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Research paper

The single server vacation queue with geometric abandonments and Bernoulli's feedbacks

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Abstract

In this article the behaviour of a single server vacation queue with geometric abandonments and Bernoulli's feedbacks is carried out and various important performance measures are derived. Some numerical experiments are presented to study how the parameters of the model influence the state of the system.

Keywords: *M/M/1 queue, geometric abandonments and Bernoulli's feedbacks.*

Mathematics Subject Classification: *68M20, 90B22 and 60K25*

1. Introduction

Queueing systems with vacations have been widely studied by numerous researches in the past few decades. A comprehensive and an excellent survey can be found in Doshi[4]. The recent research works on queueing model with vacations can be found in Keetal.[14]. The monographs of Takagi[12] and Tian and Zhang [11] provide limited virtual waiting time. The model analyse in the queueing systems with a variety of vacation schemes.

Queueing systems with feedback have been broadly carried out in queueing literature because of their vital role and applications in real life problems such as production, inventory, manufacturing, communication systems etc. Queues with feedback mechanism start from Takacs [8]. He has studied the steady state analysis of the queue size distribution and the first two movements of the distribution function of sojourn time by a customer. Nakamura [5] considered a single server Markovian queue with delay feedback mechanism. Davignon and Disney[2] have studied a single server Markovian queue where feedback is state dependent.

Disneyetal. [3] have studied a single server queue with instantaneous Bernoulli's feedback and general service time distributions through Markov renewal process. Thangaraj and Santhakumaran [13] have studied a Markovian queue with instantaneous independent Bernoulli's feedback mechanism. Thangaraj and Santhakumaran [9] have analysed a queue with two types of delayed feedback. Kalidass and Kasthuri [7] have studied a two phase service M/G/1 queue with Bernoulli's feedbacks and the customer is allowed to take only finite number of feedbacks. Kalidass and Kasthuri [6] have considered a priority retrail queue with second phase in multi-optional service and immediate finite number of Bernoulli's feedback. The readers refer to second multi optional service [6] and references there in for recent studies on queues with feedback.

The queueing models with impatient customers area phenomenon in many real life problems. For this kind of queueing models and computational analysis readers are referred to (2006,2009) Queueing models with server in vacations. Due to the number of papers being on queue the customer impatience happens.

Adanetal.[2009] [1] and Economou and Kapodistria have studied queueing models with vacations and synchronized abandonments. Dimonet al. [10] considers that a Markovian vacation queue with customer abandonments is geometric.

This paper aims at studying an M/M/1 queue vacations, geometric abandonments and Bernoulli's feedbacks. The structure of this article is given below:

Section 2 is a formal description of the model, Section 3 and 4 study the steady state behaviour of the model, Section 5 gives several kinds of performance measures, Section 6 discusses the case study of the model and, finally, Section 7 concludes with some numerical examples.

2. Model description

In queueing system customers would arrive as per the process of Poisson rate λ. The service is being offered through a single server which could be either in active or non-active mode. When the system gets on, the customers are being served on the basis of FIFO approach. The service cannot be availed of when the server is in off mode. The inter service rate of μ are exponentially distributed. When there are no customers, the server goes into a vacation state. The system appears empty during the end of the vacation and then the server relapse into vacation mode. If there is atleast one customer in the system at the end of the vacation mode, the server begins to offer service and the system functions as a standard M/M/1 queue till it turns empty again. The vacation times are exponentially distributed with the rate γ. At the time the server is on vacation, abandonment opportunities take place as per Poisson process with rate ζ. An impulsive opportunity condition, customers are taken one by one sequentially. Every one of them leaves the system with probability 'p' or rests in the system with complementary probability 'q'. The customers start sequentially for abandoning the system and a decreased number of the customers halt at the first person who remains in the system or when all other customers abandon it. Alternatively, it is assumed

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that during the abandonment opportunity condition the total number of waiting customers in the system gets reduced as per the geometric distribution. It is also assumed that all the process involved in our model is mutually independent. After the completion of each service customers can take the feedback service with probability 's' or leave the system with complementary probability 'r'.

