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Research paper

Development of FOPDT and SOPDT model from arbitrary process identification data using the properties of orthonormal basis function

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Abstract

Most of industrial process can be approximately represented as first-order plus delay time (FOPDT) model or second-order plus delay time (FOPDT) model. From a control point of view, it is important to estimate the FOPDT or SOPDT model parameters from arbitrary process input as groomed test like step test is not always feasible. Orthonormal basis function (OBF) are class of model structure having many advantages, and its parameters can be estimated from arbitrary input data. The OBF model filters are functions of poles and hence accuracy of the model depends on the accuracy of the poles. In this paper, a simple and standard particle swarm optimisation technique is first employed to estimate the dominant discrete poles from arbitrary input and corresponding process output. Time constant of first order system or period of oscillation and damping ratio of second order system is calculated from the dominant poles. From the step response of the developed OBF model, time delay and steady state gain are estimated. The parameter accuracy is improved by employing an iterative scheme. Numerical examples are provided to show the accuracy of the proposed method.

Keywords: *First-Order Plus Delay Time Model, Second-Order Plus Delay Time Model, System Identification, Orthonormal Basis Function Models.*

1. Introduction

A linear process model can be represented as

$$Y(\cdot) = G(\cdot)U(\cdot) + H(\cdot)v(\cdot) \quad (1)$$

where $Y(\cdot)$, $U(\cdot)$, and $v(\cdot)$ are the process output, input, and noise, respectively and $G(\cdot)$ is the process transfer function and $H(\cdot)$ is the noise model transfer function in continuous or discrete domain.

Most industrial processes are continuous and can be reasonably described in continuous domain by a first-order or second-order transfer function model with delay. Generally, systems with highly damped dynamics are represented as first-order plus delay time (FOPDT) model, while systems with lightly damped dynamics are represented as second-order plus delay time (SOPDT) model. For FOPDT model, the continuous domain pole is real and negative, while for SOPDT model the continuous domain poles are either negative and real or complex conjugate with negative real part.

FOPDT and SOPDT models are simpler approximation of complex processes but holding the required steady state and dynamic process information. They are widely used in controller design, controller tuning, prediction, and process analysis. For PID controllers, Cohen-Coon method, Ziegler-Nichols method, CHR method, etc. [1-3] and for MPC, Shridhar and Cooper method, Iglesias method, Bagheri and Sadigh method, etc. [4-6] are prominent tuning methods using FOPDT or SOPDT model parameters.

The general continuous domain transfer function of FOPDT model is given as

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K}{\tau s + 1} e^{-Ls} \quad (2)$$

where K is the steady state (dc) process gain, τ is time constant, and L is the delay time (dead time).

The general continuous domain transfer function of SOPDT model is given as

$$G(s) = \frac{Y(s)}{U(s)} \frac{K}{\tau_n^2 s^2 + 2\zeta\tau_n s + 1} e^{-Ls} \quad (3)$$

where $\tau_n$is the natural period of oscillation and ζ is the damping ratio.

Identification of FOPDT or SOPDT models from process empirical data helps in representing complex processes in minimum parameters and simple model structure but holding the essential process dynamics without the need of knowing the complex physics involved in the process.

The simplest and easiest method for FOPDT or SOPDT model parameter estimation from empirical data is by the open-loop step test of the process [7-9]. Step test involves exciting the intended input in one direction with a significant magnitude while maintaining all other inputs at its nominal value. The output is allowed to complete its transition from the initial steady state to a new steady state. But from the view of process control, externally forcing an input in a control loop affects other loops undesirably. Also, the process operating away from its nominal operating point for a long period could result in off-spec products, raise issues in safety and stability [10]. Moreover, in step test, the input is excited only in one direction of the operating point to capture the process dynamics. For linear system this is fair but for nonlinear process the dynamics in the opposite direction is missed. The magnitude of the step signal is also significant particularly while using graphical analysis techniques for determining the process parameters. Similar methods like pulse test, relay test [11, 12] are proposed to overcome the issues with step test. But such methods still need standard and groomed input signal and disturbs the normal operation of the process. Moreover, the process need to be at steady state both initially and finally. Also, the involved graphical analysis makes the procedure offline and computationally less effective.

A method capable of using any arbitrary input signal and devoid of initial and final steady state requirements is more useful in real practical operations. The method should involve direct computation instead of graphical analysis making it computationally effective. In [13], a systematic method to estimate the FOPDT and SOPDT models from the orthonormal basis function (OBF) models developed from arbitrary process input-output data is discussed. The method involved step response analysis of noise-free OBF model and includes graphical analysis. The graphical analysis makes the method time consuming, less computational friendly, and unsuitable for recursive identification.

In this paper, we propose a method using properties of the orthonormal basis functions taking inspiration from [13] to estimate FOPDT and SOPDT model parameters from arbitrary process data. The proposed method is free from graphical analysis making it computation friendly and capable of online and recursive estimation.

