theta}^2\sin\theta\cos\theta - \left(W + \frac{Q\cos\theta}{r}\right)\left[-k_1\,\theta - k_1\dot{\theta} - k_3\,x - k_4\,\dot{x}\right]}{WP - Q^2\cos\theta^2} \quad (34)$$

$$\ddot{x}(t) = \frac{PM_pL\dot{\theta}^2\sin\theta - QM_pgL\sin\theta\cos\theta + \left(\frac{P}{r} + Q\cos\theta\right)\left[-k_1\,\theta - k_1\dot{\theta} - k_3\,x - k_4\,\dot{x}\right]}{WP - Q^2\cos\theta^2} \quad (35)$$

To build the simulation using simulink diagram for the system with equations (34) and (35), it performed four sub-systems, as can be seen in Fig. 5.

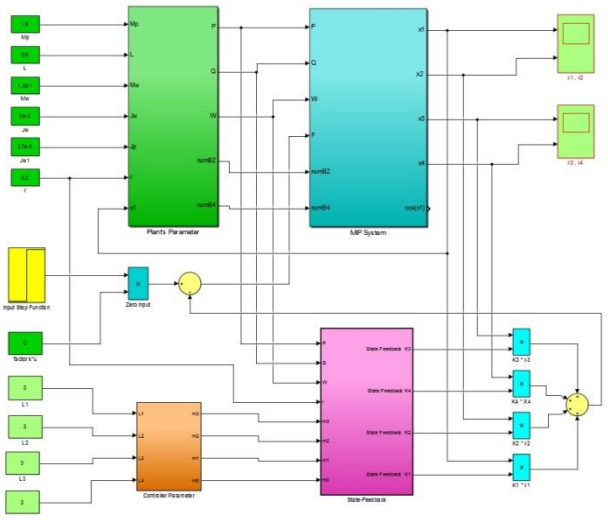


Fig 5. The Simulation of Closed Loop System

The sub-system consists of the plant's parameter, the nonlinear plant, the controller parameters and the state feedback as shown in Figs. 6 to 9.

