

Image compression using Analytical and Learned Dictionaries

Gaddam Padma Priyanka^{1*}, Mosali Geetha Priya², M. Harshali³, M. Venu Gopala Rao⁴

^{1,2}B tech Student, Dept. of ECE, K L E F, India

³SRF-CSIR, Dept. of ECE, K L E F, India

⁴Professor, Dept. of ECE, K L E F, India

*Corresponding author E-mail: gaddampadmapriya07@gmail.com

Abstract

The modern signal and image processing deals with large data such as images and this data deals with complex statistics and high dimensionality. Sparsity is one powerful tool used signal and image processing applications. The mainly used applications are compression and denoising. A dictionary contains information of the signals in the form of coefficients. Recently dictionary learning has emerged for efficient representation of signals. In this paper we study the image compression using both analytical and learned dictionaries. The results show that the effectiveness of learned dictionaries in the application of image compression.

Keywords: Analytical Dictionaries; Compression; Dictionary Learning; Sparse Representation.

1. Introduction

In modern image processing, we deal with large data that has complicated features and need to be analyzed for compression using fast algorithms, storage and transmission. Image compression deals with efficient representation of images that has large size. In the past decade, there has been much focused in the field of sparsity of signals/images that leads to various applications such as compression, detection, recognition and classification etc. Sparse representation modeling of a data deals with signals as a linear combinations of a few atoms from pre-defined dictionaries like the DCT, DWT etc. These algorithms are much faster in computation. In recent years, learned dictionaries have been emerged to deliver better results that yield a dense sparser representation than analytic ones [1, 2, 3]. Motivated by their search work of Skretting [21] and other authors [1, 2], we study and explore the basic image compression structure using various dictionary learning algorithms and compare with analytical dictionaries in this paper.

In this paper, we describe the sparse and redundant representation modeling in section-2. Section-3 discusses the analytical dictionaries and image compression. The details of learned dictionaries and corresponding image compression are provided. Experimental evaluation and results are discussed in section-4 and conclusions are provided in section-5. Finally the references are given at the end.

2. Sparse and redundant representation modeling

Consider a linear system that represent a sparse approximation of an image 'x'.

$$x = Du + \eta \quad (1)$$

Where D is a dictionary, $D \in \mathbb{R}^{n \times m}$, η noise component and $u \in \mathbb{R}^m$ is a coefficient vector. The coefficient vector is said to be sparse when most of the coefficients in the vector are zero. Here x is the test vector which is of size $n \times 1$.

$$\hat{x} = \sum_{k=1}^K u(k)d_k = Du, \quad \eta = x - \hat{x} = x - Du \quad (2)$$

A column in a Dictionary is called an atom. An image 'x' with k_0 linear atoms is generated by multiplying the Dictionary with a sparse vector 'u'. These k_0 atoms are of different weights. The sparsity is measured using the ℓ_0 norm. If the sparse approximation is linear then the signal is can be shown as-

$$x_a = u(1) d_1 + u(6) d_4 + u(8) d_8 + u(9) d_9 \quad (3)$$

$$x = x_a + r \quad (4)$$

Where d_1, d_4, d_7, d_9 are the atoms in the dictionary and $u(1), u(2), u(3), u(4)$ are the non zero elements in the sparse vector. The sparseness factor is L/M , where L is the number of non zero elements in the vector u and M is the total number of elements. Consider set of all possible placements of the sparse vector.

$$\min_u \|u\|_0 \text{ subject to } \|x - Du\|_2 \leq \varepsilon \quad (5)$$

Our aim in this section is to find the sparse vector 'u'. This vector explains us about the image 'x' which is a combination of columns from the Dictionary matrix D . During this process we need to remember that the error should not be larger than ε . This process is known as atomic decomposition [5]. If $\|u\|_0$ is replaced by $\|u\|_1$ then the problem is given below.

$$\min_u \|u\|_1 \text{ subject to } \|x - Du\|_2 \leq \epsilon \tag{6}$$

3. Analytical and Learned Dictionaries

The Dictionary is the sunshine of the Image Processing and the assumption is that the good behaved images have a good sparse representation. There are two directions of designing a Dictionary.

