



# Effect of pressure changes in sliding contact

Vera Deeva<sup>1\*</sup>, Stepan Slobodyan<sup>2</sup>

<sup>1</sup>Tomsk Polytechnic University, Tomsk, Russia

<sup>2</sup>Omsk State Technical University, Omsk, Russia

\*Corresponding author: [veradee@mail.ru](mailto:veradee@mail.ru)

## Abstract

The sliding contact when the air together with wear particles flow in contact area between commutator and brush is considered. The dynamical interaction between two surfaces is probabilistic. The behaviour of space-time-varying process is described by the differential equations, which are generally very difficult to solve. The simple numerical solution applying the method of Galerkin approximation to estimate the change in the pressure field in thin contact layer is obtained. It was found that under the leading edge of the brush the pressure change doesn't exceed 0.07 of the maximum value. The numerical simulations of the absolute error are presented for the 0.1, 0.2, 0.5, and 1 of the relative length. The relative error of pressure changes for small contact area is smaller ( $1 - 0.8e^{-0.1\tau}$ ). It is concluded that the approximate solution tends to the exact one. Moreover, it is shown that as the sliding velocity decreases, the relative error of the pressure change tends to the zero.

**Keywords:** approximate error; numerical simulation; tribological process; debris.

## 1. Introduction

The numerical simulation is performed to investigate the mechanical interaction between the sliding surfaces of different material types particularly in relation to tribological applications. A space-time-varying behaviour is the distinctive feature of sliding interaction [1–4]. The dynamical interaction between moving bodies is characterized by an abrasive effect on the surfaces and consequently wear and the change in the energy [5–7]. A part of the kinetic energy to overcome the friction in the contact area is converted to thermal energy [8,9] through the transfer of heat to the surroundings. Another part of the energy is dissipated [1,10] as heat of moving masses within the contact area, inducing the change of the boundary conditions. As a result of these two opposite processes the properties of the surfaces change the temperature fields and surface geometry, which leads to the generating wear particles [11–14]. Hence, the changes in the physics of the interaction should be taken into account to evaluate the 'third body' characteristics.

Since the nature of contact interaction is probabilistic [15–17], it is clear that differences in the structure and chemical composition of materials lead to variable responses during sliding. This means that geometry, physical properties, and wear behaviour of this dynamic system are the probability values. The pressure and temperature play an important role among these factors [18–21]. It arises from the media properties which depend on the velocity of the particle in the surrounding (gas or lubricant) and therefore the dissipation of heat away from the area [22–24].

It is obvious, the 'third body' has a specific feature as a lubricant during the sliding interaction [25–29]. The surrounding, material structure, and layer properties impact either high or low degree of lubrication and therefore the type of friction between bodies that change abruptly from sliding friction to rolling friction [30,31]. During such dramatic change, the energy dissipation values fall

within regions of interaction together with the temperature in contact area tends to move to the surrounding temperature. It is well known that the coefficient of rolling friction is much lower than that on a sliding one for the same material. The average time interval of changing friction from sliding to rolling is affected by average wear rate the surface composites [32–34] that is, the higher the average wear rate of the material, the longer the time of crossing from one type of friction to another under the same conditions.

We consider the abundance of wear particles within the air as the porous plug, and therefore the process of air flow together with the debris can be described as throttling process [35] which could lead to either an increase or a decrease in sliding contact temperature. On the other hand, the sliding contact layer is dielectric medium, thus it influences the transmission of electrical energy for example, in commutator-and-brush assembly of electrical machine and other electromechanical devices, in a particular case where the oxide layer would find.

It is obvious that the occurrence of the 'third body' should be connected predominantly with the actual surrounding for a realistic case, and should be accounted in a tribological contact model to represent the interaction properties. In the present analysis, we investigate the characteristics of the sliding layer formed by wear particles and gas, using mathematical modeling to improve our understanding of the change in the pressure in the contact area between commutator and brush.

