

Centroidal Voronoi Partitioning using Virtual Nodes for MultiRobot Coverage

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Abstract

This paper addresses the problem of Voronoi partitioning using Centroidal Voronoi configuration for a multi-robotic coverage strategy known as Voronoi Partition based Coverage (VPC) algorithm. In VPC, the area to be covered is divided into Voronoi cells and each robot covers the corresponding cell. We use the concept of Centroidal Voronoi Configuration (CVC) to achieve a more uniform load distribution among the robots in terms of the area covered. Instead of the robots moving physically into the CVC, we introduce a concept of virtual nodes, which are deployed into CVC. Once the Voronoi partition is created based on the virtual nodes, the robots cover the corresponding Voronoi cells. A gradient based control law has been used for deployment of the virtual nodes. Simulation results are provided to demonstrate the proposed deployment and partitioning scheme.

Keywords: Centroidal Voronoi Configuration, Multi-robot coverage, Virtual nodes

1. Introduction

The problem of coverage of a region of interest using autonomous robots has many applications such as in autonomous vacuum cleaning, lawn moving, landmine detection [1], etc. In these applications, the mobile robot is required to move through the region of interest such that every point in this space has come under a coverage tool (or sensor, in applications such as landmine detection) at least once. Apart from this “completeness” of coverage, the coverage path is also expected to avoid coverage overlap or repeated coverage. Several algorithms have been proposed in the literature to solve this problem both when the area to be covered is known a priori (offline algorithms) and not known a priori (online algorithms). A survey of various coverage algorithms is provided in [2] and [3].

Using multiple robots to cover a specified region is expected to reduce the coverage time, apart from possible robustness to failure of a (or a few) robot(s). However, a challenging problem when using a Multi-Robotic System (MRS) is to achieve distributed coordination between different robots so that they do not collide with each other or perform repeated coverage of regions within the environment. Several multi-robot coverage (MRC) strategies have been proposed in the literature [4], [5], [6], [7], [8], [9], [10], [11]. In most of these algorithms, fundamentally two strategies are followed to ensure cooperation and avoid coverage overlap. In the first approach, the regions already covered by all the robots are stored in a central location, and the robots continuously communicate with the central information provider ensuring an indirect cooperation. This approach, apart from increased communication overhead and increased spatial (memory) complexity, may not be suitable in situation where a central information provider cannot be used. In fact, the approach is no more distributed. In the second

approach, each robot should not only keep track of the region it has already covered, in order to avoid self-redundant coverage, but also needs to communicate the covered region to other robots in the team. Though this is a distributed approach, it results in higher communication overhead along with high memory requirement in each robot. In [9] authors use information compression to reduce the communication overhead.

A simple and elegant technique to reduce the communication requirement is to use divide and conquer approach. Here, the region to be covered is divided into cells and each robot is allotted a cell or a group of preferably contiguous cells for coverage. This leads to a passive cooperation, requiring no communication between robots while performing coverage. Further, each robot solves a single robot coverage algorithm. In [4] and [12] authors decompose the region into cells and dynamically allocate a cell (considered a task) to a robot, until all the cells are covered. In [13] authors divide the region to be covered into n polygons, where n is the number of robots, and each robot is allowed to cover exactly one cell. While in most partition and cover approaches, the region is decomposed into cells by a central computer, and not taking into account the current robot positions, authors in [11] propose a Voronoi Partition-based Coverage (VPC) strategy, where, the robots partition the region to be covered into Voronoi cells, considering their current location as nodes. This strategy is amendable for a completely distributed implementation, as Voronoi cells can be computed in a distributed manner [14], [15].

Though VPC solves problems associated with cooperation between robots, and eliminates the on-the-go communication requirement, partitioning is not guaranteed to ensure a uniform load distribution among the robots in terms of the area allocated for coverage. Thus, a robot allocated a smaller cell completes coverage faster than a robot allocated a larger cell. This increases the overall time of completion of the coverage task and underutiliza-

tion of some of the robots (allocated smaller cells). In this paper we propose a Centroidal Voronoi Partitioning methodology using virtual nodes to efficiently partition the workspace which results in a more uniform workspace allocation to the individual robots. Once the optimal partitioning is achieved, any of the single robot algorithms like Boustrophedon coverage [16], Spanning Tree Coverage (STC) [17] etc. can be used to cover the allotted portion of the workspace.