The rest of the paper is organised as follows: Section II gives an overview of various OBF models and the unifying construction of its filters. Section III discusses with necessary Algorithms the proposed iterative method for estimation of FOPDT or SOPDT model parameters from arbitrary input-output data. The efficiency of the proposed method is IV. Section V concludes the paper.

2. Preliminaries

Orthonormal basis function (OBF) model

A stable system can be represented by discrete orthonormal basis functions as

$$G(q) = \sum_{i=1}^{n} \theta_i B_i(q) \tag{4}$$

where q is the forward shift operator, $G(q)$ is the discrete transfer function of the system, $\theta_i, \forall\, i = 1, \dots, n$, are the model parameters, n is finite order of the model, and $B_i, \forall\, i = 1, \dots, n$, are orthonormal basis functions used as filters in representing the system.

Orthonormal basis function (OBF) models have several advantages and used extensively in control oriented identification. OBF models uses *a priori* knowledge of system dynamics in the form of dominant poles, and $\varepsilon_i, \forall\, i = 1, \dots, n,$ explicitly in the filters. This helps in reducing model order drastically compared to other model structures like FIR, ARX, ARMAX, etc. OBF models are independent of output feedback and hence have improved prediction capabilities. It is also independent of input-output data scaling [14]. The parameters can be reformulated to apply linear least square method for estimation [15]. Different classes of OBF models found extensive use in model predictive and adaptive control strategies [16,17].

OBF models are classified based on their filters. A unifying construction for filters of OBF models is developed by [18] and is given as

$$B_1 = \frac{\sqrt{1 - |\varepsilon_1|^2}}{q - \varepsilon_1}$$

$$B_i = \frac{\sqrt{1 - |\varepsilon_i|^2}}{q - \varepsilon_{i-1}} \left(\frac{1 - \overline{\varepsilon_{i-1}} q}{q - \varepsilon_i} \right) B_{i-1}, \forall\, i = 2, \dots, n. \tag{5}$$

Laguerre, Kautz, and generalised OBF (GOBF) filters and hence corresponding classes of OBF models can be obtained from the unified equation (5). All poles are real and equal for Laguerre models [19]. The poles are repeated in complex conjugate fashion in Kautz models [14]. Multiple system modes are represented using GOBF models [18].

The OBF model output $\hat{Y}_{(k)}$ at sampling instant k is given as

$$\hat{Y}_{(k)} = \sum_{i=1}^{n} \theta_i B_i\, U_{(k)} \tag{6}$$

where $U_{(k)}$ is the input at instant k.

Theoretically, any stable poles can be used in the filter of OBF models but at the cost of accuracy and model order [14, 18]. Combining the right class of OBF model and near accurate poles in filters can result in parsimonious (relatively fewer number of parameters) model with the best possible accuracy [13]. Parsimonious models are computationally effective and easy to implement in model based control strategies [17]. The class of OBF model is based on system dynamics and hence can be known from the characteristics of dominant poles.

3. Estimation of FOPDT and SOPDT model parameters

Estimation of dominant poles

The time constant of first order system τ, and the natural period of oscillation τ_i, and the damping ratio ζ of second order system can be calculated from the knowledge of system dominant poles.

In the development of discrete OBF models, the first step is the selection of stable dominant discrete poles, $\varepsilon_i, \forall\, i = 1, \dots, n$. The basis of OBF model converges as the dynamics of the model approaches the dynamics of the system [14, 20].

The OBF poles $\varepsilon_i\ \forall\, i = 1, \dots, n,$ for different OBF model classes are overlooked below:

- For Laguerre models,
  $\varepsilon_i = a_i, \forall\, i = 1, \dots, n,$ and $|a_1| < 1$.
- For Kautz models,
  $\varepsilon_1 = a_1 + ib_1, |a_1 + ib_1| < 1,$
  $\varepsilon_i = \varepsilon_{i-1,}\ \forall\, i = 2, \dots, n.$
- For GOBF models (we use 2 distant over-damped modes),
  $\varepsilon_i = a_1, |a_1| < 1,$ if $i \le n$ and i is odd,
  $\varepsilon_i = a_2, |a_2| < 1,$ if $i \le n$ and i is even.

Considering the repeating nature of the OBF poles, we represent the poles of all three classes of OBF models in generalised form as

$$\varepsilon_1 = a_1 + ib_1$$
$$\varepsilon_2 = a_1 + ib_2 \tag{7}$$
$$\varepsilon_i = \begin{cases} \varepsilon_1, & \text{if } i \text{ is odd} \\ \varepsilon_2, & \text{otherwise} \end{cases} \quad \forall\, i = 3, \dots, n,$$

where the characteristics of ε_1 and ε_2 and the class of OBF model depends on the system characteristics:

1) For systems with dominant first order characteristics, the OBF poles are real and equal. Hence in the generalised form of OBF poles $b_1 = b_2 = 0$ and $a_1 = a_2$. The resulting model is Laguerre model.
2) For systems with second order like characteristics, there are three conditions:
   a) For dominant over-damped characteristics, the OBF poles are real and unequal. Hence in the generalised form of OBF poles, $a_2 \neq a_1$ and $b_1 = b_2 = 0$. The resulting model is GOBF model.
   b) For dominant critically-damped characteristics, the poles are real and equal. Hence in the generalised form of OBF poles, $a_2 = a_1$ and $b_1 = b_2 = 0$. The resulting model is Laguerre model. It should be noted here that second order critically-damped characteristics and first order characteristics results in Laguerre model with repeating real poles.