The motion of wear particle and air flow in the sliding contact [36–44] is the process of major importance in modern technologies, however it has not been studied well enough. Such fact could be explained by the difficulty of observation, as demonstrated in contact mechanics measures when the probe slides across the wear track, or in commutator and brush assembly [42–46] when the brush slides on a slip ring (collector) of electrical machines. Dynamic motion and random surface interaction with the air bounda-

ry layer remain largely uninvestigated due to the dynamic complexity and the difficulty for making precise measurements. Thus, the numerical methods are the best way to study the behaviour of the surface contact system [47–50]. Thuswise, we have to apply the semi-empirical theories of turbulence (by Prandtl, Karman, Taylor, Poiseuille and others) [51,52] based on the closed system of equations [53] describing the flow and the isobaric boundary layer flow to investigate the interaction of the surfaces.

The airflow in the contact area (CA) contains the wear particle detachment from the surfaces. The literature review on aerodynamics and gas dynamics [54,55] confirms that the approach to study the motion of the particles in contact area as a continuum is justified [56–58]. Moreover, this approach can be considered to be ‘solid’ and classical one [59,60]. We can take the average characteristics of the particle motion in the air as the equivalent of the medium in differential equations, making allowance for both the physical properties of air and the wear particles. Therefore, we could consider the average velocity of the air boundary layer equals to the velocity of the entrained wear particles. Otherwise the random behaviour of the flow components, as well as the laws of distribution of their parameters, are different.

## 2. Contact pressure distribution during sliding interaction

Due to the non-stationary nature of the contact area, it is difficult to estimate the state of a stochastic process being the random of interaction between two surfaces. Any parameter of the process measured from a contact at different times will not have a unique value, due to a stochasticity arises because random surface roughness makes any tribological interaction stochastic. In the case of complexity measures, the approximate methods [49,61–65], are more frequently applied to such problems, in particular, the method of Bubnov-Galerkin [61,66–68] from continuum mechanics. In this study, we apply the method of Bubnov-Galerkin to investigate the tribological properties of the sliding contact in the commutator and brush assembly of the electric machine.

The theory of approximate methods is based on the functional analysis in applying numerical mathematics to solve different problems [49,61–65]. Setting the approximate methods serves to determine the solvability of the approximate equation [  $\tilde{A}\tilde{x} = \tilde{y}$  ] on the exact equation  $Ax=y$ , where  $A$  is a mathematical operator;  $y$  is a given data and  $x$  is a dependent variable, and the closeness between the true and approximation solution. Furthermore, it is possible to formulate the inverse problems, that is if there exist solutions to the approximate equation whether the exact equation could be found and then measured, and the convergence of the approximate solution is defined. From this finding, we can estimate the closeness between the exact and approximate solution, degree of convergence and obtain the efficient estimator of the approximate error.

Let the length of brush be  $l$  starting on  $x=0$  and the pressure at the start point be  $p_b$ , and at the end of the brush be  $p_k$ , therefore the pressure  $p(x,t)$  that occurs into the contact area changes as relative function:  $p_{rel}(x,t)=[p(x,t)-p_b]/(p_k-p_b)$ .

In order to estimate the pressure changes, we need to solve the differential equation  $\frac{\partial^2 p}{\partial x^2} = \frac{\partial p}{\partial t}$  under conditions  $p_{rel}|_{x=l}=p_{rel}|_{\tau=0}=0$ , and  $p_{rel}|_{\tilde{x}=0}=1$ . It is convenient to introduce non-dimensional parameters: the  $\tilde{x}=x/l$  is a relative longitudinal coordinate in the interval  $[0, 1]$ , and the  $\tau$  is a relative time.

### 2.1. Contact pressure distribution

If we neglect the motion inertia in sliding contact [54], the pressure field in the horizontal contact area can be defined by differential equation of second order

$$\frac{\partial^2 p}{\partial x^2} = k \frac{\partial p}{\partial t} + f(x,t) \quad (1)$$

with  $k$  a constant and  $f(x, t)$  a known function. Thus, the solution could be found subject to the initial and boundary conditions.