2. Problem Setting

We consider a problem of N robots cooperatively covering a region of interest Q . The multi-robot coverage strategy is expected to provide a complete and non-overlapping coverage of Q . We use Voronoi partitioning technique to partition Q into Voronoi cells and allot each cell to the corresponding robots for coverage. In this work, the main problem addressed is to ensure a more uniform partitioning of Q in terms of the area of the Voronoi cells. Ideally, we expect an equitable partition. That is, area of all the cells is same. But practically, we try to achieve an approximately equitable Voronoi partition. In order to achieve a more uniform partitioning it is important to select the location of nodes based which the Voronoi partition is generated. Further, the nodes should be associated with robots. That is, each node represents a unique robot. Thus problem now reduces to selection and placement of the nodes used for generating the Voronoi partition of Q .

3. The Proposed Partitioning Scheme

In this section we discuss the proposed Voronoi partitioning scheme to achieve a more uniformly sized Voronoi cells. Voronoi partitioning [21] has been widely used as an effective spatial partitioning tool in many applications including coverage optimization in multi-agent (robotic) systems [11],[22]. A standard Voronoi partitioning scheme decomposes a space using the concept of nearness to nodes. Let $I_N = \{1, 2, \dots, N\}$; $Q \subset \mathbb{R}^2$; and $P = \{p_1, p_2, \dots, p_N\}$, $p_i \in Q$, be a set of points in Q called a *node set*. The *Voronoi partition*, generated by P is the collection $\{V_i(P)\}_{i \in I_N}$ with,

$$V_i(P) = \{q \in Q \mid \|q - p_i\| \leq \|q - p_j\|, \forall j \in I_N\} \quad (1)$$

The Voronoi cell, V_i is the collection of those points which are closest to p_i . In the context of the multi-robot coverage problem addressed in this paper, N is the number of robots, p_i is the position of the i^{th} robot, and Q is the region to be covered.

In VPC [11], authors use the initial location of the robots as the nodes for generating the Voronoi partition. However, if the robots are not reasonably uniformly placed in Q , generated Voronoi cells would typically be non-uniform in size. In this work we use *virtual nodes*, where each virtual node is associated with exactly one robot. The initial location of each of the virtual nodes is same as that of the robot. We use these virtual nodes to partition Q using the Voronoi cells. The virtual nodes are deployed into optimal configuration. For simplicity, we use p_i to represent both the initial location of the i -th robot and the corresponding virtual node. The objective of optimal deployment of the virtual nodes is to obtain a more uniformly sized Voronoi cells.

3.1. Centroidal Voronoi Configuration

In centroidal Voronoi configuration, the nodes which generate the Voronoi cells are located at the centroid of the respective Voronoi cells. This concept has been used in several locational optimization, facility location [18], optimal sensor deployment [19] and multi-robot deployment [20]. In these problems, the nodes are made to move toward the respective centroids using a gradient based proportional control law (Lloyd's algorithm) while the Voronoi cells and hence the centroids are recomputed as the nodes

move. Eventually, the nodes reach the centroid of the respective Voronoi cells, asymptotically. We skip the details of formulation and theoretical results here. The details can be found in [19, 20].

Initially the virtual nodes are located at the physical location of the robots. A Voronoi partitioning is generated, based on this initial configuration of the virtual nodes. Once the partitioning is established, the virtual nodes are moved towards the centroids of the respective Voronoi cells, in small steps. However, the robot do not move physically. During this motion of the virtual nodes, the topology of the initial Voronoi cells changes. So the new centroids corresponding to the present virtual node locations are generated. The process continues until the virtual nodes reaches the centroids of their respective Voronoi cells. Once the virtual nodes reaches the respective centroids, the partitioning so obtained will be the optimal in terms of the coverage effectiveness, implying that the areas of the Voronoi cells are more or less same. Once this final partition is established, the corresponding Voronoi cells are then allotted to the robots of the respective virtual nodes. The strategy followed by each virtual node is shown in Algorithm 1.

Algorithm 1: "Deployment" strategy followed by each robot.

Step 1 Compute Voronoi cell V_i based on initial deployment.