For dominant under-damped characteristics, the poles are complex conjugate. Hence in the generalised form of OBF poles, $a_2 = a_1$ and $b_2 = -b_1$. The resulting model is Kautz model.

The OBF models using optimum OBF poles will be parsimonious

and accurate, i.e. the predicted model output $\hat{Y}_{(k)}$ will be closest to the measured output $Y_{(k)}$. The objective in this section is to estimate the optimum OBF poles directly from any arbitrary input-output data.
The objective is re-framed as a constrained optimisation problem as in expressions ($8a$ – $8g$) to minimise a quadratic error function $f(e^2)$ for the variables $a_1,\ a_2,\ b_1$. Note that the variable $b_2 = -b_1$.

$$\min_{a_1,\ a_2,\ b_1} f(e^2), \qquad (8a)$$

subject to (5), (6), and the following constraints,

$$|(a_1, a_2)| < 1, \qquad (8b)$$
$$|b_1| < 1, \qquad (8c)$$
$$|a_1 + ib_1| < 1, \qquad (8d)$$
$$b_2 = -b_1, \qquad (8e)$$
$$a_2 = a_1, \text{if } b_1 \neq 0, \qquad (8f)$$
$$\varepsilon_i = \begin{cases} \varepsilon_1, \text{if } i \text{ is odd} \\ \varepsilon_2, \text{otherwise} \end{cases} \quad \forall\ i = 3, \dots, n, \qquad (8g)$$

The quadratic error function $f(e^2)$ used is the normalised squared prediction error (NSPE) given as

$$f(e^2) = \frac{\sum_k \left(Y_{(k)} - \hat{Y}_{(k)}\right)^2}{\sum_k \left(Y_{(k)} - \bar{Y}_{(k)}\right)^2} \qquad (9)$$

where $\bar{Y}_{(k)}$ is the mean of the measured output.
To solve the optimisation problem($8a$ – $8g$) having conditional constraints, we preferred the meta-heuristic optimisation algorithm, particle swarm optimisation (PSO), for variables,$[a_1,\ b_1,\ a_2]$ with lower and upper bound of search space as $[-1, 0, -1]$and $[1, 1, 1]$, respectively.

Algorithm 1: Estimation of dominant poles and OBF model parameters

Input: Process input data $U_{(k)},\ k = 1, \dots, N$, and output data $Y_{(k)}$, $k = 1, \dots, N$, samples of apparent delay d_a (if available, else set d_a=0).
Output: Dominant poles,$\varepsilon_i, \forall\ i = 1,2$, OBF model parameters $\theta_i, \forall\ i = 1, \dots, n$.

1. Correct the input-output data for apparent delay, $U_{(k)_{k=1,\dots,N-d_a}}$ and $Y_{(k+d_a)_{k=1,\dots,N-d_a}}$;
2. Initialise the number of OBF parameters n (set a large value, i.e. $n \geq 12$), and set the PSO variables, $\mathbf{X} = [a_1, b_1, a_2]$;
3. Set upper bound of variables $\mathbf{X}^{max} = [-1,0,-1]$ and lower bound of variables $\mathbf{X}^{min} = [1,1,1]$;
4. Estimate dominant poles ε_i by minimising the optimisation function (8) using PSO;
5. Develop parsimonious OBF model, for n where NSPE converges;
6. Validate the model.

Calculation of τ, τ_n, and ζ

The time constant τ of first order continuous system can be calculated from discrete real and equal OBF pole $\varepsilon_i = a_1$ and sampling time T as

$$\tau = \left|\frac{T}{\ln(a_1)}\right| \qquad (10)$$

The natural period of oscillation τ_n and damping ratio ζ can be calculated from OBF poles ε_i and sampling time T.
For under-damped system,

$$\tau_n = \left|\frac{T}{\ln(\varepsilon_1)}\right| \qquad (11a)$$
$$\zeta = -\cos(\angle \ln(\varepsilon_1)) \qquad (11b)$$

For critically-damped and over-damped system,

$$\tau_n = \sqrt{\tau_1 \tau_2} \qquad (11c)$$
$$\zeta = \sqrt{1 + \frac{(\tau_1 - \tau_2)^2}{4\tau_1\tau_2}} \qquad (11d)$$

where,

$$\tau_i = \left|\frac{T}{\ln(\varepsilon_i)}\right|, i = 1,2.$$

From the estimated poles, it is not possible to distinguish between first order systems and critically damped second order systems as both have real and equal repeating OBF poles and results in Laguerre model, when using (5). Section III-C discusses a method based on the relation between contributed delay L_c and first order system time constant τ to choose the best among first order and second order models for critically-damped system characteristics.