1. The D can be chosen from a known set of transforms. There are very good dictionaries for example JPEG is an outstanding algorithm so we would use the cosine as a dictionary. We could even use the Fourier or Wavelet and there are many that can be used to design a dictionary.
2. The other direction is to learn the dictionary.

3.1. Image compression using analytical dictionaries

Image compression is a process that reduces and represents the data in a compact form. Transform coding is the most common approach that uses a dictionary of atomic signals[28-31]. The encoder and decoder can be built into the dictionary D in case of transform coding. Various compression techniques using analytical dictionaries are available in the literature. The basic structure of image compression is illustrated in the following figure.

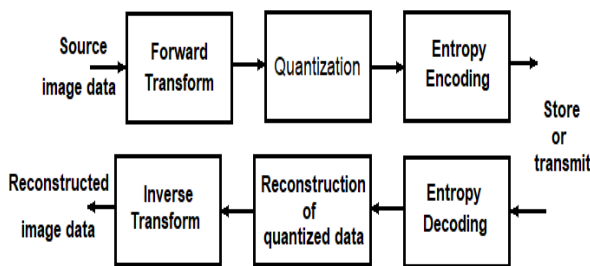


Fig. 1: Illustration of image compression algorithm using analytical dictionaries.

3.2. Image compression using Learned Dictionaries:

As described above, thresholding the transform coefficients introduce errors in the reconstructed image. Dictionary learning is a process of training a dictionary to fulfill the sparsity constraint of a signal/s to adapt for specific task or purpose. This process is useful in many applications like compression, feature extraction, modeling, denoising and classification [10].

The initial step in the dictionary learning process starts with a training set, $\{x_i\}_{i=1}^L, x_i \in \mathbb{R}^N$. Using this training set we need to find both Dictionary $D \in \mathbb{R}^{n \times m}$ and coefficient set $\{u_i\}_{i=1}^L, u_i \in \mathbb{R}^m$. X is a matrix and each column in this matrix is x_i . W is a coefficient matrix and each column in this matrix is u_i . The problem in designing the above two are formulated as shown below

$$\{D_{opt}, U_{opt}\} = \arg \min_{D, U} \sum_{k=1}^K \|u_k\|_p + \gamma \|X - DU\|_F^2$$

and $\|d_i\| = 1$

Since it is a NP hard problem, it has split into two phases that are iteratively solved:

Phase1: Sparse coding

Initialize the dictionary matrix $D^{(0)} \in \mathbb{R}^{n \times K}$ with L_2 normalized columns using orthogonal transforms such as DCT, DWT and lapped transforms (LOT) etc. Then keep the dictionary D fixed and find U according to some sparseness constraint rule.

In practice, to find the coefficients ‘w’, vector selection algorithms such as Matching Pursuit algorithm [11, 12], Basic Matching Pursuit (BMP) [13], Orthogonal Matching Pursuit (OMP) [14, 15], or Order Recursive Matching Pursuit (ORMP) [16] are used. This stage is computationally intensive, since it needs to update each dictionary, particularly for large training sets.

Phase2: Dictionary update

Keeping U fixed and find D. In this phase the coefficients are fixed and update the dictionary. Various dictionary learning methods are illustrated briefly as below.

MOD (Method of Optimized Directions)

MOD uses a finite learning set x of size N x L, with sparseness either by L_0 -norm or L_1 -norm. Dictionary normalization is used in this method. The corresponding solution is $D = (XW^T)(WW^T)^{-1}$. Some variants of MOD such as large-MOD are developed to improve the learning speed for large training sets. The ILS-DLA is capable of reconstructing the generating dictionary vectors from a sparsely generated data set.

An iterative least squares dictionary learning algorithm (ILS-DLA) is developed alternate to large MOD which may be easier to implement.

