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Research paper

A New 3-D Chaotic System with Conch-Shaped Equilibrium Curve and its Circuit Implementation

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Abstract

This paper reports the finding a new chaotic system with a conch-shaped equilibrium curve. The proposed system is a new addition to existing chaotic systems with closed curves of equilibrium points in the literature. Lyapunov exponents of the new chaotic system are studied for verifying chaos properties and phase portraits of the new system via MATLAB are unveiled. An electronic circuit simulation of the new chaotic system with conch-shaped equilibrium curve is shown using MultiSIM to check the model feasibility.

Keywords: *Chaos, chaotic systems,circuit simulation, hidden attractors, Lyapunov exponents*

1. Introduction

Chaos theory has applications in several branches such as Tokamak system [1], economy [2], ecology [3], random bit generators [4], chemical reactions ([5]-[9]), robotics ([10]-[11]), weather systems ([12]-[14]), oscillators ([15]-[24]), text encryption [25], image encryption [26], voice encryption [27], and secure communication systems ([29]-[31]).

According to a new classification has been made for chaotic attractors ([32]-[34]), chaotic systems with no equilibrium points ([35]-[38]), with infinite number of equilibrium points ([39]-[40]) and with stable equilibrium points ([41]-[42]) are designated as chaotic systems with hidden attractors. Chaotic studies with open curves of equilibrium points such as line equilibrium ([43]-[44]) and closed curves of equilibrium points are well-studied in the literature ([45]-[48]). Chaotic systems with hidden attractors have many applications in science and engineering ([49]-[52]).

In Section 2, we provide details of the new chaotic system with conch-shaped equilibrium curve. Lyapunov exponents, Kaplan-Yorke dimension and phase plots via MATLAB are provided for the new chaotic system. In Section 3, using MultiSIM, we design the electronic circuit of the new system and verify the feasibility of the new chaotic oscillator.

2. A new chaotic system with conch-shaped equilibrium curve

We report a new three-dimensional dynamical system given by

$$\begin{cases} \dot{x} &= z \\ \dot{y} &= -z(ay+bxz) \\ \dot{z} &= -x^2+8|y|-8x^3+x^4 \end{cases} \tag{1}$$

which has a total of six nonlinearities in the dynamics.
We show that the system (1) is *chaotic* for the parameter values $(a,b)=(1,0.05)$.
For numerical simulations of phase portraits and for the calculation of Lyapunov chaos exponents, we take the initial values as $X(0)=(0.001,0.01,0.1)$ and parameter set as $(a,b)=(1,0.05)$.
The Lyapunov chaos exponents are determined as $(L_1,L_2,L_3)=(0.0034,0,-0.0045)$ using Wolf's algorithm [53]. Since $L_1>0$, the new system (1) is chaotic. By adding L_1,L_2 and L_3, we get the sum as -0.0011, which is negative. This shows that the new system (1) is dissipative. The Kaplan-Yorke dimension is determined as

$$D_{KY}=2+(L_1+L2)/|L_3|=2.7556 \tag{2}$$

which is a high value showing the complexity of the new system.
The equilibrium points of the new system (1) are tracked by solving the following system:

$$\begin{cases} z &= 0 \\ -z(ay+bxz) &= 0 \\ -x^2+8|y|-8x^3+x^4 &= 0 \end{cases} \tag{3}$$

Simplifying (3), we see that the equilibrium points of the system (1) are characterized by the two equations

$$z = 0 \text{ and } 8|y| = x^2(1 + 8x - x^2) \tag{4}$$

which is a conch-shaped curve in the (x, y)-plane as shown in Figure 1.

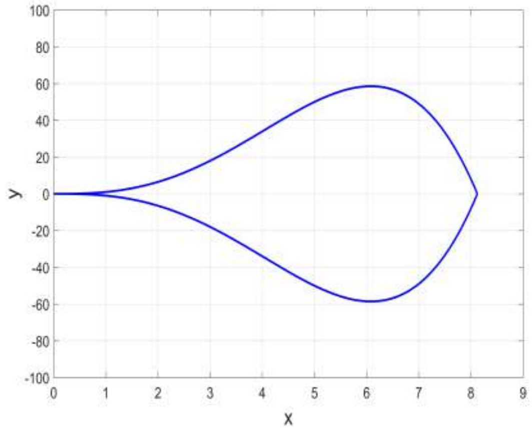

Figure 1: Conch-shaped curve of equilibrium points of the new system (1)

The phase portraits of the new chaotic system (1) with conch-shaped equilibrium curve are displayed in Figures 2-5.
The Lyapunov chaos exponents of the new chaotic system (1) are displayed in Figure 6.