Estimation of delay time

In process industry, delay time or dead time L (equivalent discrete samples d) is mostly contributed by the lag in transportation of materials. Delay contributed by the transportation lag is clearly visible in the process responses from steady state and is therefore called as apparent delay L_a (equivalent discrete samples d_a). Higher order or over damped process shows sigmoidal or S-curve shaped responses contributing a lag that is counted as contributed delay L_c (equivalent discrete samples d_c). The total delay time L in the approximated FOPDT or SOPDT model of the process is the sum of apparent delay L_a and contributed delay L_c and an illustration is given in Fig.1.

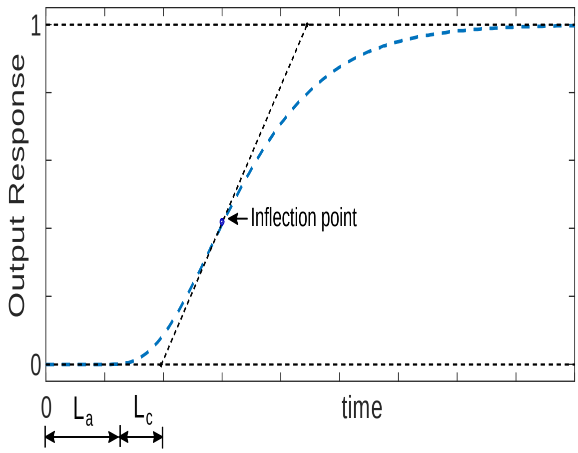

Fig.1. Illustration of apparent delay L_a and contributed delay L_c in a higher order system

For accurate estimation of optimal poles from input-output data, it is better to use apparent delay corrected data. So it is essential to find the apparent delay separately, and correct the input-output data accordingly to improve the estimation of optimal OBF poles and then the estimation of FOPDT and SOPDT model parameters.
The most commonly used maximum slope method uses the intersection of tangent drawn at the inflection point with the time axis to calculate the approximate delay time [3,13].
The maximum slope method uses extensive graphical analysis to calculate the delay.
Here in this work, a rather simple and non-graphical method is used to calculate both apparent delay L_a and contributed delay L_c, separately.

1. *Estimation of apparent delay L_a*: Using Algorithm 1, a near approximate discrete pole(s) are determined. To estimate the remaining parameters of FOPDT or SOPDT model and to improve the estimated pole(s), step response analysis of the developed OBF model is used.

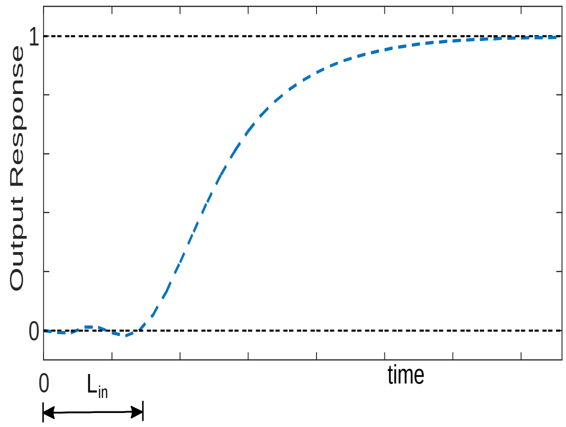

Fig.2. Illustration of inverse response in the step response of OBF model of a process with delay

An OBF model approximates the process delay by non-minimum phase zeros [21, 22]. So the step response of OBF model of process with delay shows inverse response comparable to the delay in the original process.

The duration of inverse response L_{in} (equivalent discrete samples d_{in}) is always observed less than or equal to the apparent delay L_a with a maximum observed error of $0.2\,\tau/T$ for first order systems and $0.2\,\tau_n/T$ for second order systems. The discrete samples of apparent delay d_a is further fine tuned in the space $\{d_{in}, d_{a_{max}}\}$, where $d_{in} = \lceil L_{in}/T \rceil$ and $d_{a_{max}} = d_{in} + \lceil 0.2\,\tau/T \rceil$ for first order systems and $d_{a_{max}} = d_{in} + \lceil 0.2\,\tau_n/T \rceil$ for second order systems. The fine tuning of d_a can be represented as an optimisation problem to

$$\min_{d_a} \text{NSPE}, \ \forall\, d_a = d_{in}, \dots, , d_{a_{max}} \tag{12}$$

subject to (5), (6), and using the OBF poles $\varepsilon_i \ \forall\, i = 1, \dots, n$, as estimated from (8). Apparent delay in unit time can be then calculated as

$$L_a = d_a T \tag{13}$$

Apparent delay estimation can be further fine-tuned with a better estimate of OBF poles and OBF model in an iterative manner till convergence.

The input-output data is corrected for the estimated L_a in estimating OBF poles and hence OBF model parameters.

2. *Estimation of contributed delay L_c*: For any FOPDT system, the normalized step response can be expressed as

$$Y^*_{(t)} = 1 - e^{-\frac{t-L}{\tau}} \tag{14}$$

For pure first order systems the output reaches 63.2% of its final value after $t = \tau$ units of time. τ can be directly calculated from real dominant pole using (10).