Step 2 Compute centroid C_{V_i} of V_i .

Step 3 Move virtual node towards C_{V_i} .

Step 4 If position of the virtual node $p_i \approx C_{V_i}$

GO TO Step 5

Else GO TO Step 1.

Step 5 END

The uniformity in area allotment to each robots can be calculated using

$$\eta = \frac{\max(A_{r_i})}{\left(\frac{A(Q)}{N}\right)} \quad (1)$$

Here, A_{r_i} refers to the area covered by i -th robot (that is the area of the i -th Voronoi cell), $A(Q)$ is the total area to be covered and N is the number of robots in multi-robot system. For optimal partitioning $\eta = 1$. Suboptimal solutions are obtained when $\eta > 1$. Figures 1-5 illustrate the process of deployment of the virtual nodes and the Voronoi partitioning in each stage in a multi-robot system consisting of 3 robots. The robots are positioned randomly in the workspace and Voronoi partitioning based on the initial positions is generated. This is shown in Figure 1. The centroids of the corresponding Voronoi cells are computed and the virtual nodes are moved towards the centroids. At each iteration the position of virtual nodes changes so as the Voronoi cell boundaries which results in new centroids. This process is illustrated in Figures 2-4. Optimum partitioning occurs when the virtual nodes reaches the corresponding centroid positions (Fig 5). Once the optimum partitioning is generated the actual physical robots move towards their respective Voronoi cells.

The proposed Partitioning technique has the following advantages.

- i. Since the virtual nodes are moving instead of physical robots, the battery usage can be minimized and can be used for more important tasks like exploration and coverage.
- ii. The final partition will be optimal in the sense that the the area allotted to each robot will be nearly uniform so that the resources available can be utilized to maximum extend.
- iii. The introduction of the concept of virtual nodes eliminates the coverage overlap issue since the physical robots move towards their respective Voronoi cells only at the end of partitioning process.

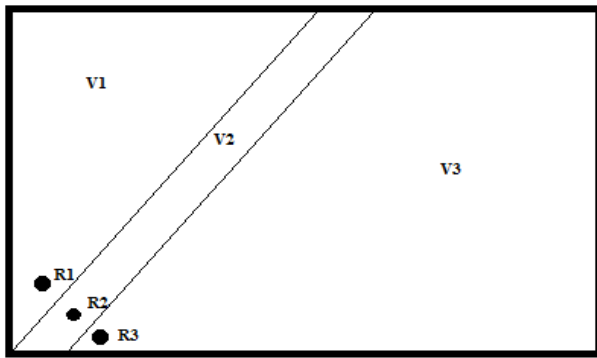


Figure 1. Initial deployment of robots, R_1, R_2 and R_3 with their corresponding Voronoi cells V_1, V_2 and V_3

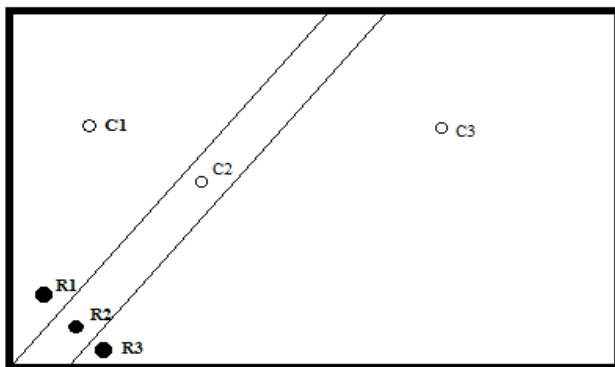


Figure 2. The centroids C_1, C_2 and C_3 of the Voronoi cells V_1, V_2 and V_3 respectively

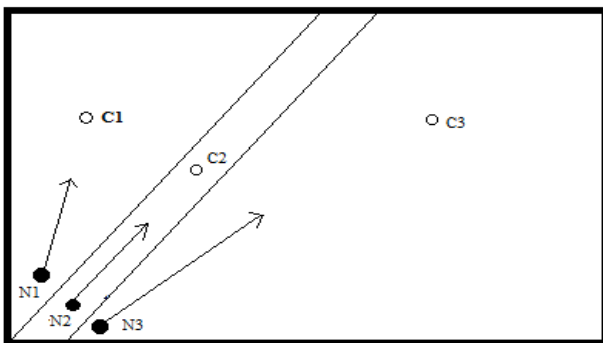


Figure 3. The virtual nodes N_1, N_2, N_3 (initially the positions will be same as that of physical robots) move towards the respective centroids.