KSVD

Since the singular value decomposition (SVD) make sure that the dictionary atoms has unit norm, dictionary normalization is not needed in KSVD. After a group (a whole set) of training vector has been processed, the dictionary is updated. And this dictionary can be improved when the training set is used iteratively. The K-SVD dictionary update step do not use L_0 -norm and modifies the L_1 -norm of the coefficients, therefore the actual coefficients will be neither L_0 nor L_1 sparse. The K-SVD will keep non-zero positions in W constant and D is calculated using SVD decompositions [2].

RLS-DLA

In recursive least squares dictionary learning algorithm, a single training vector is used in each iteration. The coefficients w_i are found using the current dictionary D_{i-1} . The improvement in RLS-DLA, is that the matrix inversion is used to update the Dictionary but in MOD the dictionary is updated by calculating the least squares solution in each step. The advantage of RLS-DLA is that it improves the sparse representation capabilities and removes blocking artifacts where the overlapping atoms are present [20]. The basic process of image compression using dictionary learning is illustrated below

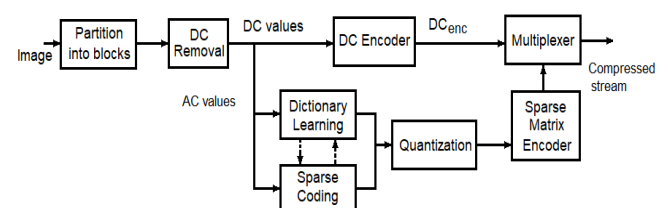


Fig. 2: Illustration of image compression algorithm using learned dictionaries

4. Result

We conducted two experiments for image compression using analytical dictionaries and learned dictionaries. The results are discussed as follows:

Experiment-1

In the first experiment, the image compression is performed using the analytical dictionaries such as DCT, LOT, M97(Wavelet) and SPHIT, for three different images flinstones, man, lighthouse images. The results are compared with JPEG and JP2K. The visual results are shown in Fig3, Fig4 and Fig5. Fig6 illustrates the plots of bit rate versus PSNR for three test images. From this results, we observe that DCT and JPEG performance is poor than all other

methods. Further JPEG 2000 is better than SPHIT and Wavelet methods.

Experiment-2

In the second experiment, we conducted image compression using three algorithms namely K-SVD, RLS-DLA, ILS-DLA based on the dictionary learning. We have chosen eight natural images shown in Fig 7 for training the dictionary. ORMP is used in this experiment. A M97dictionary trained on patches from natural images is shown in Fig8. Fig9 display the plot of SNR against number of iterations. From this experiment, we observe that ILS-DLA performs better than RLS-DLA and K-SVD because ILS-DLA can identify all atoms even for a small angle where K-SVD and RLS-DLA have atoms far away from the true atom.

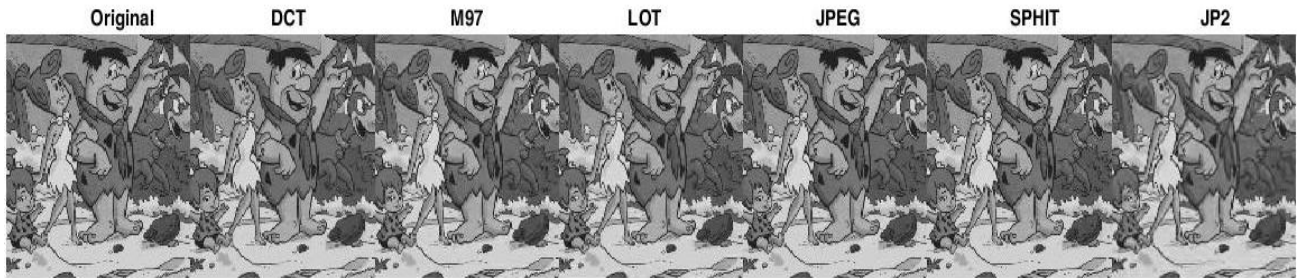


Fig. 3: To the left we see the original image of flinstones and to the right the image is compressed using different algorithms such as DCT, M97, LOT, JPEG, SPIHT and JP2K.