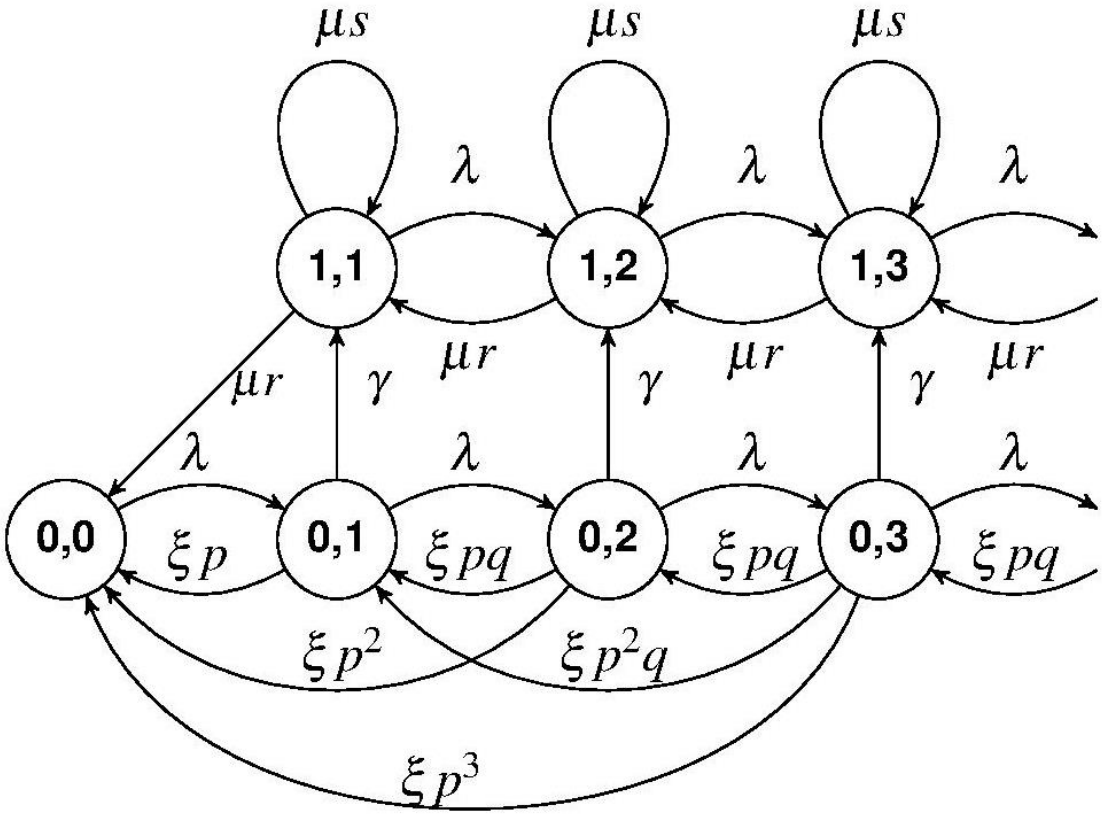


Figure 2.1: State-transition rate of the model diagram

3. Balance stow equations

$$P_0(u) = \sum_{n=0}^{\infty} u^n P_{0,n}$$

Define

From the steady state balance equations, we have for the state $(0,0)$

$$(\lambda + \zeta) P_{0,0} = \mu r P_{1,1} + \zeta \sum_{n=0}^{\infty} p^n P_{0,n}$$

$$= \mu r P_{1,1} + \zeta P_0(p) \quad (1)$$

In state $(n,0)$, we have

$$(\lambda + \gamma + \zeta) P_{n,0} = \lambda P_{0,n-1} + \zeta q \sum_{j=n}^{\infty} p^{j-n} P_{0,j}, \qquad 1 < n \quad (2)$$

In state $(1,1)$, we have

$$(\lambda + \mu r) P_{1,1} = \gamma P_{0,1} + \mu r P_{1,2} \quad (3)$$

In state $(1,n)$, we have

$$(\lambda + \mu r) P_{1,n} = \gamma P_{0,n} + \mu r P_{1,n+1} + \lambda P_{1,n-1}, \qquad 1 < n \quad (4)$$

The normalization condition is

$$P_{0,0} + \sum_{n=0}^{\infty} \left(P_{1,n-1} + P_{0,n} \right) = 1 \quad (5)$$

has a unique solution which is introduced and the main result obtained below section.