For calculating contributed delay L_c, the normalised step response of OBF model with estimated apparent delay L_a is used.

Rearranging the (14),

$1 - Y^*_{(t)} = e^{-\frac{t-L}{\tau}}$,

and taking natural logarithm on both sides of the equation,

$$\ln(1 - Y^*_{(t)}) = -\frac{t-L}{\tau},$$

then rearranging the equation,

$$L = \tau * ln(1 - Y^*_{(t)}) + t \tag{15}$$

Let the time, $t = \hat{t}_{0.632}$ when the normalised step response of OBF model $\hat{Y}^*_{(t=kT)} \approx 0.632$, then

$$L = \tau * \ln(1 - 0.632) + \hat{t} = -0.9997 * \tau + \hat{t}_{0.632}$$

Since $L = L_c + L_a$,

$$L_c = -0.9997 * \tau + \hat{t}_{0.632} - L_a \tag{16}$$

Equation (16) is used for estimating the contributed delay L_c in a first-order system. It is to be noted that instead of $\hat{Y}^*_{(t)} = 0.632$, any value preferably $0.5 < \hat{Y}^*_{(t)} < 0.8$ can be used for the calculation of L_c.

Algorithm 2: Estimation of apparent delay L_a

Input: OBF poles, $\varepsilon_i, \forall\, i = 1, \dots, n$, OBF model parameters, $\theta_i \ \forall\, i = 1, \dots, n$, sampling time T, time constant τ for first-order system (pole) dynamics or period of oscillation τ_n for second-order system (poles) dynamics.

Output: Apparent delay $\mathbf{L_a}$.

1. Excite the OBF model with unit step signal;
2. Find duration of inverse response, L_{in}. From the final steady state value of step response use zero crossing algorithm to find d_{in} samples of inverse response. $L_{in} = d_{in} * T$;
3. Calculate $d_{a_{max}}$;
4. Estimate optimum apparent delay in time unit $L_a = d_a * T$ by solving (12);
5. For fine tuning of estimated parameters go to Step 1 of Algorithm 1.

If $L_c > \tau$, for better approximation our recommendation is to model the system as critically damped second order system.

For under-damped second order system the contributed delay L_c can be easily calculated from the peak time t_p, natural period of oscillation τ_n and damping ratio ζ. The peak time t_p is

$$t_p = \frac{\pi\,\tau_n}{\sqrt{1-\zeta^2}} + L. \tag{17}$$

Rearranging,

$$L = t_p - \frac{\pi\,\tau_n}{\sqrt{1-\zeta^2}}$$

Let the time, $t = \hat{t}_p$ when the normalised step response of OBF model $\hat{Y}^*_{(t=kT)}$ reaches the peak magnitude, then

$$L = \hat{t}_p - \frac{\pi\,\tau_n}{\sqrt{1-\zeta^2}}$$

Since $L = L_c + L_a$,

$$L_c = \hat{t}_p - \frac{\pi\,\tau_n}{\sqrt{1-\zeta^2}} - L_a \tag{18}$$

For critically-damped second order system the normalised step response can be expressed as

$$Y^*_{(t)} = 1 - \left(1 + \frac{(t-L)}{\tau_n}\right) e^{\left\{-\frac{(t-L)}{\tau_n}\right\}} \tag{19}$$

At $t = \tau_n + L$,

$Y^*_{(t)} = 1 - (1+1)e^{-1}$,

$Y^*_{(t)} = 0.2642$.

Let the time, $t = \hat{t}_{0.2642}$ when the normalised step response of OBF model $\hat{Y}^*_{(t=kT)} \approx 0.2642$, then

$L_c = \hat{t}_{0.2642}$-τ_n

Since $L = L_c + L_a$,

$$L_c = \hat{t}_{0.2642} - \tau_n - L_a. \tag{20}$$

For over-damped systems, the normalised step response is given as

$$Y_{(t)}^{*} = 1 - \left(\frac{\tau_1}{\tau_1 - \tau_2}\right) e^{-(t-L)/\tau_1} - \left(\frac{\tau_2}{\tau_2 - \tau_1}\right) e^{-(t-L)/\tau_2}, \qquad (21)$$

where,

$$\tau_1 = \left(\zeta + \sqrt{\zeta^2 - 1}\right) \tau_n$$

and

$$\tau_2 = \left(\zeta - \sqrt{\zeta^2 - 1}\right) \tau_n.$$

For estimating contributed delayL_c, if $\tau_1 \gg \tau_2$ use (16) with $\tau = \tau_2$, else use (20).

Algorithm 3: Estimation of contributed delay L_c

Input: OBF poles, $\varepsilon_i, \forall\, i = 1, \dots, n$, OBF model parameters, $\theta_i\ \forall\, i = 1, \dots, n$, sampling time T, time constant τ for first-order system (pole) dynamics or period of oscillation τ_n for second-order system (poles) dynamics, apparent delayL_a.
Output: Contributed delay L_c.