Fig. 4: To the left we see the original image of man and to the right the image is compressed using different algorithms such as DCT, M97, LOT, JPEG, SPIHT and JP2K.

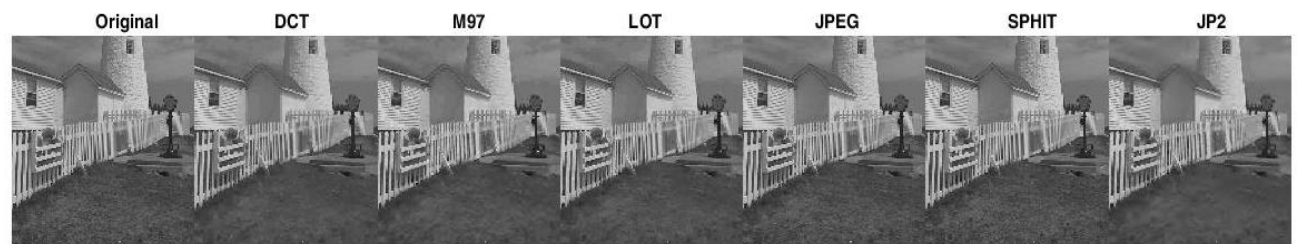


Fig. 5: To the left we see the original image of lighthouse and to the right the image is compressed using different algorithms such as DCT, M97, LOT, JPEG, SPIHT and JP2K.

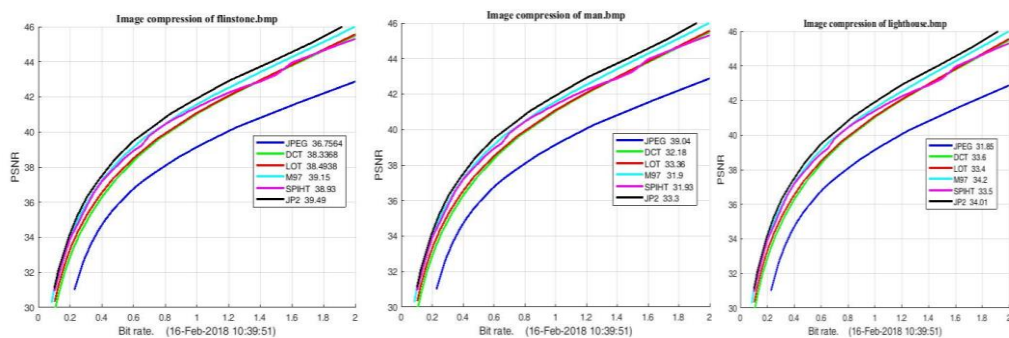


Fig. 6: Bit rate versus PSNR for three test images flinstone, man and lighthouse with the methods JPEG, DCT, LOT, M97, SPIHT and JP2K.