4. Steady state analysis

Suppose the system attains the steady state probabilities equilibriums $(P_{i,n})$ is know by

$$P_{0,0} = \frac{q(r_2 - 1)(\mu r - \lambda)}{r_2 (q\mu r + \lambda p - \lambda r_1)} \quad (6)$$

$$P_{0,n} = \frac{P_{00}}{r_2^{\,n}}, \qquad n \geq 0 \quad (7)$$

$$P_{1,n} = \left\{ \begin{array}{ll} \frac{\rho(1-r_1) r_2 P_{0,0}}{q(1-\rho r_2)} \left[\left(\frac{1}{r_2} \right)^n - \rho^n \right] & if\ r_2 \neq \frac{1}{\rho} \\ \frac{(1-r_1) P_{0,0}}{q} n \rho^n & if\ r_2 = \frac{1}{\rho} \end{array} \right\} n \geq 1 \quad (8)$$

Where $r_1 = \frac{\lambda + \lambda p + \gamma + \zeta p - \sqrt{(\lambda + \lambda p + \gamma + \zeta p)^2 - 4\lambda p(\lambda + \gamma + \zeta)}}{2\lambda}$ (9)

$$r_2 = \frac{\lambda + \lambda p + \gamma + \zeta p + \sqrt{(\lambda + \lambda p + \gamma + \zeta p)^2 - 4\lambda p(\lambda + \gamma + \zeta)}}{2\lambda} \quad (10)$$

$$0 < r_1 < 1, r_2 > 1$$

Proof

Both sides of equation (2) multiplying by u^n and adding for $1 \leq n$, we find

$$(\lambda+\gamma+\zeta)\left(P_0(u)-P_{0,0}\right)=\lambda u P_0(u)+\zeta q\sum_{j=1}^{\infty}p^j P_{0,j}\sum_{n=1}^{j}\left(\frac{u}{p}\right)^n \quad (11)$$

We discuss different two cases.

Case (i): $u \neq p$, then replacing the sum $\sum_{n=1}^{j}\left(\frac{u}{p}\right)^n$ in (11) by $\frac{u}{p-u}\left[1-\left(\frac{u}{p}\right)^j\right]$ and multiplying with $(u-p)$ yields

$$(u-p)(\lambda+\gamma+\zeta-\lambda u)P_0(u)=(u-p)(\lambda+\gamma+\zeta)P_{0,0}+\zeta pu\left(P_0(u)-P_0(p)\right) \quad (12)$$

Case (ii): $u = p$, equation (11) assumes the form

$$(\lambda+\gamma+\zeta-\lambda p)P_0(p)=(\lambda+\gamma+\zeta)P_{0,0}+\zeta pq\sum_{j=1}^{\infty}p^{j-1}P_{0,j}\ j$$

ie. $$(\lambda+\gamma+\zeta-\lambda p)P_0(p)=(\lambda+\gamma+\zeta)P_{0,0}+\zeta pq P_0'(p) \quad (13)$$

Solving (1) for $P_0(p)$, we get

$$P_0(p)=\frac{(\lambda+\zeta)P_{0,0}-\mu r P_{1,1}}{\zeta} \quad (14)$$

From (14), we obtain

$$P_0(z)=\frac{[(u-p)(\lambda+\gamma+\zeta)-qu(\lambda+\zeta)]P_{0,0}+\mu rquP_{1,1}}{(u-p)(\lambda+\gamma+\zeta-\lambda u)-\zeta u q}=\frac{\pi_0(u)}{\pi_1(u)}(say) \quad (15)$$

We see that $\pi_1(z)=0$ if and only if u –a(u) = 0, where $a(u)=a_0+a_2u^2$ with

$$a_0=\frac{p(\lambda+\gamma+\zeta)}{\lambda+\lambda p+\gamma+\zeta p} \text{ and } a_2=\frac{\lambda}{\lambda+\lambda p+\gamma+\zeta p}$$

Since a(1) < 1 we can apply a result of Rouche's theorem and we find that the equation $\pi_1(u)=0$ has two different roots r_1 and r_2, one inside the open unit disk and the other outside the closed unit disk. The different roots r_1 and r_2 are set by equation (9) and (10) respectively. It is clear that $1>r_1>0$ and $1<r_2$.