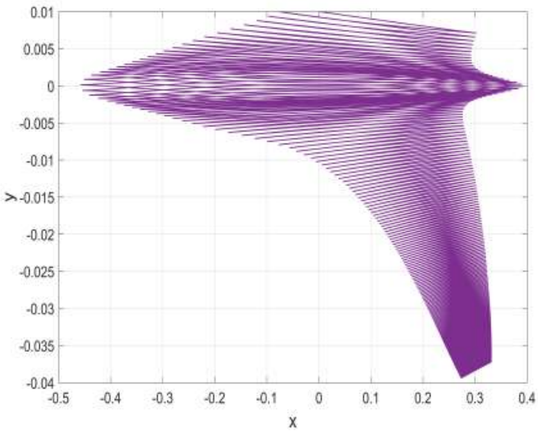

Figure 2: Numerical simulations of phase portraits of the new chaotic system (1) for $X(0) = (0.001, 0.01, 0.1)$ and $(a, b) = (1, 0.05)$ in $x - y$ plane

3. Circuit Simulation

This work describes the design and implementation of an circuit electronic of system (1). Circuit design of chaotic systems is an interesting application for practical implementation ([54]-[56]). Circuit implementation of the system (1) is shown in Fig. 7. The circuit is designed with parameters $(a, b) = (1, 0.05)$. The circuit consists of three capacitors (C_1, C_2, C_3), 18 resistors $(R_1, \ldots, R_{18})$, 8 op-amp (TL082CD), and 6 multipliers (AD633JN) and 2 diodes (1N4148). The diodes (1N4148) are used to realise absolute nonlinear terms. In this study, a linear scaling is considered as follows:

$$\begin{cases} \dot{x} &= z \\ \dot{y} &= -z(2ay + 2bxz) \\ \dot{z} &= -2x^2 + 16|y| - 32x^3 + 8x^4 \end{cases} \tag{5}$$

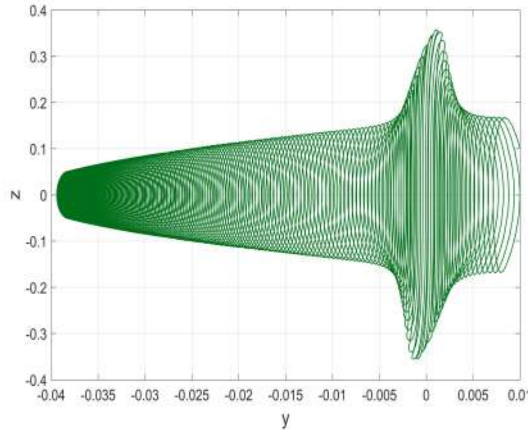

Figure 3: Numerical simulations of phase portraits of the new chaotic system (1) for $X(0) = (0.001, 0.01, 0.1)$ and $(a, b) = (1, 0.05)$ in $y - z$ plane

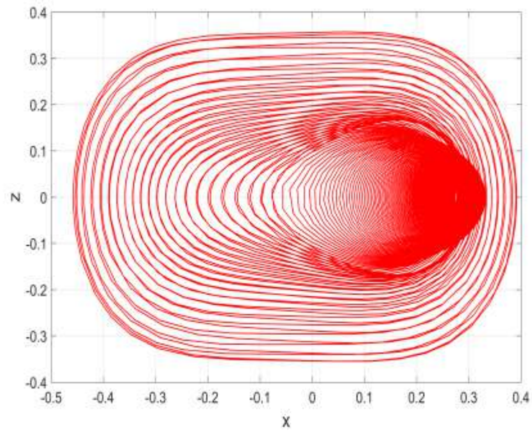

Figure 4: Numerical simulations of phase portraits of the new chaotic system (1) for $X(0) = (0.001, 0.01, 0.1)$ and $(a, b) = (1, 0.05)$ in $x - z$ plane

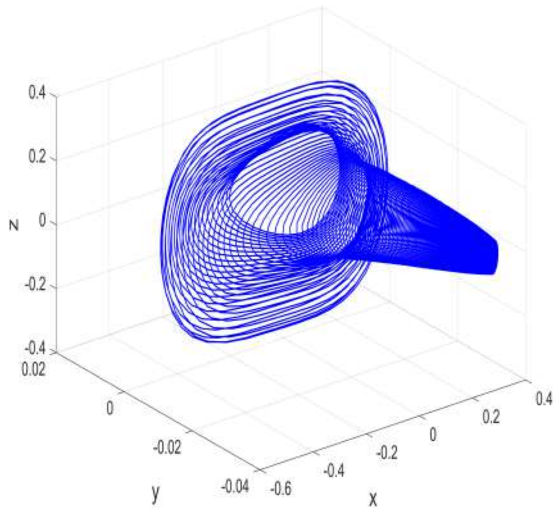

Figure 5: Numerical simulations of phase portraits of the new chaotic system (1) for $X(0) = (0.001, 0.01, 0.1)$ and $(a, b) = (1, 0.05)$ in $\mathbf{R}^3$