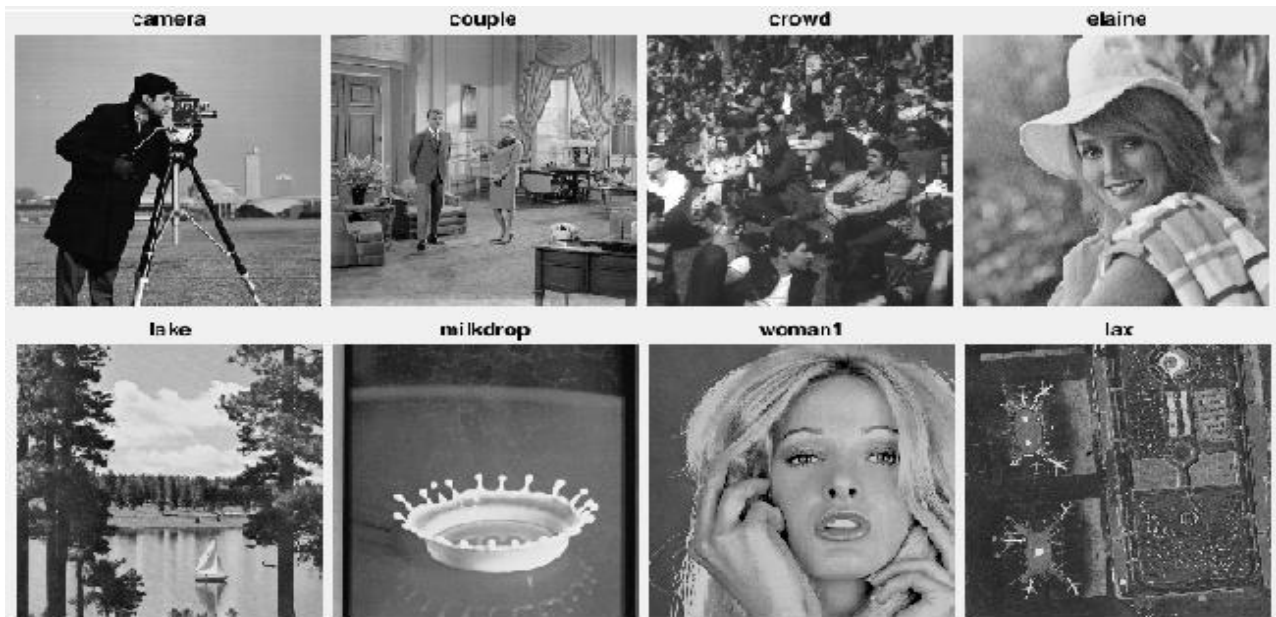


Fig. 7: The above figure shows the training set of 8 images

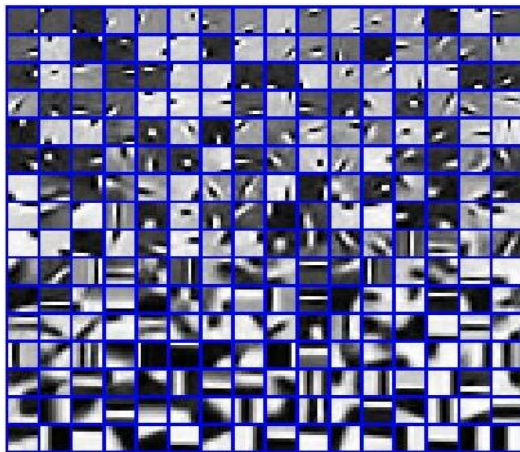


Fig. 8: AM97dictionarytrained on patches from natural images.

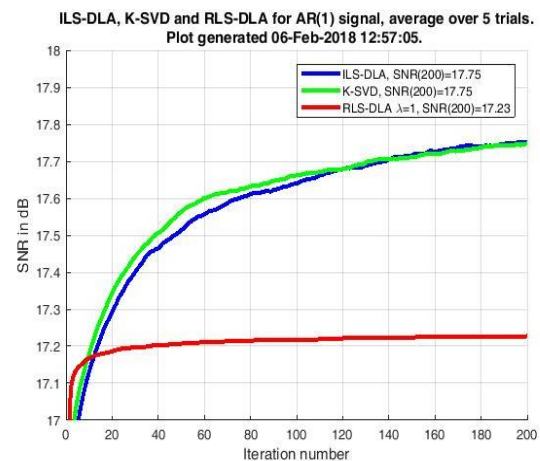


Fig. 9: Iteration number vs. SNR value

5. Conclusion

We conducted two experiments and presented image compression using both analytical and learned dictionaries. In the first experiment we observed that the DWT based techniques produce better performance than DCT / JPEG based techniques. Further JPEG2000 outperforms of all other techniques. In the second experiment, the dictionary is trained using eight natural images initially. Then image compression is performed using three algorithms KSVD, RLS-DLA and ILS-DLA. On comparing among the learnt dictionary algorithms, ILS- DLA and K- SVD produce high SNR value compared to RLS-DLA for 200 iterations.

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