However, $P_0(u)$ converges in the closed unit disk as a probability generating function, we have essentially that $\pi_0(r_1)=0$; hence $\pi_0(u)=K(u-r_1)$. Next, we make simpler the term $u-r_1$ from the numerator and the denominator in(15) and $P_0(u)$ assumes the form $P_0(u)=K/\{\lambda(r_2-u)\}$. Solving $P_0(u)$ at 0 yields $K=\lambda r_2 P_{0,0}$ and $P_0(u)$ can be written as

$$P_0(u)=\frac{r_2}{r_2-u}P_{0,0} \quad (16)$$

Multiplying equation (3) by u and (4) by u^n and adding them for $n>1$ implies that

$$P_1(u)=\frac{\gamma u}{\lambda u+\mu r u-\lambda u^2-\mu r}P_0(u)-\frac{(\gamma P_{0,0}+\mu r P_{1,1})u}{\lambda u+\mu r u-\lambda u^2-\mu r}P_{1,0} \quad (17)$$

In this position we need a relation between $P_{0,0}$ and $P_{1,1}$.

Equation $\pi_0(r_1)=0$ yields simply that

$$P_{1,1}=\frac{p(1-r_1)(\lambda+\zeta)+\gamma(p-r_1)}{\mu r q r_1}P_{0,0} \quad (18)$$

Substituting $P_0(z)$ from (16) and $P_{1,1}$ from (18) in (17) yields

$$P_1(u)=\frac{\lambda(1-r_1)u-(\lambda r_2-\lambda r_1 r_2-\gamma q)}{q(r_2-u)(1-u)(\lambda u-\mu r)}r_2 u P_{0,0} \quad (19)$$

Let $M(u)$ be the numerator of $P_1(u)$ and applying that $r_1r_2=(\lambda p+\gamma p+\zeta p)/\lambda$ *and* $r_1+r_2=(\lambda+\lambda p+\gamma+\zeta p)/\lambda$, we can simply that $M_0(1)=0$.

Hence $M_0(u)=\lambda(1-r_1)(u-1)$ and $P_1(u)$ assumes the form

$$P_1(u)=\frac{r_2(1-r_1)\rho}{q}P_{0,0}\frac{u}{(r_2-u)(1-\rho u)} \quad (20)$$

Now proceed to obtain the expansion of $P_1(u)$ by using partial fractions.

Discuss two different types.

Type(i): Suppose $r_2\rho\neq 1$, we get

$$P_1(u)=\frac{\rho(1-r_1)r_2}{q(1-\rho r_2)}P_{0,0}\sum_{n=0}^{\infty}\left[\left(\frac{1}{r_2}\right)^n-\rho^n\right]u^n \quad (21)$$

Type (ii):Suppose $r_2\rho=1$, we get`

$$P_1(u)=\frac{\rho(1-r_1)r_2}{q}P_{0,0}\sum_{n=1}^{\infty}n\rho^n u^n \quad (22)$$

Equation (21) and (22) yield readily (13).By using the normalization condition, $P_0(1)+P_1(1)=1$, and substituting (16) and (19) we obtain $P_{0,0}$ in the form given in (11).

We have,

$$P_{0,0}=\frac{q(1-\rho)(r_2-1)}{r_2\left[q(1-\rho)+\rho(1-r_1)\right]} \quad (23)$$

$$P_{0,n}=\frac{q(1-\rho)(r_2-1)}{r_2^{n+1}\left[q(1-\rho)+\rho(1-r_1)\right]} \quad (24)$$

The number of expected customers in the system is $E(N)=P_0'(1)+P_1'(1).$

Differentiating formulas (16) and (20) at $u=1$ and replacing $P_{0,0}$ from (11) yields

$$E(N)=\frac{r_2}{(r_2-1)^2}P_{0,0}\left[1+\frac{\rho(1-r_1)[(r_2-1)+(1-\rho)]}{q(1-\rho)^2}\right] \quad (25)$$

5. Performance measures

According to the distribution of the steady state various system performance measures can be adopted.

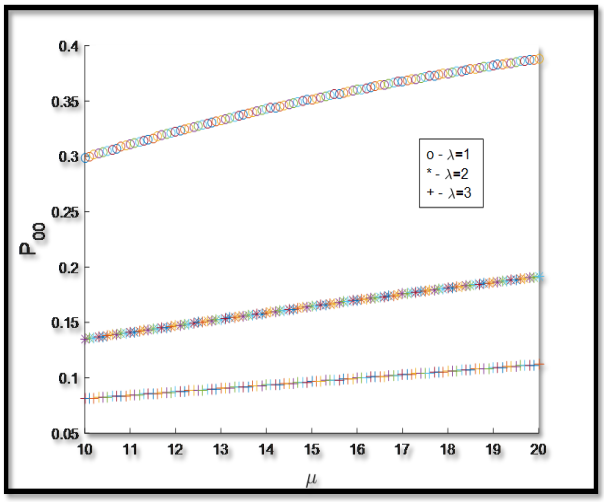
1. Number of expected customers in the system = $E(N)$.
2. Expected waiting time of the system = $E(W)=\frac{E(N)}{\lambda}$.
3. Probability to the server being busy = $P_1(1)$
4. Probability to the server being vacation = $P_0(1)$