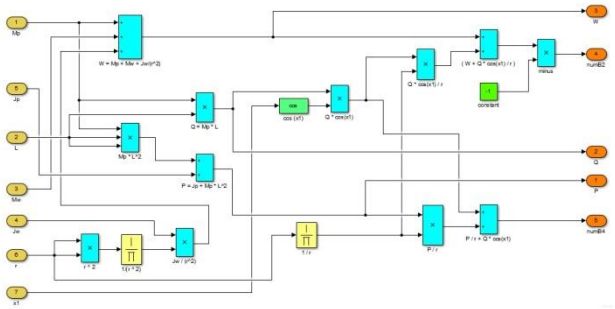


Fig. 6. The Plant's Parameter of Fig. 5

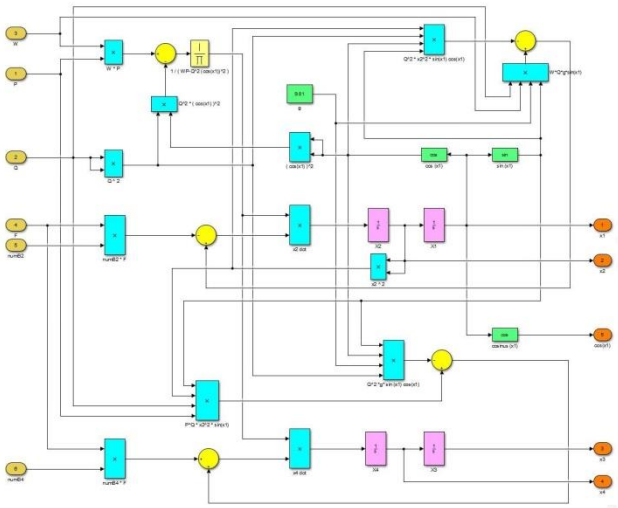


Fig. 7. The Nonlinear Plant of Fig. 5

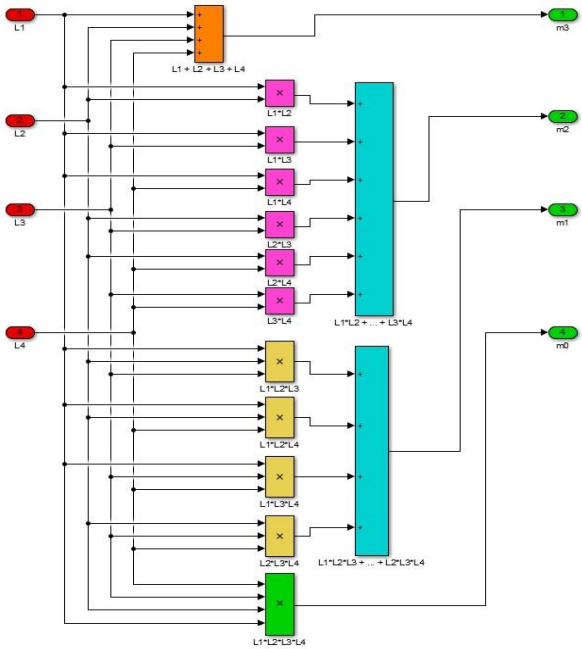


Fig. 8. The Controller Parameters of Fig. 5

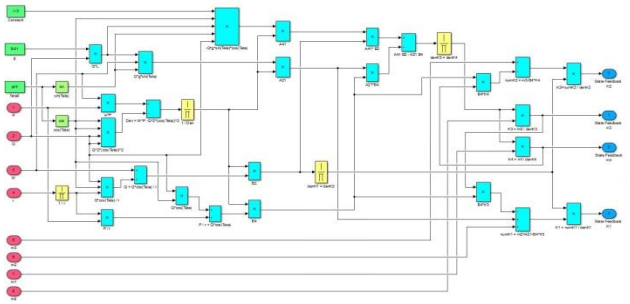


Fig. 9. The State Feedback of Fig. 5

There are two figures that captured by scope in the simulation, those are x_1, x_2 as can be seen in Fig. 10 and x_3, x_4 as can be seen in Fig. 11.

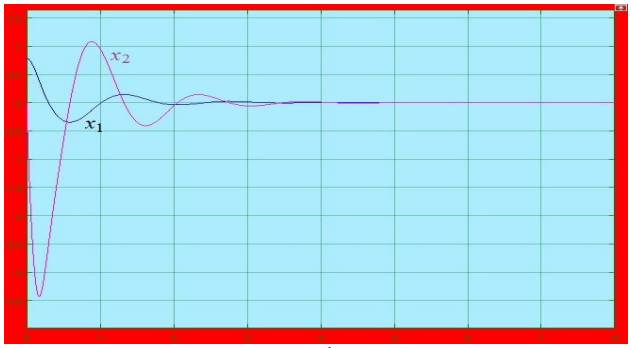


Fig 10. Response of $\theta(t)$ and $\dot{\theta}(t)$ on Closed Loop System

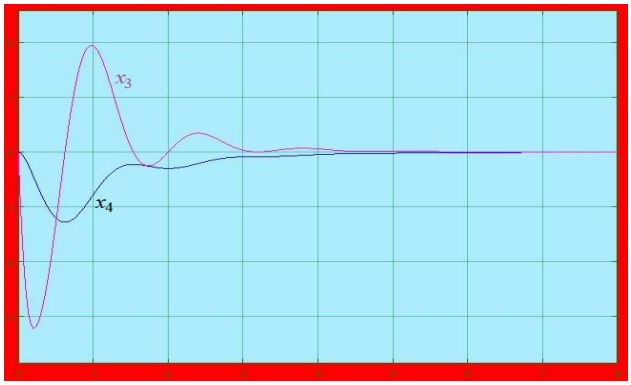


Fig 11. Response of $x(t)$ $\dot{x}(t)$ on Closed Loop System

Based on the results in Fig. 10 and 11, all the responses of the nonlinear plant will run toward stable with oscillation damped until the responses toward zero, and all the responses are eligible.

5. Conclusion

Based on the result, it can be described that responses of linearization of the nonlinear plant has been adequate. Then the controller has been produced for the nonlinear plant by using the local stability analysis. The process needed the special feedback pole placement method like equation (21) to (32) that should be applied at simulink process. Lastly, the controller has been simulated by using simulink techniques. All the behavior of the responses was stable revering to the poles given. The results of these responses on the Fig. 10 and 11 have been taken after choosing the best poles by the trial and error process..

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