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Research Paper

Mathematics reasoning and proving of students in generalizing the pattern

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Abstract

The purpose of this study was to identify students' reasoning in generalizing the patterns that proved by generalizing the structural generalizations with involve the mathematical structures and empirical generalizations that emphasize perceptions or evidence derived from the found regularities. The subjects in this research were the 7th semester students of Mathematics Education of University of Madura, Indonesia. The research steps in this research were (1) giving the reasoning tests to the research subjects, (2) analyzing the results of reasoning tests to identify reasoning and mathematical proofs, (3) conducting in-depth interviews as the triangulation method, and (4) summarizing the tendencies of reasoning and proof of student in generalize pattern. Based on the results and discussion can be obtained that in the process of reasoning and verification, students in identifying the same pattern with trial and error, so by using trial and error students find many ways to generalize the existing pattern. However, sometimes through the use of ways of trial and error students find the right pattern. Therefore, the student only identifies a reasonable pattern and does not identify mathematical patterns, then makes reasonable assumptions about finding a relationship but only hypothetical and needs to prove the allegations and only do a few stages of reasoning and not doing the stages of proof, giving no argument and not doing a validation of the evidence.

Keywords: *Mathematics Reasoning, Mathematical Proof, Generalize the Pattern*

1. Introduction

The learning of mathematics is the same as thinking about the patterns, communicating between patterns, or learning based on patterns [1]. These patterns originate from factual arrangements that are then used to formulate conjectures. This alleged test will be new evidence, a revision of the supposition to overcome the possibility of contradictory examples, and how to understand and give an argument [2, 3]. Therefore, it is very important for students to have reasoning ability in studying and understanding mathematical objects.

In teaching patterns to students, the students need to be given the knowledge of how to learn a pattern. Several studies have shown that they often face serious difficulties to gain efficiency in these math activities [4]. Therefore, the process of learning mathematics, not only to help students in finding a pattern, but also to see why a generalization is applicable. In the mathematics, proving is the core of mathematical thinking [5], because there is the research that found that 40% of algebraic assignments, number theory, and geometry units are designed to engage students in reasoning and proving, and that 25% of these tasks involve pattern identification [6].

Some experts have defined the notion of patterns. Patterns are a setting of either verbal, numerical, or recognizable or predictable forms [7]. While the other definition about the patterns in mathematics can be described as regularities of predictable or predictable objects, involving numerical numbers, and logical relationships between objects [8] Therefore, in identifying a pattern, it takes a conjecture that will eventually produce generalizations of the pattern. Generalization as part of reasoning that consists of several contexts but the focus are not only on the context itself but on patterns, procedures, structures, and relationships between the forms [9]. Furthermore, the generalization of the pattern as the activity of defining possible objects into a conjecture of a sequence of numbers [10]. While, the ability of students to make generalizations based on mathematical structures (structural generalizations) and not based on the perceptions or evidence offered by regularity found in some tests (empirical generalizations) [11].

Understanding a pattern in a mathematical activity needs to have reasoning abilities. Mathematical activity involves two kinds of reasoning inter alia logical reasoning where reasoning is generated by conjecture, and demonstrative (deductive) reasoning produced through proven mathematical knowledge [3]. Reasoning in mathematics has four stages inter alia identifying patterns, making conjectures, giving evidence, and giving unsubstantiated (non-proof) arguments. In this study, the aimed of this research is to identify students' reasoning in generalizing patterns proved by structural generalizations involving mathematical structures and empirical generalizations that emphasize perceptions or evidence derived from the found regularities.

2. Research Method

This research is a qualitative research by using descriptive approach. The subject of this research is 7th semester of students majoring in mathematics education. Data were collected by giving tests and interviews. The test given is a reasoning test, then the researcher do categorization based on the answers given by the subject. In this study the researcher also observed at students' reasoning in empirical generalizations and structural generalizations. researcher conducted interviews with research subjects after classification. Interviews were recorded and the results were transcribed and encoded.