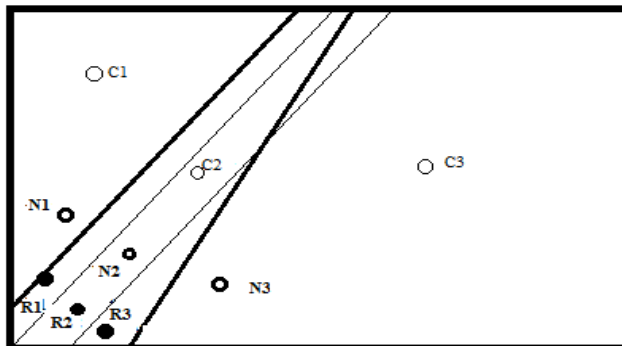


Figure 4. The position of virtual nodes after first iteration, N_1, N_2, N_3 . The new Voronoi partitioning at this stage is represented in bold lines and old partitioning in thin lines. C_1, C_2 and C_3 corresponds to the respective new centroids.

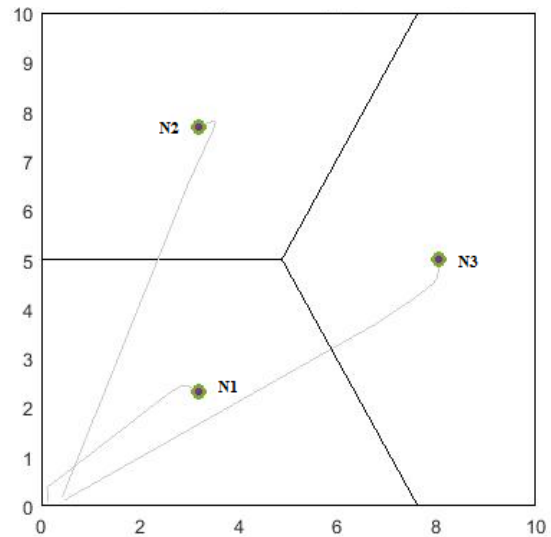


Figure 5. Final positions of virtual nodes and the optimal Voronoi partitioning generated.

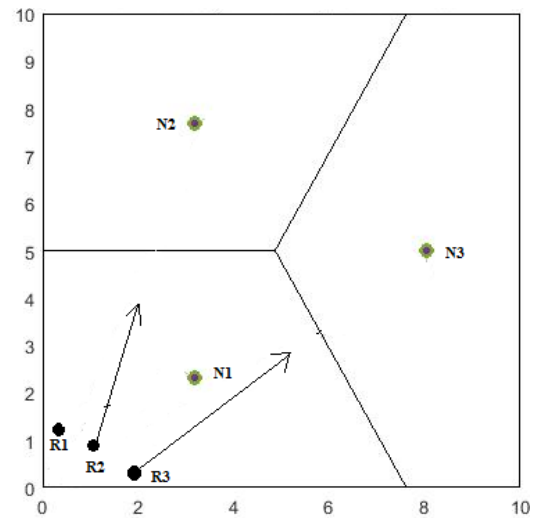


Figure 6. Physical robots moves towards their respective Voronoi cells for covering.

4. Result and Discussions

In this section we provide simulation results to demonstrate the proposed deployment and optimal partitioning strategy for VPC. The simulation is carried out in Matlab environment with ten robot system. Total of 100 iterations are done. Figures 7-11 show the scenario after various iterations.

In all the Figures, small circles indicate the centroids of the Voronoi cell and '+' indicate the current position of the virtual nodes. Note that the initial location of the virtual nodes is identical to that of the physical robots. During the deployment process only the virtual nodes move, while the physical robots are stationary. It can be observed that from an initial non-uniform partition as shown in Figure 7, the deployment of the virtual nodes into centroidal Voronoi configuration (that is, the virtual nodes are at or sufficiently close to the respective centroids) leads to a uniform Voronoi partitioning.