With the use of Kirchhoff's circuit laws, we derive the mathematical model of the new chaotic system (1) with conch-shaped equilibrium

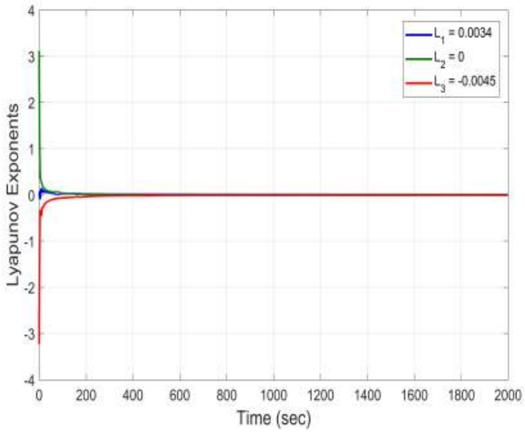

Figure 6: Lyapunov chaos exponents of the new chaotic system (1) for $X(0) = (0.001, 0.01, 0.1)$ and $(a, b) = (1, 0.05)$

curve , which is described by the following equations:

$$\begin{aligned} \dot{x} &= \tfrac{1}{C_1R_1}z \\ \dot{y} &= -\tfrac{1}{C_2R_2}yz - \tfrac{1}{C_2R_3}xz^2 \\ \dot{z} &= -\tfrac{1}{C_3R_4}x^2 + \tfrac{1}{C_3R_5}|y| - \tfrac{1}{C_3R_6}x^3 + \tfrac{1}{C_3R_7}x^4 \end{aligned} \quad (6)$$

where x, y and z correspond to the voltages on the integrators (U1A–U3A), respectively, while the power supply is $\pm$ 15 V. The values of circuit components are $R_1 = 400$ kΩ, $R_2 = R_4 = 200$ kΩ, $R_3 = 4$ MΩ, $R_5 = 25$ kΩ, $R_6 = 12.5$ kΩ, $R_7 = 50$ kΩ, $R_8 = R_9 = R_{10} = R_{11} = R_{12} = R_{13} = R_{14} = R_{15} = R_{16} = R_{17} = R_{18} = 100$ kΩ and $C_1 = C_2 = C_3 = 1$ nF. The MultiSIM projections of chaotic attractors in different planes are reported in Fig. Fig. 8. It is seen from Fig. Fig. 8 that the attractor plots of the new chaotic system (1) obtained using circuit implementation matches with the MATLAB simulation results(Fig. 2-4).

4. Conclusion

In this research paper, we found a new chaotic system with a conch-shaped equilibrium curve. The proposed system is a new addition to existing chaotic systems with closed curves of equilibrium points in the literature. Lyapunov exponents of the new chaotic system were investigated for verifying chaos properties and phase portraits of the new system via MATLAB are unveiled. An electronic circuit simulation of the new chaotic system with conch-shaped equilibrium curve was carried out using MultiSIM to check the model feasibility. As future research work, application of the new chaotic system for encryption and secure communication systems can be investigated.

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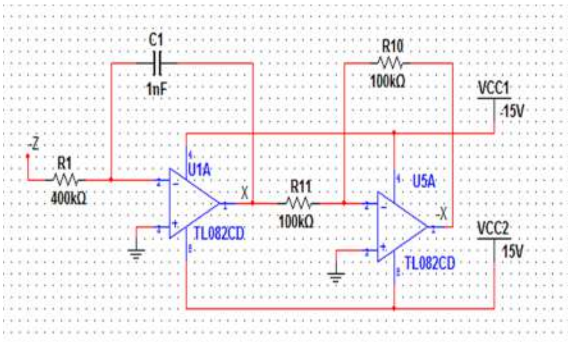

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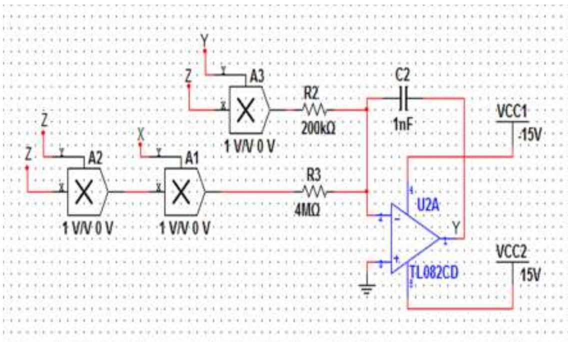