1. Excite the OBF model with unit step signal;
2. For first order system, use (16);
3. For second order under-damped system, use (18);
4. For second order critically-damped system, use (20);
5. For second order over-damped system, if $\tau_1 \gg \tau_2$ use (16), else use (20).

Estimation of steady state gain

OBF model being linear, the parameters have a linear relation to steady state gain K. From successive iterations of Algorithm 1, 2, and 3 all the parameters except K of the FOPDT or SOPDT model can be estimated or calculated. The steady state gain K can be easily found from the ratio of corresponding parameters of OBF model of the process to the OBF model of the unity gain FOPDT or SOPDT model. We recommend θ_1 among $\theta_i \forall\, i = 1, \dots, n$, , as being the first parameter of OBF model, is non-zero for stable systems. Steady state gain can also be estimated from the final steady state value of unity step response of OBF model.

Algorithm 4: Estimation of steady state gain K

Input: OBF poles, $\varepsilon_i, \forall\, i = 1, \dots, n$, OBF model parameters, $\theta_i\ \forall\, i = 1, \dots, n$, estimated from apparent delay corrected process input-output data, time constant τ for first-order system (pole) dynamics or period of oscillation τ_n and damping ratio ζ for second-order system (poles) dynamics.
Output: Steady state gain K.

1. Develop the first-order or second-order model with $K = 1$ and $L = 0$;
2. Estimate the OBF model parameters of the above first-order or second-order system $\theta_i^{K=1}\ \forall\, i = 1, \dots, n$ using the same dominant poles ε_i of the process;
3. Calculate the steady state gain$K = \theta/\theta_1^{K=1}$.

FOPDT or SOPDT model can be improved using an iterative approach as shown by the flowchart in Fig.3.

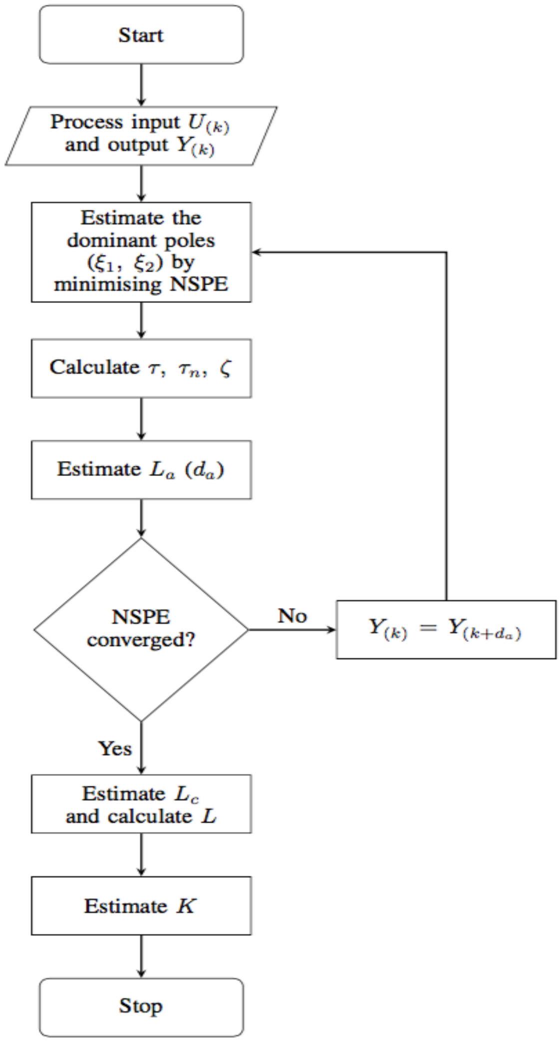

Fig.3. Flowchart for developing a FOPDT or SOPDT model from arbitrary input-output data

4. Results and discussion

Consider the system studied by [13].

$$Y(s) = \frac{13.5\, e^{-12s}}{(17s+1)(1.4s+1)(1.2s+1)(0.6s+1)} U(s) \qquad (22)$$

The system is sampled at $T = 1$ time unit and the discrete poles are0.9429, 0.4895, 0.4346, and 0.1889. Clearly, the pole 0.9429 is the dominant pole and the system have an over-damped dynamics. For identification pseudo random binary signal (PRBS) of band [0 0.01], and level $[-0.5\ 0.5]$ is used. Three thousand data points are used for identification. Initially we start with the OBF order, $n = 12$.
Table I provides the estimation of dominant poles with each iteration. Fig.4 shows the inverse response of OBF model with poles estimated in the first iteration. Finally, it estimate the dominant poles as $\varepsilon_1 = 0.9462$ and $\varepsilon_2 = 0.4261$, corresponding to time constant of $\tau_1 = 18$ and $\tau_2 = 1.17$. As $\tau_1 >> \tau_2$, we can easily approximate the model as FOPDT. The parsimonious model order $n = 6$, and with $\varepsilon_i = 0.9462$ the OBF parameters are $\theta_i = [1.9677\ 0.5354\ -0.4379\ 0.2950\ -0.1663\ 0.0569]$. The contributed delay is estimated as $L_c = 2$, and the steady state gain is estimated as $K = 13.54$.