After collecting the data, the researcher analyzed the data. The process in analyzing inter alia (1) colecting the data that the process of gathering the data collected from the test and interview, (2) reducting the data that the process of selecting the data, (3) organizing the data that processing to organize the data, and (4) making the conclusion of the research.

3. Results and discussion

The subject is asked to do the identifier by looking for the same pattern between picture 1, picture 2 and picture 3. The subject looks at the pattern by looking at how many triangles, in picture 1 there are 2 triangles, in picture 2 there are 8 triangles and picture 3 there are 18 triangles.

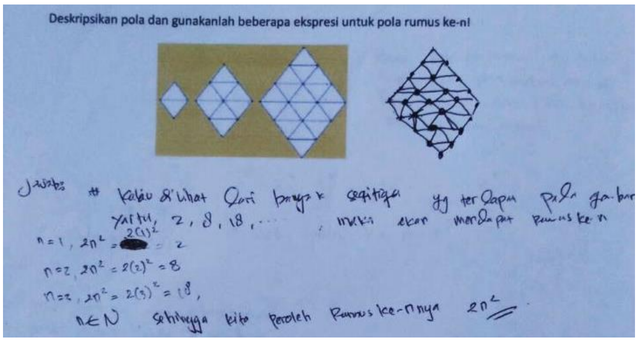

Fig. 1

The subject states that the pattern is already formed. Then, the subject does the allegation by looking at the pattern that is formed. however, the truth of the pattern that is formed still needs proof. Furthermore, in the process of substantiation, the subject can not give mathematically but only the empirical argument in which the subject is unsure of the mathematical claims he has made. In the end the subject generalizes the pattern with the nth formula is $2n^2$. But this subject, finding a different pattern. The first looks at the pattern of the triangle, the next the subject sees from many points and from the many awakening of the rhombus. Patterns formed from many points are as follows picture 1 there are as many as 4 points, picture 2 there are 9 points, and picture 3 there are as many as 16 points.

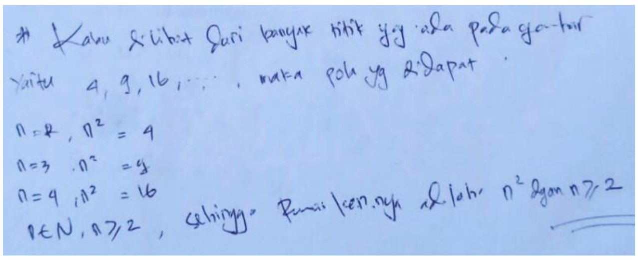

Fig. 2

Using the previous experience, the subject gives the nth formula conclusion of the pattern is n^2, where n is greater than or equal to 2. Next, another viewpoint in identifying the pattern by looking at many rhinos. So the pattern that is formed is 1, 4, and 9 etc, with this subject pattern summed up the formula n is n^2. However, different from the n^2 seen from the point of view of many points earlier.

The second subject in identifying the pattern by looking at the number of lines contained in the image. In figure 1 there are as many as 5, for image 2 there are as many as 16 and figure 3 there are as much as 33. However, the subject does not proceed to the next pattern and claim with existing data 5, 16, and 33, this will not form a pattern. Subsequent subjects to experiment with alleged by counting many lines on the image without calculating the diameter. Subjects found many lines in figure 1 of 4 lines, 2 drawings of 14 lines and figure 3 there were 27 lines, but the subject claimed with such data would not form a pattern and would not find the generalization of the pattern. After doing an experiment that would not find a pattern, the subject saw the image with the viewpoint of many rhombus. Subjects found 1 rhombus in Fig 1, there are 4 rhombus in Fig 2 and there are 9 rhombus in Fig. 3.