6. Particular case

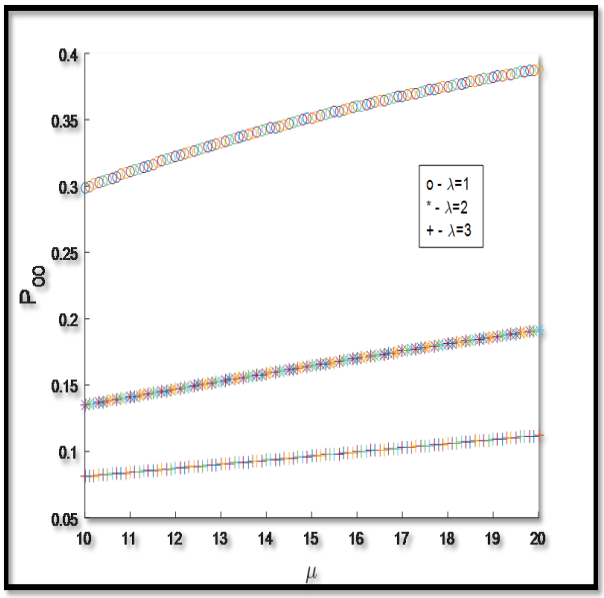
Suppose $r=1$ *and* $s=0$. That is, there is no feedback service. The value of $P_{0,0}$ coincides with $P_{0,0}$ of the model discussed in the single server vacation queueing model with geometric abandonments.

7. Numerical examples

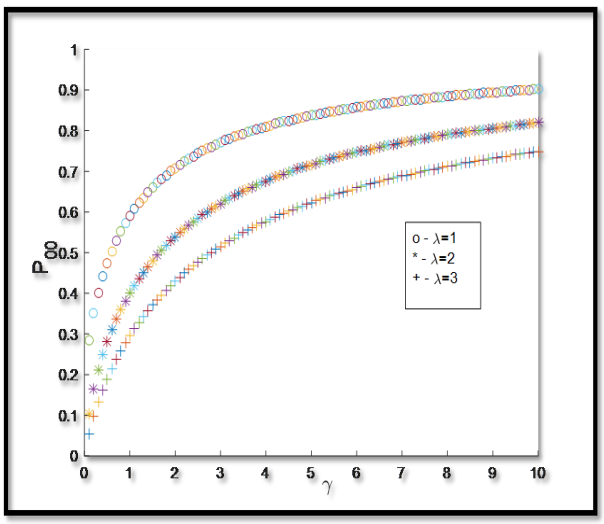
Numerical experiments of various main performance indices are provided.