Table I. Estimation of Dominant Poles

Iteration	ε_1	ε_2	L_{in}	L_a
1	0.8939	0.8035	12	12
2	0.9462	0.4261	0	12

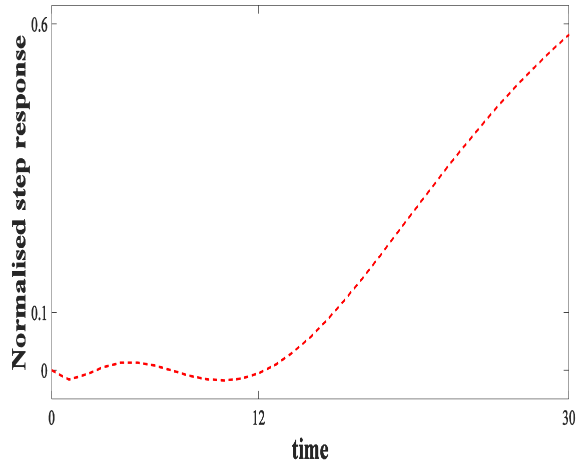

Fig.4. Step response of OBF model after first iteration showing inverse response

The estimated FOPDT model is

$$\hat{G}(s) = \frac{13.54\, e^{-14s}}{18.1s + 1} \qquad (23)$$

The step response of the original process (22) and its approximated FOPDT model (23) is shown in Fig. 5. The bode plot comparing the original process and approximated FOPDT model is shown in Fig. 6. Bode plot is very close in the operating low frequency. The two figures shows the closeness of the approximated FOPDT model to the original model.

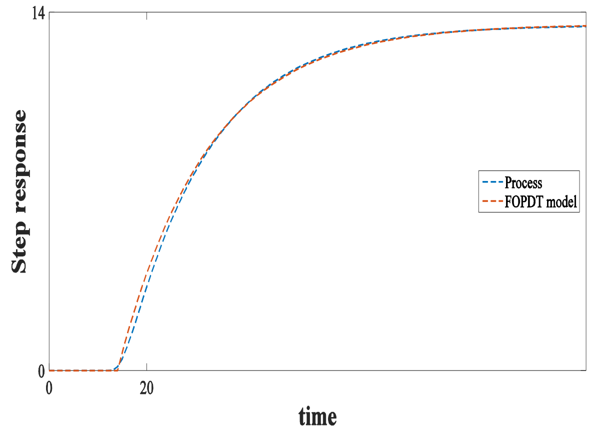

Fig.5. Step response of process and FOPDT model.

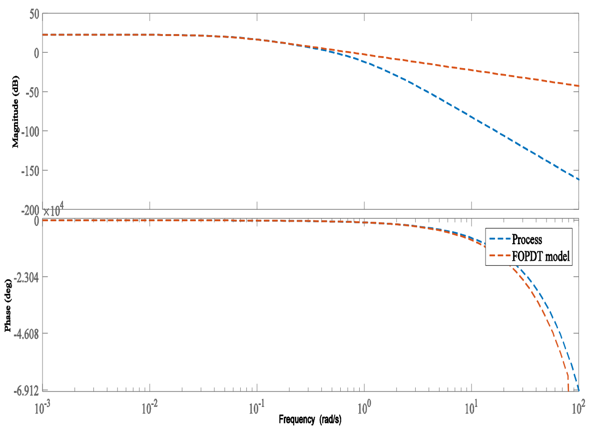

Fig.6. Bode plot of process and FOPDT model

In order to further assess the effect of noise signal in estimation of parameters, the process output of (24) is corrupted with white noise filtered through a model and is given by

$$Y(s) = \frac{13.5\, e^{-12s}}{(17s+1)(1.4s+1)(1.2s+1)(0.6s+1)} U(s) + \frac{3}{6s+1} v(s) \qquad (24)$$

The noise is having a mean of 0.0875 and standard deviation of 0.9840. The signal to noise ratio is 16.7. Fig. 7 shows the input-output data used for estimation.

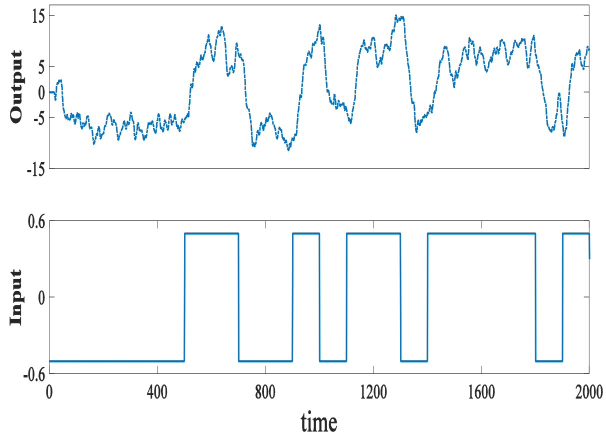

Fig.7. Input-output data for identification

The parameters estimated in each iterations are given in Table II. The parsimonious model order $n = 8$, and with $\varepsilon_i =$ 0.938$ the OBF parameters are $\theta_i = [1.9544\ 0.8058\ -0.5033\ 0.2244\ 0.0318\ -0.0353\ -0.0494\ -0.0086]$. The steady state gain is estimated as $K = 13.43$. The approximated model is FOPDT and is given as

$$\hat{G}(s) = \frac{13.54\, e^{-13s}}{15.1s + 1}$$

Table II. Estimation of Dominant Poles

Iteration	ε_1	ε_2	L_{in}	L_a
1	0.9781	0.9465	11	11
2	0.9527	0.6789	0	13
3	0.9389	0.3992	0	13

The estimated FOPDT model output is validated with the process output in Fig.8. The FOPDT output is able to capture the complex dynamics near to satisfaction.