A five robot scenario is shown in Figure 12. A similar result as before is obtained here.

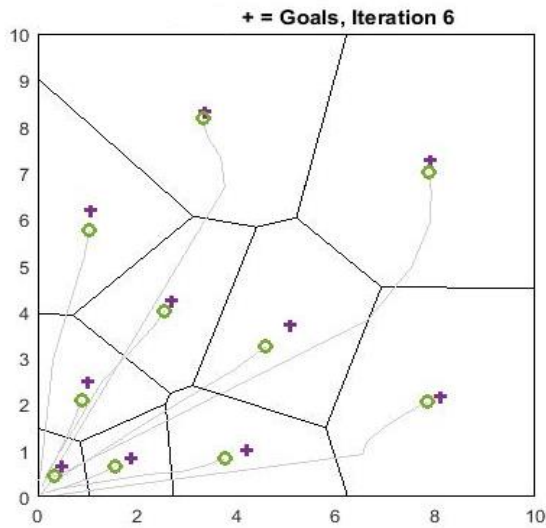


Figure 7. The scenario after 6 iterations. The circles and the '+' sign represents virtual nodes and centroids (goals) respectively

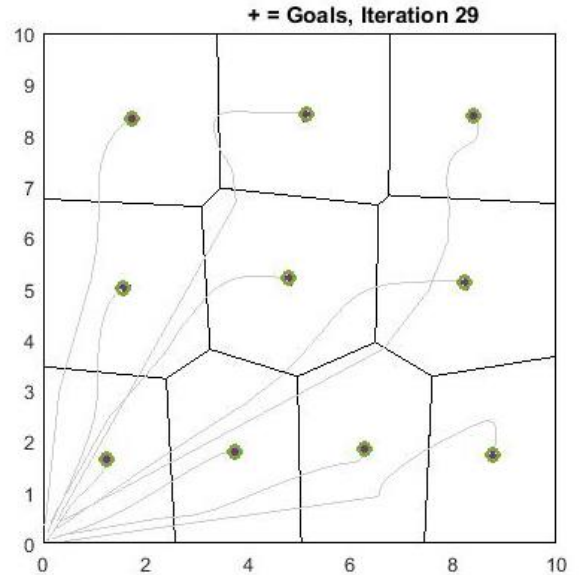


Figure 10. The scenario after 29 iterations.

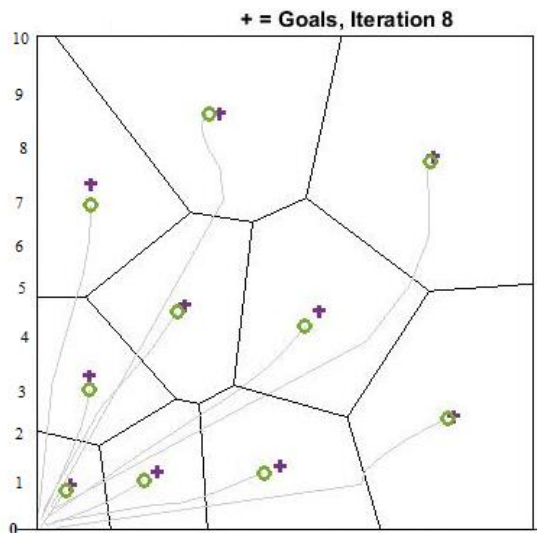


Fig 8 The scenario after 8 iterations.