$Fig\ 7.1 - P_{00}\ vs\ \lambda$



$Fig\ 7.2 - P_{00}\ vs\ \mu$



$Fig\ 7.3 - P_{00}\ vs\ \gamma$

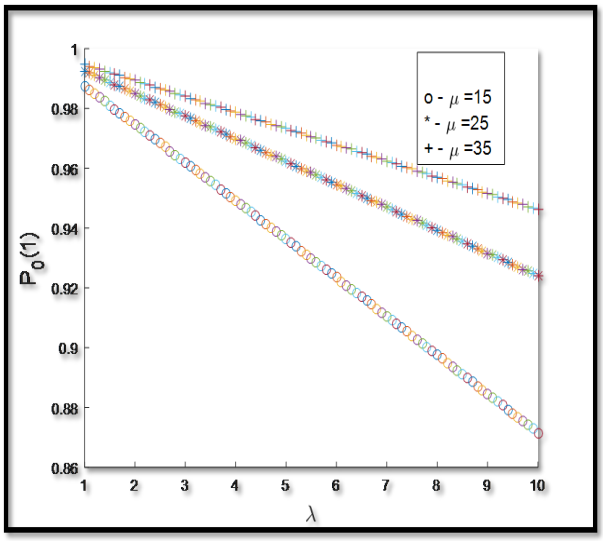


Fig $7.4 - P_0(1) vs \lambda$

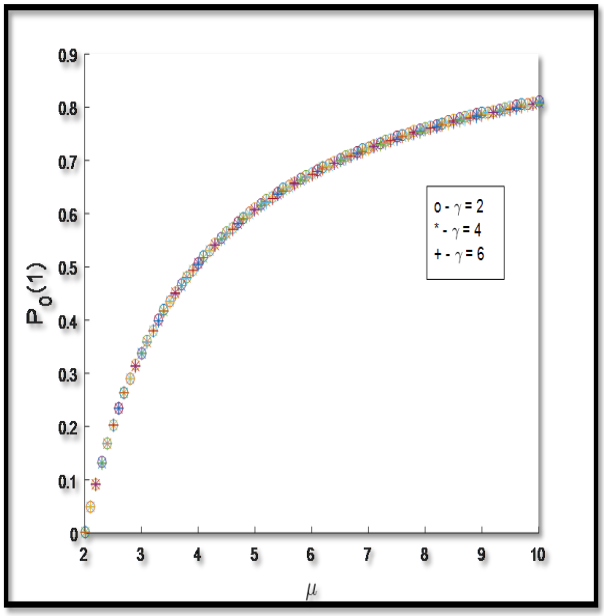


Fig $7.5 - P_0(1) vs \mu$

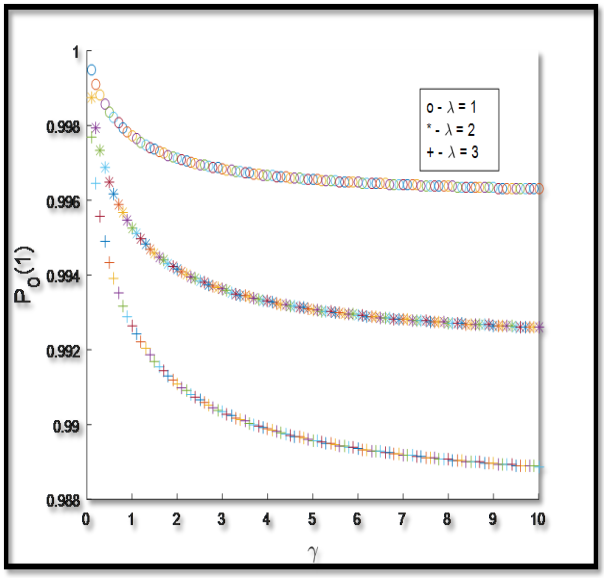


Fig $7.6 - P_0(1) vs \gamma$

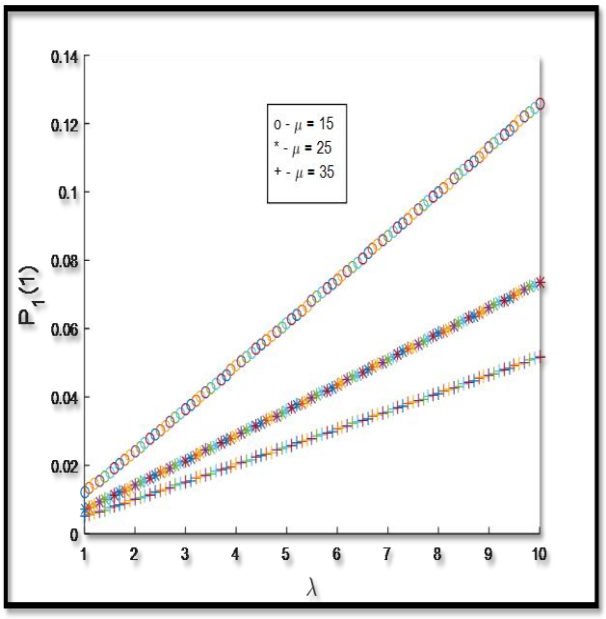


Fig $7.7 - P_1(1) vs\, \lambda$

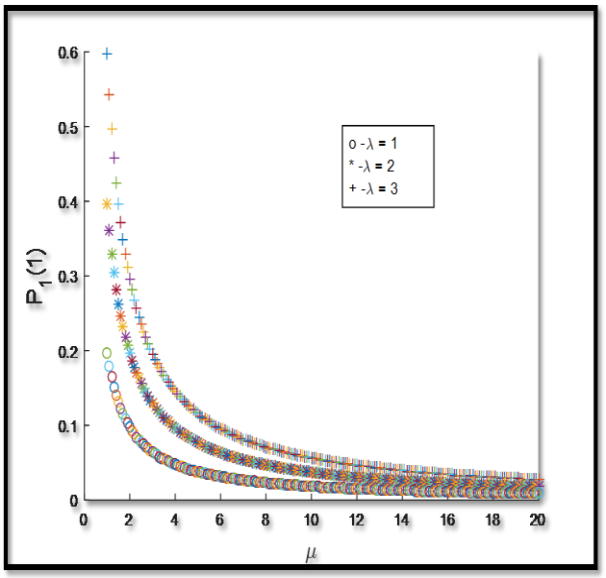


Fig $7.8 - P_1(1) vs\, \mu$

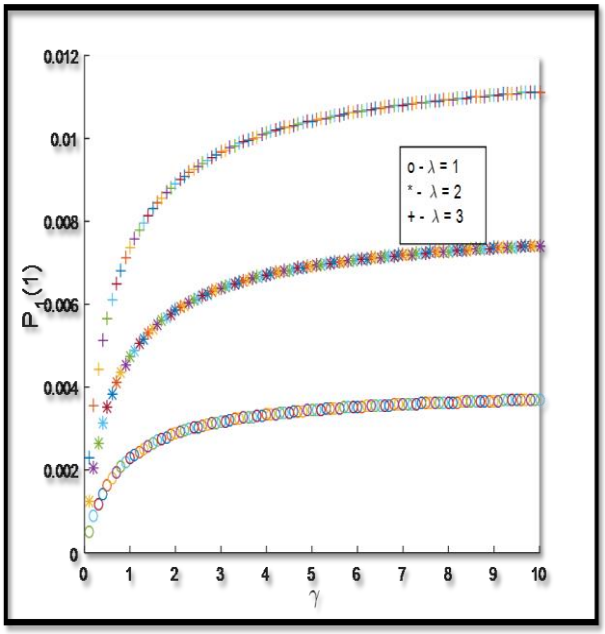


Fig $7.9 - P_1(1) vs\, \gamma$

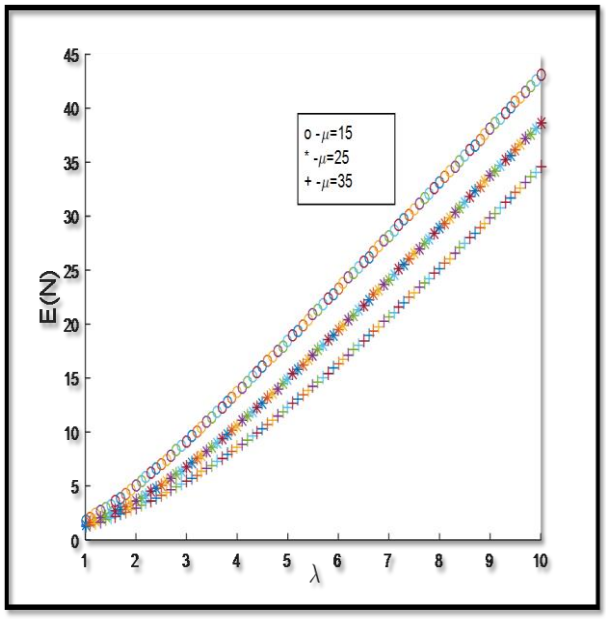


Fig 7.10 – E(N) vs λ

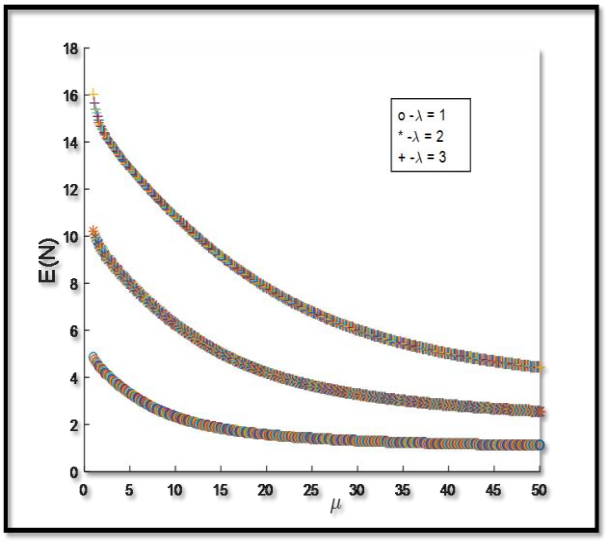


Fig 7.11 – E(N) vs μ

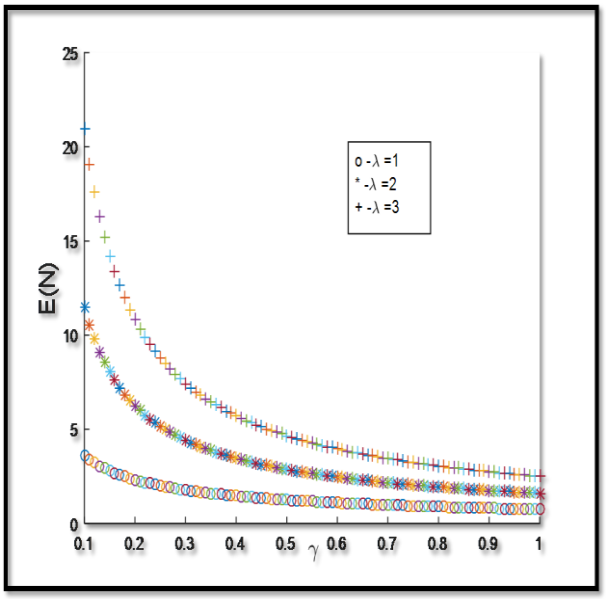


Fig 7.12 – E(N) vs γ

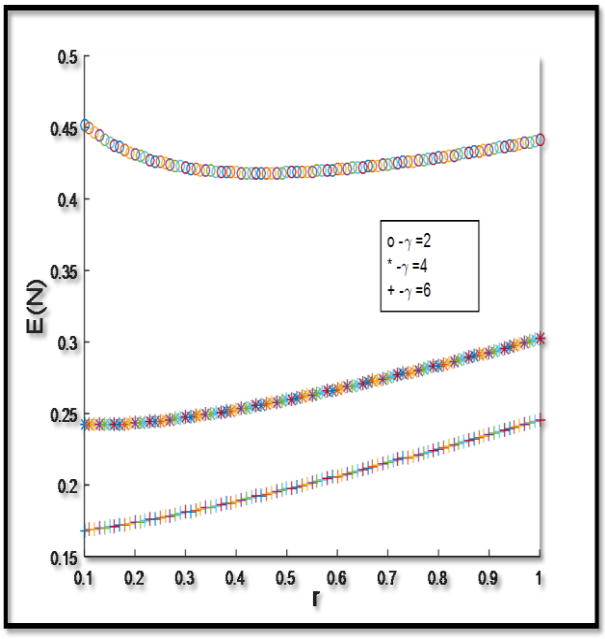


Fig 7.13 – $E(N)$ *vs* r