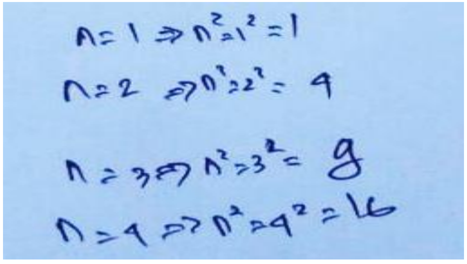

Fig. 3

With data 1, 4, and 9, the subject tries to find the next data and finds 16 rhombus. This is obtained by drawing many diamonds and finding as many as 16. On the basis of that belief, the subject claims that the generalization of this pattern is n^2 with n being the original number. Based on the data obtained that subject 2 identifies the pattern by using trial and error, from several experiments the subject finds a pattern matching the problem. However, the subject only after finding a pattern, the subject does not prove anything either proof or non-proof argument.

The third subject in the identification, he saw the number of rhombus. In Figure 1, there is 1 rhombus, in Figure 2, there are 4 rhombus, and in Figure 3, there are 9 rhombus. Then the subject makes a mathematical pattern of data 1, 4, and 9, the subject gets the pattern of square numbers. Then the subject estimates the next pattern or figure 4 there are as many as 16 rhombus. Subsequently the subject generalizes the pattern by formulating the nth term n is n^2 where n is the original number. After getting the pattern the subject identifies by searching the number of small triangles, the subject sees from a small triangle because in Figure 1 there is no big triangle and there is only a small triangle. For Figure 1, there are 2 triangles, 2 there are 8 triangles and 3 there are 18 triangles, so the pattern is 2, 8, 18. So it can be concluded that the nth term formula is $2n^2$ with n element of the original number. Next the subject looks at the number of parallelograms, but the subject is difficult to find the same pattern. By reason of Figure 1 there is no parallelogram so difficult to find the same pattern. Based on the data obtained that subject 3 identifies the pattern using trial and error, the subject finds a suitable pattern by looking at the number of small triangles and many rhombus. However, the subject only finds a similar pattern, but does not verify either proof or non-proof argument.

Based on the results analysis, there are some reasoning done by students. Analysis of reasoning and proving in generalizing patterns based on mathematical components are (1) identifying a pattern, (2) making conjectures, (3) providing evidence, (4) providing unproven arguments. There are some subject that will be discuss in this research. The first subject implements three components in mathematics in reasoning and proof. The subject does not perform and does not provide mathematical evidence, but finds more ways to generalize the pattern. The second subject performs many conjectures in identification of the same pattern and only finds one generalization pattern. While, the third subject identifies in two ways and tries to do re-conjecture for the same pattern, but it does not find and believe the two ways are correct. The subjects do not prove and do not provide with the non-proof arguments.

The results of this study indicated that the students are still doing the reasoning without doing the verification process correctly. Students in the process of reasoning and proof, only identify identical patterns, and just make conjecture. For the proving process students have not been able to provide validity and argument as evidence. This is in line with the other results that most of mathematicians spend time exploring and conjecturing, they do not to find evidence of good propositions, and certainly not a real proposition [5]. Furthermore, the research result also is in line that in the development of mathematical knowledge should include several stages, from the initial exploration stage of the idea, to the final stage of a reasonable argument, which is usually a proof [12]. But what the students do is just a few stages of reasoning, giving no argument and not doing a validity of proof.

4. Conclusion

Based on the discussion result, there are some conclusions in this research, inter alia.

1. Students in identifying the same pattern with trial and error. By using trial and error students find many ways to generalize the existing pattern. However, the use of this method makes it difficult for students to find the right pattern. Therefore, the student only identifies a reasonable pattern and does not identify the pattern mathematically
2. Students in identifying patterns also make reasonable conjecture to seek relationships between the forms, but they need to proof the conjectures that they have been made.
3. Students only do some stage of reasoning without followed by the stage of proving. They can not provide a reasonable argument and there is no validity of the evidence.

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