With respect to the contact area of the brush length of  $l [ 0, l ]$ , an initial condition is  $p(x,t)|_{t=0} = \phi(x)$  and boundary conditions are the following:

$$\begin{cases} f_0(p) = a_1 p(0,t) + a_2 \frac{\partial p(0,t)}{\partial x} = a_1 p_b + a_2 q_b = A \\ f_l(p) = b_1 p(l,t) + b_2 \frac{\partial p(l,t)}{\partial x} = b_1 p_k + b_2 q_k = B \end{cases} \quad (2)$$

where  $p_b, p_k$  are the pressure at the start point 0 and the end point  $l$  respectively;  $q_b, q_k$  are the rate of change of pressure at the start point 0 and the end point  $l$  respectively; the coefficients  $a_1^2 + a_2^2 \neq 0$ ;  $b_1^2 + b_2^2 \neq 0$ ; and  $p(x,0) = f(x)$ .

It is apparent that we can use previously defined formalization for easy calculation. Assuming that  $a_2=b_2=0$ , we obtain the flow profile entrained air in the area formed under the leading edge of the brush and the commutator for the given pressure  $p_b$ , and exhaust air in the area formed under the trailing edge of the brush and the commutator for the given pressure  $p_k$ . Then assuming  $a_1=b_1=0$ , we obtain the flow profile entrained air in the area formed under the leading edge of the brush and the commutator for the given rate of pressure change  $q_b$ , and the exhaust air in the area formed under the trailing edge of the brush and the commutator for the given rate of pressure change  $q_k$ .

So, the solution of the equation (1) corresponding to the initial and boundary conditions (2) is given by the infinite sum [52]:

$$p(x,t) = p_c(x,t) + \sum_{i=1}^{\infty} A_i(t) \sin(\omega t + \phi) \quad (3)$$

where  $p_c(x,t)$  is a solution component. Clearly, it would be hard to estimate the numerical value using (3). However, regardless of the calculation difficulties, it is necessary to do is to substitute  $n$  in the upper bounds for making partial sums, that is, sums of finitely many terms of the series, we obtain the approximation with good accuracy:

$$p_n(x,t) = p_c(x) + \sum_{i=1}^n B_i(t) p(x) \quad (4)$$

In formula (4) the functions  $p_c(x)$  and  $\phi(x)$  are prescribed by the boundary conditions, and in addition, the coefficients  $B_i(t)$  determine the accuracy to the analytical solution due to the infinite series. It follows that the sum can be made as large as we please by taking enough terms as a finite number. Since we can take a sufficient number of the terms ( $n = 1, 2, 3$ ) to get the desired estimates for the solution and to facilitate the calculation.

### 2.2. Estimation accuracy of approximation

The papers [61,66–68] confirm that the method of Bubnov-Galerkin allows obtaining a good approximation of the continuum behaviour. This helps to estimate numerical parameters of the motion. We demonstrate that the method could be applied to the motion of nanoparticles to estimate the tribological properties of the sliding contact and thereafter we will illustrate that our findings are corresponding to the results calculated analytically.

Now we employ method of Bubnov-Galerkin to find the function  $\bar{p}_{rel}(\tilde{x}, \tau)$ , that satisfies the boundary conditions (2) as follows:

$$p_{rel}(\bar{x}, \tau) = 1 - x + \sum_{i=1}^m \varphi_i(\tau) \bar{x}^i (1 - \bar{x}) \tag{5}$$

where  $\varphi_i(\tau)$  is a function satisfying a normalization condition

$$\int_0^1 I(\bar{x}, \varphi_i) \bar{x}^i (1 - \bar{x}) d\bar{x} = 0 ; i = 1, 2, \dots, m ; I(\bar{x}, \varphi_i) = \frac{\partial^2 p_{rel}}{\partial \bar{x}^2} - \frac{\partial p_{rel}}{\partial \tau}$$

Furthermore, let us take first term of the series in (5) on its own, that is  $m=1$ . Essentially, this is enough to solve the differential equation  $\dot{\varphi}_1(\tau) + 10\varphi_1(\tau) = 0$ , and to find the well-known solution

[53,61]:  $\varphi_1(\tau) = Ce^{-10\tau}$ . Hence, we can find  $2\varphi_1(0) = 5$  from

normalization condition  $\int_0^1 I[\bar{p}|_{\tau=0} - p|_{\tau=0}] \bar{x}^i (1 - \bar{x}) d\bar{x} = 0$ . Defini-

tely, a constant  $C$  is found from the initial condition  $\varphi_1(0)$ . Thus,  $C = -2.5$ .