Figure 7.1 displays the relation between P_{00} and the arrival rate λ when $\gamma = 0.2,\ \mu = 25,\ \alpha = 2, \zeta = 1, N = 1$ *and* $r = 0.2$. It can be noticed that P_{00} decreases with an increasing value of λ when μ = 15,25,35.

From figure 7.2 it is observed that, the probability of being server ideal in normal state increases with an increasing value of μ for different values of λ=1,2,3.

Figure 7.3 illustrates the relation of P_{00} and the vacation rate γ. It can be noticed that P_{00} and γ both increasing for λ=1,2,3.

Figure 7.4 exhibits the relation of $P_0(1)$ by varying arrival rate λ for service rate μ = 15,25,35. It is seen that $P_0(1)$ decreases with the increase arrival rate λ.

Figure 7.5 displays the relation between $P_0(1)$ and service rate μ by varying vacation rate γ = 2,4,6. It can be observed that $P_0(1)$ increases along with the increase of μ.

Figure 7.6 displays the relation between $P_0(1)$ and vacation rate γ by varying arrival rate $\lambda = 1, 2, 3.$ It can be observed that $P_0(1)$ decreases along with the increase of γ.

Figures 7.7, 7.8, and 7.9 gives the relation between $P_1(1)$and λ, μ, and γ respectively.

Figures 7.10, 7.11 and 7.12 displays the relation between E[N] and λ, μ, and γ.

Figure 7.13 demonstrates the relation between number of expected customer in the system E(N) and probability r, the customer leaving the system without feedback the for γ =2,4,6.

8. Conclusion

In this article, a single server vacation queue with geometric abandonments and Bernoulli's feedbacks scheme has been discussed. Various main performance measures are derived. A few numerical examples are given to demonstrate how the various parameters of the model influenced the behaviour of the system. It would be interesting to develop this research further by including the concept of finite capacity with this model.

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