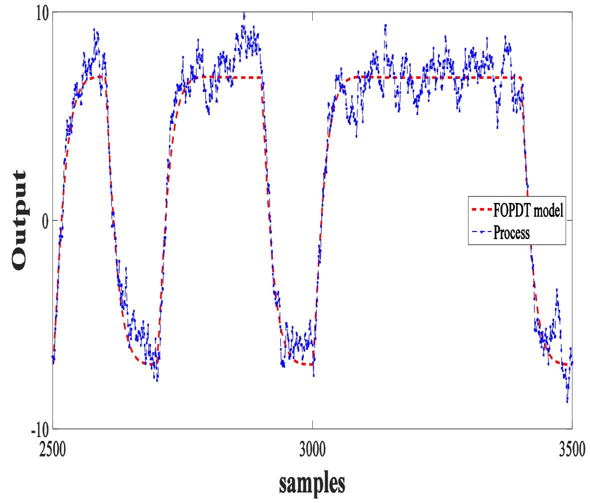

Fig.8. Validation of FOPDT model output to process output.

The closed-loop identification of an under-damped process is done to verify the algorithm simultaneously for closed-loop identification and estimation of under-damped process. The simulation block diagram is shown in Fig. 9. The process is

$$\hat{G}(s) = \frac{0.5\, e^{-9s}}{s^2 + 0.13s + 0.01}$$

The controller is a PI controller tuned using Z-N closed loop method and the sensor $G_s(s) = 1$. The reference $R_{(t)}$ is changed in arbitrary manner (here, pulse signal of magnitude 1 and duration 100 time unit and 60% duty cycle). Additive white noise having a $SNR = 20$ is added. The process input-output data is sampled at 1time unit, and the equivalent discrete dominant poles are $0.9344 \pm 0.0711i$.

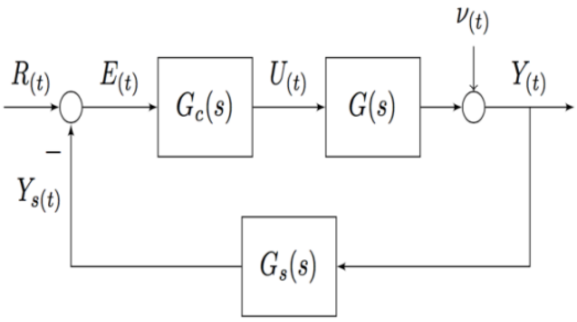

Fig.9. Block diagram of closed loop identification setup.

The NSPE converged after three iterations. Using the algorithms, a SOPDT model for $G(s)$ is estimated and given as

$$\hat{G}(s) = \frac{0.42\, e^{-9s}}{s^2 + 0.119s + 0.013}$$

and the estimated parameters of the SOPDT model from closed-loop identification is closely matching to the original system.

Table III. Estimation of Dominant Poles

Iteration	ε_1	ε_2	L_{in}	L_a
1	0.8917+0.0326i	0.8917-0.0326i	6	6
2	0.9215+0.011i	0.9215-0.011i	2	8
3	0.9382+0.092i	0.9382-0.092i	0	9

5. Conclusion

In this paper, the properties of OBF model and its parameters are used to estimate FOPDT or SOPDT model parameters from arbitrary process input and corresponding output data. The accuracy of the discrete OBF model depends on the accuracy of the discrete dominant poles used in the OBF model filters. The problem of estimation of accurate poles is formulated as an optimisation problem and PSO is used to estimate the poles. The estimation of poles is affected by the delay and a delay corrected output data is found to increase the accuracy. Step test of estimated OBF model is used to separately estimate apparent delay and contributed delay. Then the output data is corrected for apparent delay and poles are estimated with further improved accuracy. Time constant, period of oscillation, and damping ratio is calculated directly from dominant pole and sampling time. From the OBF model, steady state gain is calculated as the parameters are linearly changing to steady state gain.

Compared to step test, the proposed method is more process friendly as it hardly disturbs the normal operation and free from any graphical analysis. Normal operating data is mostly sufficient to estimate the FOPDT or SOPDT parameters using the proposed method. The method is free from any initial guess and with standard optimisation parameters it is found to provide consistent results. Effect of measurement noise, controller, and changes in closed-loop input are also analysed and the estimated models are showing closely matching dynamics to the process.

The closed-loop identification capability will be further explored for the design of adaptive controllers.

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