We write the solution as  $\varphi_1(\tau) = 2.5e^{-10\tau}$ . Substituting this result into (5) and solving it for  $m = 1$  we get

$$p(x, t) = \bar{p}_{rel}(\bar{x}, \tau) = 1 - \bar{x} - 2.5\bar{x}(1 - \bar{x})e^{-10\tau} \tag{6}$$

For small contact area ( $\bar{x} \ll 1$ ) the function (6) is reduced to a simpler form  $p(x, t) \cong 1 - 2.5\bar{x}e^{-10\tau}$ . Hence, an absolute error of pressure changes estimated in the area formed under the leading edge of the brush and the commutator for  $x \ll 1$  is

$$\Delta p(x, t) = 2.5xe^{-10\tau} (1 - 0.8e^{0.1\tau}) \tag{7}$$

### 3. Discussion and analysis results

In order to investigate the change in the pressure in the sliding contact of modelling tribological mechanism, the adiabatic process during which the air together with wear particles flow in contact area between two moving bodies is considered. In this case the pressure gradient is low. The absolute error of pressure changes in the area formed under the leading edge of the brush describing by the Eq.7 is shown in Fig. 1. The results of the numerical simulations are presented for several cases of the relative length:  $\bar{x} = 1$ ,  $\bar{x} = 0.5$ ,  $\bar{x} = 0.2$ ,  $\bar{x} = 0.1$ . Certainly, any choice of the relative length of the contact leads to the same consequence, so one may conclude that all lines converge to 0 that means the change in pressure ( $\Delta p$ ) is almost unchanged and just pressure achieves a very close match to a stable state.

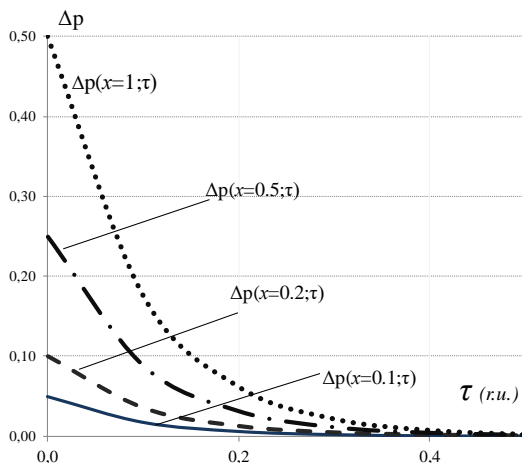


Fig. 1: The change in absolute error of the pressure changes with the time of an arbitrary-profile

Comparing the lines in Fig. 1 one can see that the relative length determines how fast the change in pressure diverges. It is evident that from the one-third relative length the pressure into the sliding contact area approximately equal to static pressure. Consequently, the latter half relative time the pressure of the debris together with air flow pressure tends to the outside environment. In addition, as the speed of sliding between bodies is reduced, so the pressure of the boundary flow can be reduced, furthermore the point where it reaches its steady-state position as comes to the inlet of the sliding contact region. This is strong proof that at sufficiently low sliding speed, the contact area is stable.

It should be noted that we consider the relative parameter  $\tau$ , this was achieved by assuming that  $\tau = c^2 t / 2al^2$ , where  $c$  is a speed of sound in sliding layer,  $a$  is a non-dimensional coefficient depends on the contact area geometry. Let us emphasize here that the sliding layer is a homogeneous two-phase medium and while the speed of sound in various materials is changed due to the different density caused by the contact area structure being the probability value changes the relative time. It is possible to consider the abundance of wear particles as the porous plug, no doubt to significant variations in the form, size and amount of the pores across the porous structure cause the velocity and mass transfer in the sliding contact. In the contact layer, the boundary flow is reduced, and the measure of the boundary layer drag is determined by the term ( $e^{-10\tau}$ ) in the Eq.7. Under this designation it was found that under the leading edge of the brush (no more than fifth part of the length of the contact area) the change in pressure ( $\Delta p$ ) did not exceed 0.07 of the maximum value and hence, the air drag hardly influences the physics of the interaction only at the inlet of the sliding contact region and does not affect the outlet.