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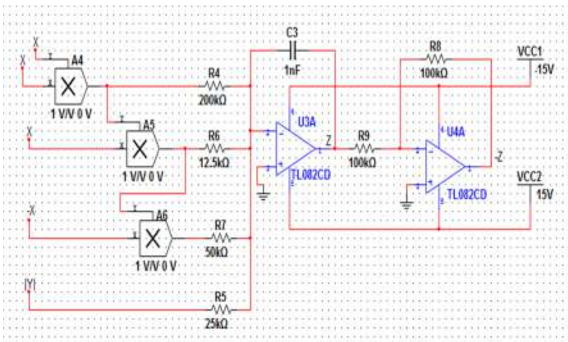

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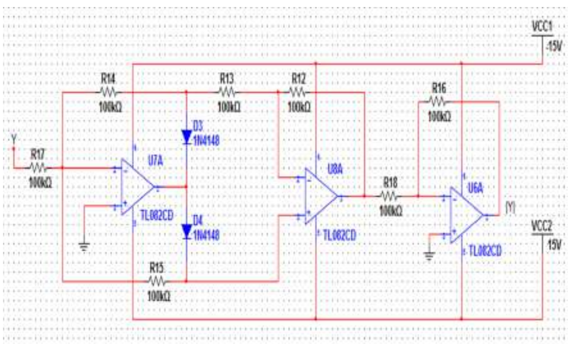

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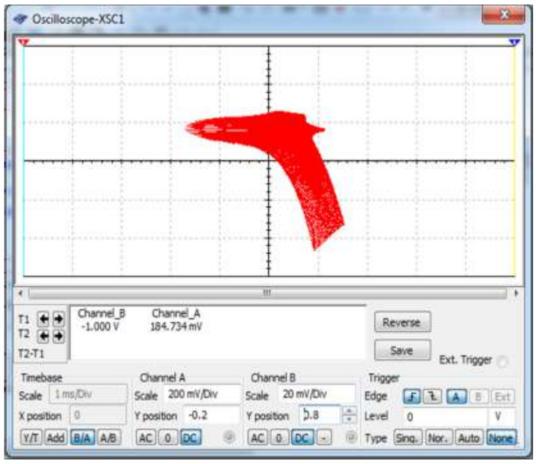
Figure 7: Circuit design of the new chaotic system (1) with conch-shaped equilibrium curve (a) x signal, (b) y signal, (c) z signal, and (d) $|y|$ signal



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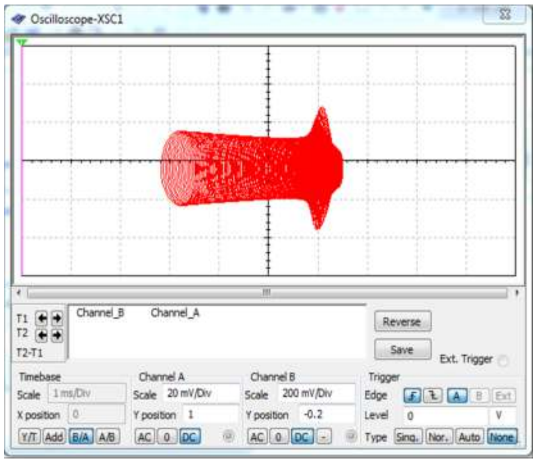

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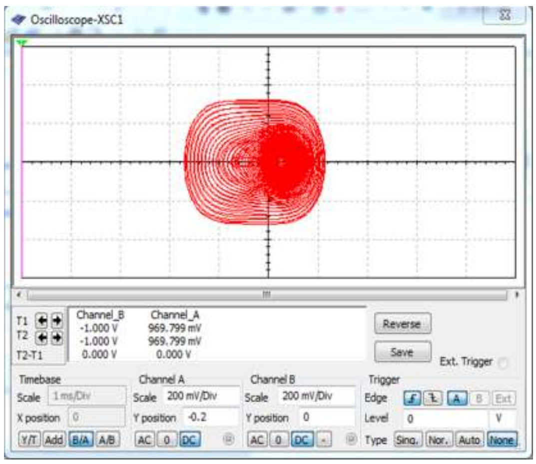

(c)

Figure 8: Chaotic attractors of the new chaotic system (1) with conch-shaped equilibrium curve using Multisim circuit simulation: (a) $x-y$ plane, (b) $y-z$ plane, and (c) $x-z$ plane.