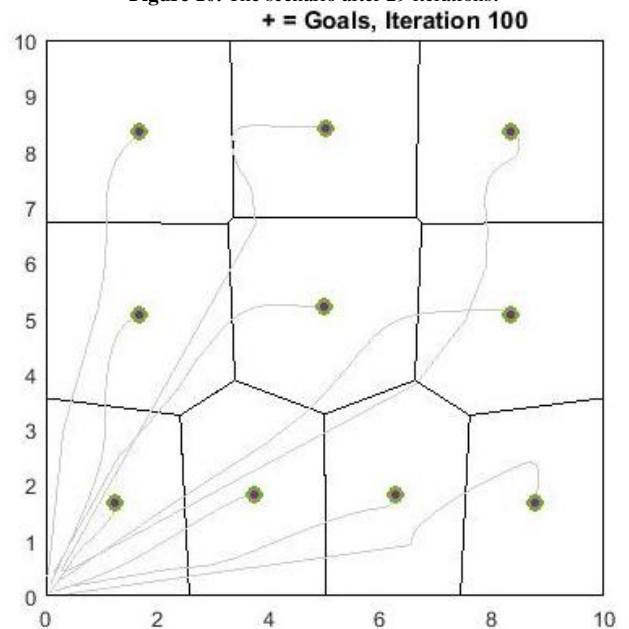


Figure 11. The final scenario after 100 iterations. The circles and the '+' sign represents virtual nodes and centroids respectively. The partitioning obtained is optimum with respect to area allotted to each robots.

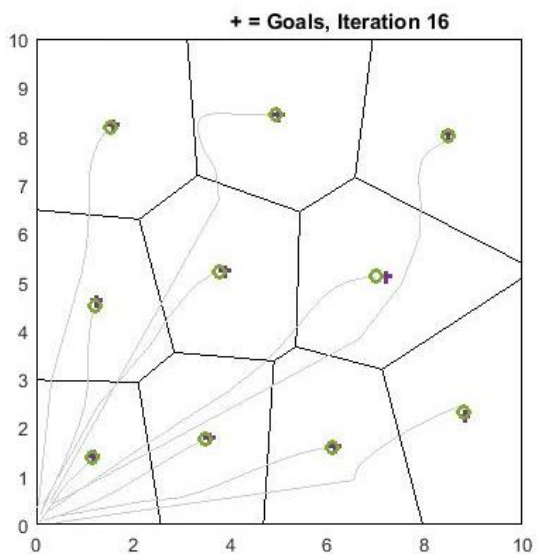


Figure 9 The scenario after 16 iterations.

Here $N=10$, $A(Q) = 100$ and $\max(A_{i_i})=10.5$.So $\eta = 1.05$, which is very close to the optimal value of 1.

5. Conclusions

We proposed an optimal deployment strategy to obtain a uniform voronoi partition for a multi-robot coverage algorithm such as vpc by introducing a concept of virtual nodes. The virtual nodes are deployed into a centroidal voronoi configuration, which is shown to be an optimal configuration in the context of sensor coverage in the literature. Instead of the robots getting physically deployed, the use of virtual nodes reduces the battery usage as well as coverage time. Further, the use of virtual nodes eliminates the coverage overlap issue since the physical robots move towards their respective voronoi cells only at the end of partitioning process. With the help of illustrative examples, we have demonstrated that the proposed partitioning scheme provides an optimal partitioning in the sense of uniformly sized voronoi cells to be covered by robots, leading to a uniform load distribution

among the robots. This further reduces the time of completion of the coverage tasks as all the robots are utilized to same extent.

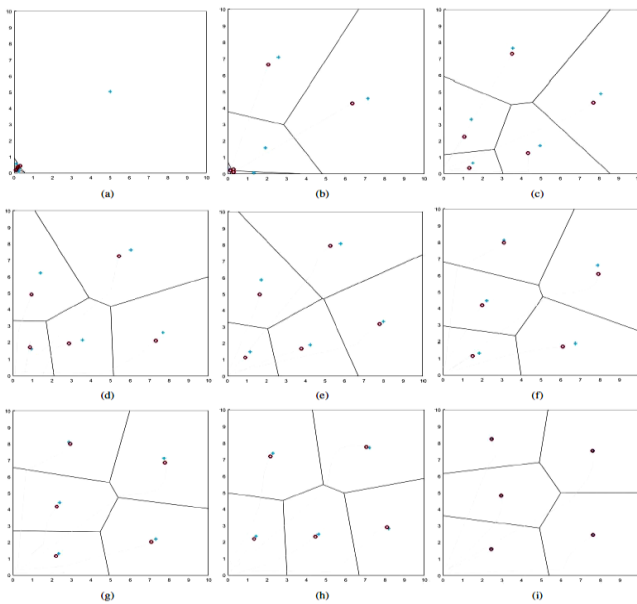


Figure 12. Five robot scenario. Process of deployment of virtual nodes from (a) an initial configuration into (i) a centroidal voronoi configuration. The centroid of each voronoi cell is marked with 'o' and the position of the node is marked with '+'.

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