As the pressure force is the sum of the tangential and normal components, the overturning moments arising from the air intrusion of the contact point depends on the normal reaction, while the penetration of air into contact area is determined by the tangential reaction. The air stream greatly raises the stochasticity of the tribological interaction being unfavourable for keeping the contact stability. The both reactions increase instability of motion of wear particles and influence the changes in thermodynamics on the sliding electrical contact [69–71] particularly significant effect is on the contact area formed under the front of the brush. So, we can claim that the heat outflow in the inlet zone is higher than one in the delivery end due to the air stream temperature change, namely the ‘third body’ in the front of the brush is much colder than in the delivery end of the brush. This temperature jump occurs during the dynamic interaction and may be caused by the Joule-Thomson effect [72,73]. The higher temperature could lead to wear acceleration, resulting in damage of the delivery end of the brush. So, this phenomenon elucidates the accelerated brush surface damage in electrical machines [18,36,46,74].

Now we can evaluate a relative error of the pressure change in the contact area being the function of time:  $\Delta p(x, t) \cong 1 - 0.8e^{0.1\tau}$ . Fig. 2 illustrates how the relative error  $\Delta p$  varies at a time. There is a clearly evidence, that the change in relative error of the pressure ( $\Delta p$ ) tends to decrease linearly from 0.2 to 0.12 with the time (in relative units  $\tau$  from 0 to 1) from the initial point. It is shown that the calculation of pressure changes is in good agreement with the theoretical solution and the results well matches with the solution of [61].

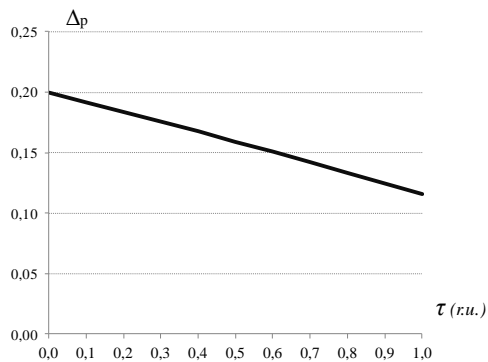


Fig. 2: The change in relative error of the pressure changes with the time of an arbitrary-profile

It is interesting to note that the largest error occurs in the area formed under the leading edge of the brush and the commutator that is where a highest sliding velocity at the contact area, whereas the smallest error occurs in the area formed under the trailing edge of the brush where the sliding velocity decreases. Moreover, the approximate solution tends to the exact one in length of time.

It is generally known that the error is an inverse function of the accuracy. One can see in Fig. 2 that the relative error of the contact pressure in the entrance to the contact area is 0.2 which determines the accuracy of the estimates is 20%. Moreover, Fig. 2 shows the accuracy of the pressure changes increases almost twice as fast as the size of the contact area with time. It means that the more the contact area size, the more precisely the contact pressure evaluation in the sliding layer. This fact following from the approximate solution is in agreement with the results of the exact solution and findings [61,75,76].

#### 4. Conclusion

In this paper, we numerically investigated the change in the pressure in contact area formed between the commutator and brush. The calculations are based on method of Bubnov-Galerkin providing opportunity to estimate numerical parameters of particle motion based on the change in the pressure field. This modeling approach gives a reasonably accurate description of error of pressure changes in the sliding layer for studying the tribological properties. Pressure changes equations are determined and solved for the motion of wear particles during sliding interaction between two surfaces.

We have taken the differential equation of second order to define the convergence of the approximate solution, estimate the closeness between the exact and approximate solution, and the efficiency of the approximate error. In order to make the problem simply tractable and deduce simple expressions, we had taken only the first term of the series in the partial sums of the terms, assuming that desired estimates of the solution are adequately approximated. It should be noted that the number of terms has no significant effects on the accuracy of the error estimates. In the present study, we determined the solvability of the approximate equation, estimated the absolute and relative error of pressure changes and calculated the relative error for small contact area ( $x \ll 1$ ) which is smaller than and in good agreement with the theoretical solution.

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